Let $X_1, X_2, X_3$ be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} 
\frac{1}{3}, & \text{for } x = 0, \\
\frac{2}{3}, & \text{for } x = 1, \\
0, & \text{otherwise}.
\end{cases}$$

Determine the moment generating function $M(t)$ of $Y = X_1X_2X_3$.

A. $\frac{19}{27} + \frac{8}{27}e^t$  
B. $1 + 2e^t$  
C. $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^3$  
D. $\frac{1}{27} + \frac{8}{27}e^{3t}$  
E. $\frac{1}{3} + \frac{2}{3}e^{3t}$

Solution.

Let $f_Y$ be the probability function for $Y = X_1X_2X_3$. Note that $Y = 1$ if and only if $X_1 = X_2 = X_3 = 1$.

Otherwise, $Y = 0$. We also know that

$$\Pr(Y = 1) = \Pr\left(\{X_1 = 1\} \cap \{X_2 = 1\} \cap \{X_3 = 1\}\right) = \Pr(X_1 = 1)\Pr(X_2 = 1)\Pr(X_3 = 1) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}.$$

We conclude that

$$f_Y(y) = \begin{cases} 
1 - \frac{8}{27} = \frac{19}{27}, & \text{for } y = 0, \\
\frac{8}{27}, & \text{for } y = 1,
\end{cases}$$

and is zero otherwise. $Y$ is a Bernoulli Trial random variable and
\[ M(t) = M_Y(t) = E(e^{\lambda t}) = \frac{19}{27} + \frac{8}{27} e'. \]

Answer A.

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