An insurance policy pays for a random loss $X$ subject to a deductible of $C$, where $0 < C < 1$. The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 2x, & \text{for } 0 < x < 1, \\ 0, & \text{otherwise}. \end{cases}$$

Given a random loss $X$, the probability that the insurance payment is less than 0.5 is equal to 0.64. Calculate $C$.

A. 0.1 \hspace{1cm} B. 0.3 \hspace{1cm} C. 0.4 \hspace{1cm} D. 0.6 \hspace{1cm} E. 0.8

Solution.

Denote the insurance payment by the random variable $Y$. Then

$$Y = \begin{cases} 0, & \text{if } 0 < X \leq C, \\ X - C, & \text{if } C < X < 1. \end{cases}$$

This relationship is illustrated in the graph below:

Based on the information given:
0.64 = \Pr(Y < 0.5) = \Pr(0 \leq X < 0.5 + C) = \int_{0}^{0.5+C} 2x \, dx = x^2 \bigg|_{x=0}^{x=0.5+C} = (0.5 + C)^2.

Therefore, solving for \( C \), we find \( C = \pm 0.8 - 0.5 \). Since \( 0 < C < 1 \), we conclude that \( C = 0.3 \).
Answer B.

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