May 2003 Course 1 Examination, Problem No. 29, also Study Note P-09-08, Problem No. 72

An investment account earns an annual interest rate \( R \) that follows a uniform distribution on the interval \([0.04, 0.08]\). The value of a 10,000 initial investment in this account after one year is given by \( V = 10,000e^R \). Determine the cumulative distribution function, \( F(v) \), of \( V \), for values of \( v \) that satisfy \( 0 < F(v) < 1 \).

\[
A. \frac{10,000e^{\frac{v}{10,000}} - 10,408}{425} \quad B. 25e^{\frac{v}{10,000}} - 0.04 \quad C. \frac{v - 10,408}{10,833 - 10,408}
\]

\[
D. \frac{25}{v} \quad E. 25\left(\ln\left(\frac{v}{10,000}\right) - 0.04\right)
\]

Solution.

We are given that \( R \) is uniform on the interval \([0.04, 0.08]\) and \( V = 10,000e^R \). Therefore, the distribution function of \( V \) is given by

\[
F(v) = \Pr(V \leq v) = \Pr(10,000e^R \leq v) = \Pr\left(e^R \leq \frac{v}{10000}\right) = \Pr\left(R \leq \ln\left(\frac{v}{10,000}\right)\right) = \int_{0.04}^{\ln(\frac{v}{10,000})} \frac{1}{0.08 - 0.04} dr = \frac{\ln\left(\frac{v}{10,000}\right) - 0.04}{0.08 - 0.04} = 25\left(\ln\left(\frac{v}{10,000}\right) - 0.04\right).
\]

Answer E.