A. 3466  B. 3659  C. 4159  D. 8487  E. 8987

Solution.
Let $Y$ be the insurance company payout. Let $X$ be the amount of loss given that an accident has happened. We are given that $X$ is an exponential random variable with mean 3000. We are given that $Y = 0$ if there is no accident, which happens with probability 0.8, and if there is an accident and given that accident, the loss is less than 500, which happens with probability

$$Pr\left(\text{There is an accident}\right) \cdot Pr\left(X < 500\right) = 0.2 \cdot \left(1 - e^{-\frac{500}{3000}}\right) = 0.2 \cdot \left(1 - e^{-\frac{1}{6}}\right).$$

Otherwise, $Y$ is positive, and given that an accident has happened, it is equal to $X - 500$.

We are looking for the 95-th percentile of $Y$, i.e., a number $y_{0.95}$ such that

$$Pr\left(Y < y_{0.95}\right) = 0.95.$$

But

$$Pr\left(Y = 0\right) = 0.8 + 0.2 \cdot \left(1 - e^{-\frac{1}{6}}\right) = 0.8307.$$

Therefore, the 95-th percentile of $Y$ occurs in its range of values resulting from the situation that an accident happened and the loss is greater than 500. Note that

$$Pr\left(Y > y_{0.95}\right) = 0.05.$$
\[0.05 = \Pr(Y > y_{0.95}) = \Pr(\text{There is an accident}) \cdot \Pr(X - 500 > y_{0.95}) =
\]
\[= 0.2 \cdot \Pr(X > 500 + y_{0.95}) = 0.2 \cdot e^{-\frac{500+y_{0.95}}{3000}} = 0.2 \cdot e^{-\frac{1}{\delta}} \cdot e^{-\frac{y_{0.95}}{3000}}.
\]
Therefore,
\[e^{-\frac{y_{0.95}}{3000}} = \frac{0.05 \cdot e^{\frac{1}{\delta}}}{0.2} = \frac{e^{\frac{1}{\delta}}}{4}.
\]
This results in
\[-\frac{y_{0.95}}{3000} = \ln\left(\frac{\frac{1}{\delta}}{4}\right) = \frac{1}{6} - \ln 4,
\]
or
\[y_{0.95} = 3000\left(\ln 4 - \frac{1}{6}\right) = 3658.8831.
\]
Answer B.

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