Exercise for February 16, 2008

November 2000 Course 1 Examination, Problem No. 28, also Study Note P-09-05, Problem No. 12

A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.
(ii) 22% have low blood pressure.
(iii) 15% have an irregular heartbeat.
(iv) Of those with an irregular heartbeat, one-third have high blood pressure.
(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

What portion of the patients selected have a regular heartbeat and low blood pressure?

A. 2% B. 5% C. 8% D. 9% E. 20%

Solution.

We are directly given that \( \Pr(\text{Irregular heartbeat}) = 0.15, \) \( \Pr(\text{High blood pressure}) = 0.14, \) and \( \Pr(\text{Low blood pressure}) = 0.22. \) We calculate other relevant probabilities and place all probabilities in a table (below):

\[
\begin{align*}
\Pr(\text{Normal blood pressure}) & = 1 - 0.14 - 0.22 = 0.64, \\
\Pr(\{\text{High blood pressure}\} \cap \{\text{Irregular heartbeat}\}) & = \\
& = \Pr(\text{High blood pressure}|\text{Irregular heartbeat}) \cdot \Pr(\text{Irregular heartbeat}) = \\
& = \frac{1}{3} \cdot 0.15 = 0.05, \\
\Pr(\{\text{Irregular heartbeat}\} \cap \{\text{Normal blood pressure}\}) & = \\
& = \Pr(\text{Irregular heartbeat}|\text{Normal blood pressure}) \cdot \Pr(\text{Normal blood pressure}) = \\
& = \frac{1}{8} \cdot 0.64 = 0.08, \\
\Pr(\text{Regular heartbeat}) & = 1 - \Pr(\text{Irregular heartbeat}) = 1 - 0.15 = 0.85,
\end{align*}
\]

Therefore, the correct answer is C. 8%.
Pr(\{\text{High blood pressure}\} \cap \{\text{Regular heartbeat}\}) = 0.14 - 0.05 = 0.09,
Pr(\{\text{Normal blood pressure}\} \cap \{\text{Regular heartbeat}\}) = 0.64 - 0.08 = 0.56,
Pr(\{\text{Low blood pressure}\} \cap \{\text{Irregular heartbeat}\}) = 0.15 - 0.05 - 0.08 = 0.02,
Pr(\{\text{Low blood pressure}\} \cap \{\text{Regular heartbeat}\}) = 0.22 - 0.02 = 0.20,

<table>
<thead>
<tr>
<th></th>
<th>High blood pressure</th>
<th>Low blood pressure</th>
<th>Normal blood pressure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular heartbeat</td>
<td>0.09</td>
<td>0.20</td>
<td>0.56</td>
<td>0.85</td>
</tr>
<tr>
<td>Irregular heartbeat</td>
<td>0.05</td>
<td>0.02</td>
<td>0.08</td>
<td>0.15</td>
</tr>
<tr>
<td>Total</td>
<td>0.14</td>
<td>0.22</td>
<td>0.64</td>
<td>1.00</td>
</tr>
</tbody>
</table>

We see from this table or from the last calculation that 20% of patients have a regular heartbeat and low blood pressure.
Answer E.

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