Exercise for December 29, 2007

May 2001 Course 1 Examination, Problem No. 35, also Study Note P-09-07, Problem No. 63

The warranty on a machine specifies that it will be replaced at failure or age 4, whichever occurs first. The machine’s age at failure, $X$, has density function

$$f_X(x) = \begin{cases} \frac{1}{5}, & \text{for } 0 < x < 5, \\ 0, & \text{otherwise}. \end{cases}$$

Let $Y$ be the age of the machine at the time of replacement. Determine the variance of $Y$.

A. 1.3  
B. 1.4  
C. 1.7  
D. 2.1  
E. 7.5

Solution.

The random variable $Y$ is effectively defined as the following function of:

$$Y = h(X) = \begin{cases} X, & \text{if } 0 \leq X \leq 4, \\ 4, & \text{if } 4 < X \leq 5. \end{cases}$$

Therefore,

$$E(Y) = E(h(X)) = \int_0^5 h(x) f_X(x) \, dx = \int_0^4 x \cdot \frac{1}{5} \, dx + \int_4^5 4 \cdot \frac{1}{5} \, dx = \frac{1}{10} x^2 \bigg|_{x=0}^{x=4} + \frac{4}{5} = \frac{16}{10} + \frac{4}{5} = \frac{12}{5},$$

$$E(Y^2) = E(h(X))^2 = \int_0^4 x^2 \cdot \frac{1}{5} \, dx + \int_4^5 16 \cdot \frac{1}{5} \, dx = \frac{1}{15} x^3 \bigg|_{x=0}^{x=4} + \frac{16}{5} = \frac{64}{15} + \frac{16}{5} = \frac{112}{15},$$

and

$$\text{Var}(Y) = E(Y^2) - (E(Y))^2 = \frac{112}{15} - \left(\frac{12}{5}\right)^2 = 1.71.$$ 

Answer C.

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