An insurance policy is written to cover a loss, \( X \), where \( X \) has a uniform distribution on \([0,1000]\). At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

A. 250  B. 375  C. 500  D. 625  E. 750

Solution.
Expected payment with no deductible is the mean of a uniform distribution on \([0, 1000]\), i.e., 500. With deductible \( d \), the amount paid on a loss of amount \( X \) is

\[
W = \begin{cases} 
0, & X \leq d, \\
X - d, & X > d.
\end{cases}
\]

The expected payment is

\[
E(W) = \int_0^{1000} (x - d) \cdot 0.001 \, dx = 0.0005 (1000 - d)^2.
\]

In order for this to be 25% of the expected payment with no deductible, note that such expected payment is 500 (mean of the uniform distribution on \([0,1000]\)) and therefore

\[
0.0005 (1000 - d)^2 = 0.25 \cdot 500.
\]

This implies

\[
d = 1000 - \sqrt{\frac{0.25 \cdot 500}{0.0005}} = 500.
\]

Answer C.