An American male 100 meters runner is getting ready for the 2008 Olympic Games. After repeated runs, he determines that his 100 meters spring run time is normally distributed with mean 9.88 seconds and standard deviation $\sigma$. He has been also observing his Russian competitor and determined that the Russian’s spring time is normally distributed with mean 9.99 seconds, and the same standard deviation $\sigma$. Given that the probability that the American beats the Russian is 0.9015, and that their sprint times are independent, determine $\sigma$.

A. 0.2512  
B. 0.1025  
C. 0.0875  
D. 0.0603  
E. 0.0499

Solution.
Let $X$ be the sprint time of the American athlete, and $Y$ be the sprint time of the Russian sprinter. Then

$$X \sim N(9.88, \sigma^2), \quad Y \sim N(9.99, \sigma^2), \quad \text{and} \quad Y - X \sim N(0.11, 2\sigma^2).$$

Therefore, if we write $Z$ for a standard normal random variable, we have

$$\Pr(Y > X) = \Pr(Y - X > 0) = \Pr\left(\frac{Y - X - 0.11}{\sigma\sqrt{2}} > -\frac{0.11}{\sigma\sqrt{2}}\right) = \Pr(Z > -\frac{0.11}{\sigma\sqrt{2}}) = 0.9015.$$ 

This implies (from the standard normal distribution table) that

$$-\frac{0.11}{\sigma\sqrt{2}} = -1.29,$$

$$\sigma = \frac{0.11}{1.29\sqrt{2}} \approx 0.0602959.$$ 

Answer D.

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