Exercise for June 14, 2008

May 2000 Course 1 Examination, Problem No. 3, also Study Note P-09-05, Problem No. 37

The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

A. 6,321  
B. 7,358  
C. 7,869  
D. 10,256  
E. 12,642

Solution

The exponential distribution with a mean of 2 has density \( f_X(x) = \frac{1}{2} e^{-\frac{x}{2}} \), for \( x > 0 \), and

the cumulative distribution function \( F_X(x) = 1 - e^{-\frac{x}{2}} \), for \( x > 0 \). The probability that a printer will fail in the first year is

\[
\Pr(X \leq 1) = F_X(1) = 1 - e^{-\frac{1}{2}} = 0.39347,
\]

so that the expected number of failures in the first year out of 100 printers is 39.347. The probability that a printer will fail in the second year is

\[
\Pr(1 < X \leq 2) = F_X(2) - F_X(1) = e^{-\frac{1}{2}} - e^{-\frac{2}{2}} = 0.23865,
\]

so that the expected number of failures in the second year out of 100 printers is 23.865. The expected amount the manufacturer will pay in refunds is

\[
200 \cdot 39.347 + 100 \cdot 23.865 = 10256.
\]

Answer D.