November 2000 Course 1 Examination, Problem No. 23, also Study Note P-09-05, Problem No. 50

A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5. What is the expected amount paid to the company under this policy during a one-year period?

A. 2,769 B. 5,000 C. 7,231 D. 8,347 E. 10,578

Solution.
Let $N$ be the number of major snowstorms per year, and let $P$ the amount paid to the company under the policy. Then

$$\Pr(N = n) = \frac{\left(\frac{3}{2}\right)^n e^{-\frac{3}{2}}}{n!}, \ n = 0,1,2,...$$

and

$$P = \begin{cases} 
0, & \text{for } N = 0, \\
10,000(N - 1), & \text{for } N \geq 1.
\end{cases}$$

It follows that
\[ E(P) = \sum_{n=1}^{\infty} 10000 \cdot (n-1) \cdot \frac{1.5^n e^{-1.5}}{n!} = \]
\[ = - (0 - 1) \cdot 10000 \cdot \frac{1.5^0 e^{-1.5}}{0!} + \sum_{n=0}^{\infty} 10000 \cdot (n-1) \cdot \frac{1.5^n e^{-1.5}}{n!} = \]
This is what we added to the series to the right, so we subtract it here to make up for that
\[ = 10000e^{-1.5} + E(10000(N-1)) = \]
\[ = 10000e^{-1.5} + E(10000N) - E(10000) = \]
\[ = 10000e^{-1.5} + 10000 \cdot 1.5 - 10,000 \approx 7231. \]

Answer C.

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