Let $X$ and $Y$ be continuous random variables with joint density function
$$f(x,y) = \begin{cases} 
15y, & \text{for } x^2 \leq y \leq x, \\
0, & \text{otherwise}.
\end{cases}$$

Let $g$ be the marginal density function of $Y$. Which of the following represents $g$?

A. $g(y) = \begin{cases} 
15y, & \text{for } 0 < y < 1, \\
0, & \text{otherwise}.
\end{cases}$

B. $g(y) = \begin{cases} 
\frac{15y^2}{2}, & \text{for } x^2 < y < x, \\
0, & \text{otherwise}.
\end{cases}$

C. $g(y) = \begin{cases} 
\frac{15y^2}{2}, & \text{for } 0 < y < 1, \\
0, & \text{otherwise}.
\end{cases}$

D. $g(y) = \begin{cases} 
15y^2\left(1 - \frac{y}{2}\right), & \text{for } x^2 < y < x, \\
0, & \text{otherwise}.
\end{cases}$

E. $g(y) = \begin{cases} 
15y^2\left(1 - \frac{y}{2}\right), & \text{for } 0 < y < 1, \\
0, & \text{otherwise}.
\end{cases}$

Solution.
The shaded portion of the graph below is the region over which $f(x,y)$ is positive:
Therefore the marginal PDF of \( Y \) is

\[
g(y) = \int_{y}^{\sqrt{y}} 15y \, dx = 15y(\sqrt{y} - y) = 15y^{\frac{3}{2}}(1 - y^{\frac{1}{2}}),
\]

for \( 0 < y < 1 \), or, if written in the form desired:

\[
g(y) = \begin{cases} 15y^{\frac{3}{2}}(1 - y^{\frac{1}{2}}), & 0 < y < 1, \\ 0, & \text{otherwise}. \end{cases}
\]

Answer E.

© Copyright 2004-2009 by Krzysztof Ostaszewski. 
All rights reserved. Reproduction in whole or in part without express written permission from the author is strictly prohibited. 
Exercises from the past actuarial examinations are copyrighted by the Society of Actuaries and/or Casualty Actuarial Society and are used here with permission.