P Sample Exam Questions, Problem No. 153 and Dr. Ostaszewski’s online exercise posted May 21, 2011
Let $X$ represent the number of customers arriving during the morning hours and let $Y$ represent the number of customers arriving during the afternoon hours at a diner. You are given:

i) $X$ and $Y$ are Poisson distributed.

ii) The first moment of $X$ is less than the first moment of $Y$ by 8.

iii) The second moment of $X$ is 60% of the second moment of $Y$.

Calculate the variance of $Y$.

A. 4  B. 12  C. 16  D. 27  E. 35

Solution.
Recall that if a random variable $N$ is Poisson, then $E(N) = \text{Var}(N)$. Let us write

$E(X) = \lambda_X$ and $E(Y) = \lambda_Y$. We are given that

$\lambda_X = \lambda_Y - 8$

and

$E(X^2) = \text{Var}(X) + (E(X))^2 = \lambda_X + \lambda_X^2 = \lambda_Y - 8 + (\lambda_Y - 8)^2 = 0.6E(Y^2) = 0.6\left(\text{Var}(Y) + (E(Y))^2\right) = 0.6(\lambda_Y + \lambda_Y^2)$.

By substituting $\lambda_X = \lambda_Y - 8$ into the equation given by the relationship of the second moments, we obtain

$\lambda_Y - 8 + (\lambda_Y - 8)^2 = 0.6(\lambda_Y + \lambda_Y^2)$.

This is a quadratic equation, which simplifies to

$0.4\lambda_Y^2 - 15.6\lambda_Y + 56 = 0$.

The two solutions of this quadratic equation are $\lambda_Y = 35$ and $\lambda_Y = 4$. They both appear feasible, but if we recall that $\lambda_X = \lambda_Y - 8$, we realize that $\lambda_Y = 4$ would result in $\lambda_X = -4$, an impossibility. We conclude that
\[
\text{Var}(Y) = \lambda_y = 35.
\]

Answer E.

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