A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let \( X \) denote the disease state of a patient, and let \( Y \) denote the outcome of the diagnostic test. The joint probability function of \( X \) and \( Y \) is given by:

\[
\begin{align*}
\Pr(X = 0, Y = 0) &= 0.800, \\
\Pr(X = 1, Y = 0) &= 0.050, \\
\Pr(X = 0, Y = 1) &= 0.025, \\
\Pr(X = 1, Y = 1) &= 0.125.
\end{align*}
\]

Calculate \( \text{Var}(Y|X = 1) \).

A. 0.13  
B. 0.15  
C. 0.20  
D. 0.51  
E. 0.71

Solution.
First note that

\[
\Pr(Y = 0|X = 1) = \frac{\Pr(X = 1, Y = 0)}{\Pr(X = 1)} =
\]

\[
= \frac{\Pr(X = 1, Y = 0)}{\Pr(X = 1, Y = 0) + \Pr(X = 1, Y = 1)} = \frac{0.05}{0.05 + 0.125} \approx 0.2857,
\]

and

\[
\Pr(Y = 1|X = 1) = 1 - \Pr(Y = 0|X = 1) \approx 1 - 0.2857 = 0.7143.
\]

The random variable \( Y|X = 1 \) is just a Bernoulli Trial with the probability of success of \( p = 0.7143 \), and thus its variance is

\[
p \cdot (1 - p) = 0.7143 \cdot (1 - 0.7143) \approx 0.2041.
\]

Answer C.

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