November 2001 Course 1 Examination, Problem No. 38, also Study Note P-09-05, Problem No. 103
In a small metropolitan area, annual losses due to storm, fire, and theft are assumed to be independent, exponentially distributed random variables with respective means 1.0, 1.5, and 2.4. Determine the probability that the maximum of these losses exceeds 3.

A. 0.002  B. 0.050  C. 0.159  D. 0.287  E. 0.414

Solution.
Let $X_1$, $X_2$, and $X_3$ denote annual loss due to storm, fire, and theft, respectively. Let

$$Y = \max\{X_1, X_2, X_3\}.$$ 

Based on the information about the distributions of $X_1$, $X_2$, and $X_3$ we have:

$$\Pr(Y > 3) = 1 - \Pr(Y \leq 3) = 1 - \Pr(\max\{X_1, X_2, X_3\} \leq 3) =$$

$$= 1 - \Pr(\{X_1 \leq 3\} \cap \{X_2 \leq 3\} \cap \{X_3 \leq 3\}) =$$

$$= 1 - \Pr(X_1 \leq 3) \cdot \Pr(X_2 \leq 3) \cdot \Pr(X_3 \leq 3) =$$

$$= 1 - \left(1 - e^{-\frac{3}{1}}\right) \cdot \left(1 - e^{-\frac{3}{1.5}}\right) \cdot \left(1 - e^{-\frac{3}{2.4}}\right) \approx 0.414.$$ 

Answer E.

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