The total claim amount for a health insurance policy follows a distribution with density function $f(x) = \frac{1}{1000} e^{-\frac{x}{1000}}$ for $x > 0$. The premium for the policy is set at 100 over the expected total claim amount. If 100 policies are sold, what is the approximate probability that the insurance company will have claims exceeding the premiums collected?

A. 0.001  B. 0.159  C. 0.333  D. 0.407  E. 0.460

Solution.

While the problem does not say so directly, you are expected to understand that the 100 policies are independent. This is the PDF of an exponential distribution with a mean of 1000. The expected claim per policy is 1000, and the variance is $1000^2$. The premium collected is 1100 per policy. For 100 policies, a total of 110,000 are collected in premiums. The total claim is $W = X_1 + X_2 + \cdots + X_{100}$, and

$E(W) = 100E(X) = 100,000,$

while, due to independence of the 100 policies,

$\text{Var}(W) = 100\text{Var}(X) = 100 \cdot 1000^2.$

By the Central Limit Theorem $W$ has an approximately normal distribution. Thus (as always, $Z$ denotes a standard normal random variable)

$$\Pr(W > 110,000) = \Pr\left( \frac{W - E(W)}{\sqrt{\text{Var}(W)}} > \frac{110,000 - E(W)}{\sqrt{\text{Var}(W)}} \right) \approx \Pr\left( Z > \frac{110,000 - 100,000}{\sqrt{100 \cdot 1000^2}} \right) = \Pr(Z > 1) = 1 - 0.8413 = 0.1587.$$