P Sample Exam Questions, Problem No. 149, also Dr. Ostaszewski’s online exercise posted March 13, 2010
A motorist makes three driving errors, each independently resulting in an accident with probability 0.25. Each accident results in a loss that is exponentially distributed with mean 0.80. Losses are mutually independent and independent of the number of accidents. The motorist’s insurer reimburses 70% of each loss due to an accident. Calculate the variance of the total unreimbursed loss the motorist experiences due to accidents resulting from these driving errors.

A. 0.0432 B. 0.0756 C. 0.1782 D. 0.2520 E. 0.4116

Solution.

Every time this motorist makes an error, there could be an accident (call it a “success”) or no accident (“failure”). Thus each error is a Bernoulli Trial, with probability of success of $p = 0.25$. There is total of $n = 3$ of those Bernoulli Trials, so that the total number of accidents, call it $N$, is binomial with $n = 3$ and $p = 0.25$. Note that $E(N) = n \cdot p = 0.75$ and $Var(N) = n \cdot p \cdot (1 - p) = 0.5625$. Let $X$ be the loss resulting from a given accident.

Then $X$ is exponential with mean 0.80, so that $E(X) = 0.80$ and

$$Var(X) = \left( E(X) \right)^2 = 0.80^2 = 0.64.$$ 

The unreimbursed portion of the loss, given that there is an accident, is $Y = 0.3X$. Therefore,

$$E(Y) = E(0.3X) = 0.3E(X) = 0.3 \cdot 0.8 = 0.24,$$

and

$$Var(Y) = Var(0.3X) = 0.3^2 Var(X) = 0.3^2 \cdot 0.64 = 0.0576.$$ 

But this is only the variance for a single accident. For a total of $N$ accidents, if we write $Y_i$ for the unreimbursed loss due to the $i$-the accident, with $Y_0 = 0$ by definition, and $S$ for the total losses due to $N$ accidents, then we have $S = Y_0 + Y_1 + \ldots + Y_N$ and
\[
\text{Var}(S) = \text{Var}(E(S|N)) + E(\text{Var}(S|N)) = \\
= \text{Var}(E(Y_0 + Y_1 + \ldots + Y_N)) + E(\text{Var}(Y_0 + Y_1 + \ldots + Y_N)) = \\
= \text{Var}(NE(Y_i)) + E(N\text{Var}(Y_i)) = \text{Var}(0.25N) + E(0.0576N) = \\
= 0.24^2 \cdot 0.5625 + 0.0576 \cdot 0.75 = 0.0756.
\]
Answer B.

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