A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let $X$ denote the number of luxury cars sold in a given day, and let $Y$ denote the number of extended warranties sold.

\[
\begin{align*}
\Pr(X = 0, Y = 0) = & \frac{1}{6}, & \Pr(X = 1, Y = 0) = & \frac{1}{12}, & \Pr(X = 2, Y = 0) = & \frac{1}{12}, \\
\Pr(X = 1, Y = 1) = & \frac{1}{6}, & \Pr(X = 2, Y = 1) = & \frac{1}{3}, & \Pr(X = 2, Y = 2) = & \frac{1}{6}.
\end{align*}
\]

What is the variance of $X$?

A. 0.47  
B. 0.58  
C. 0.83  
D. 1.42  
E. 2.58

**Solution.**

The marginal distribution of $X$ is found by summing probabilities over all possible values of $Y$.

\[
\begin{align*}
f_X(0) = & \Pr(X = 0) = \sum_{y=0}^{0} \Pr(X = 0, Y = y) = \frac{1}{6}, \\
f_X(1) = & \Pr(X = 1) = \sum_{y=0}^{1} \Pr(X = 1, Y = y) = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}, \\
f_X(2) = & \Pr(X = 2) = \sum_{y=0}^{2} \Pr(X = 2, Y = y) = \frac{1}{12} + \frac{1}{3} + \frac{1}{6} = \frac{7}{12}.
\end{align*}
\]

Therefore, the first moment of $X$ equals

\[
E(X) = \sum_{y=0}^{2} x \cdot \Pr(X = x) = 0 \cdot \frac{1}{6} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{7}{12} = \frac{17}{12},
\]

and the second moment equals

\[
E(X^2) = \sum_{y=0}^{2} x^2 \cdot \Pr(X = x) = 0^2 \cdot \frac{1}{6} + 1^2 \cdot \frac{1}{4} + 2^2 \cdot \frac{7}{12} = \frac{31}{12}.
\]
so that the variance equals

$$\text{Var}(X) = E(X^2) - (E(X))^2 = \frac{31}{12} - \left(\frac{17}{12}\right)^2 = 0.576.$$ 

Answer B.

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