Each time a hurricane arrives, a new home has a 0.4 probability of experiencing damage. The occurrences of damage in different hurricanes are independent. Calculate the mode of the number of hurricanes it takes for the home to experience damage from two hurricanes.

A. 2  B. 3  C. 4  D. 5  E. 6

Solution.
Each hurricane arrival is a Bernoulli Trial with success defined as damage inflicted on a home, so that \( p = 0.4 \). The number of hurricanes it takes for the home to experience damage from two hurricanes is the number of times you have to perform this Bernoulli Trial until you have two successes, and that random variable, call is \( X \), is a sum of a negative binomial random variable, let’s call in \( K \), with \( r = 2 \) and a deterministic number \( 2 \), as two damages is what we are waiting for. Its probability function is

\[
\Pr(X = n) = \Pr(K = n - r) = \binom{n - 1}{r - 1} p^r (1 - p)^{n-r} = (n - 1) \cdot 0.4^2 \cdot 0.6^{n-2}.
\]

We see that

\[
\frac{\Pr(X = n + 1)}{\Pr(X = n)} = \frac{n \cdot 0.4^2 \cdot 0.6^{n-1}}{(n - 1) \cdot 0.4^2 \cdot 0.6^{n-2}} = \frac{0.6n}{(n-1)},
\]

Note that this ratio equals one if \( \frac{0.6n}{(n-1)} = 1 \), i.e., if

\[
n = \frac{1}{0.4} = 2.5.
\]

Of course this value of \( n \) is not possible, but we see that \( \frac{0.6n}{(n-1)} > 1 \) when \( n < 2.5 \), and

\[
\frac{0.6n}{(n-1)} < 1 \text{ for } n > 2.5.
\]

This means that as long as \( n < 2.5 \), \( \Pr(X = n + 1) > \Pr(X = n) \), while as soon as \( n > 2.5 \), subsequent values of probability function decline in relation to
the current value. In other words,

\[ \Pr(X = 0) < \Pr(X = 1) < \Pr(X = 2) < \Pr(X = 3) > \Pr(X = 4) > \ldots \]

We conclude that the mode occurs at \( n = 3 \).

Answer B.

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