Two life insurance policies, each with a death benefit of 10,000 and a one-time premium of 500, are sold to a couple, one for each person. The policies will expire at the end of the tenth year. The probability that only the wife will survive at least ten years is 0.025, the probability that only the husband will survive at least ten years is 0.01, and the probability that both of them will survive at least ten years is 0.96. What is the expected excess of premiums over claims, given that the husband survives at least ten years?

A. 350  B. 385  C. 397  D. 870  E. 897

Solution.

Start by labeling the events. Let $W$ be the event that wife survives at least 10 years, and $H$ be the event that husband survives at least 10 years. Let us also denote by $B$ the amount of benefit paid, and $\Pi$ be the profit (i.e., excess of premiums over claims) from selling policies. We have:

$$\Pr(H) = \Pr(H \cap W) + \Pr(H \cap W^c) = 0.96 + 0.01 = 0.97.$$ 

Furthermore:

$$\Pr(W^c | H) = \frac{\Pr(H \cap W^c)}{\Pr(H)} = \frac{0.01}{0.97} = 0.0103.$$ 

It follows that:

$$E(\Pi | H) = E((1000 - B) | H) = 1000 - E(B | H) =$$

$$= 1000 - \left(0 \cdot \Pr(W | H) + 10000 \cdot \Pr(W^c | H)\right) = 1000 - 10000 \cdot 0.0103 = 897.$$ 

Answer E.