November 2001 Course 1 Examination, Problem No. 16, also P Sample Exam Questions, Problem No. 123, and Dr. Ostaszewski’s online exercise posted May 15, 2010

You are given the following information about $N$, the annual number of claims for a randomly selected insured: $\Pr(N = 0) = \frac{1}{2}$, $\Pr(N = 1) = \frac{1}{3}$, $\Pr(N > 1) = \frac{1}{6}$. Let $S$ denote the total annual claim amount for an insured. When $N = 1$, $S$ is exponentially distributed with mean 5. When $N > 1$, $S$ is exponentially distributed with mean 8. Determine $\Pr(4 < S < 8)$.

A. 0.04  B. 0.08  C. 0.12  D. 0.24  E. 0.25

Solution.
Recall that if $X$ has an exponential distribution with mean $\mu$ then

$$\Pr(a < X < b) = F_X(b) - F_X(a) = s_X(a) - s_X(b) = e^{-\frac{a}{\mu}} - e^{-\frac{b}{\mu}}.$$ 

We use the law of total probability and the information on the distribution of $S$ when $N = 1$ and $N > 1$ (when $N = 0$, $S = 0$ and this case can be disregarded), to obtain

$$\Pr(4 < S < 8) = \Pr(4 < S < 8|N = 1) \cdot \Pr(N = 1) + \Pr(4 < S < 8|N > 1) \cdot \Pr(N > 1) =$$

$$= \left( e^{-\frac{4}{5}} - e^{-\frac{8}{5}} \right) \cdot \frac{1}{3} + \left( e^{-\frac{4}{8}} - e^{-\frac{8}{8}} \right) \cdot \frac{1}{6} \approx 0.12.$$

Answer C.