Exercise for September 13, 2008

November 2001 Course 1 Examination, Problem No. 5, also Study Note P-09-05, Problem No. 52

An insurance company sells a one-year automobile policy with a deductible of 2. The probability that the insured will incur a loss is 0.05. If there is a loss, the probability of a loss amount \( N \) is \( \frac{K}{N} \), for \( N = 1, \ldots, 5 \) and \( K \) a constant. These are the only possible loss amounts and no more than one loss can occur. Determine the net premium for this policy.

A. 0.031  B. 0.066  C. 0.072  D. 0.110  E. 0.150

Solution.
The probability distribution of the amount of the loss \( N \), given that there is a loss is

\[
\begin{align*}
N = 1 & \quad \text{with probability } \frac{K}{1}, \\
N = 2 & \quad \text{with probability } \frac{K}{2}, \\
N = 3 & \quad \text{with probability } \frac{K}{3}, \\
N = 4 & \quad \text{with probability } \frac{K}{4}, \\
N = 5 & \quad \text{with probability } \frac{K}{5}.
\end{align*}
\]

Therefore

\[
1 = \frac{K}{1} + \frac{K}{2} + \frac{K}{3} + \frac{K}{4} + \frac{K}{5} = K \cdot \left( \frac{60 + 30 + 20 + 15 + 12}{60} \right) = K \cdot \frac{137}{60}.
\]
This gives $K = \frac{60}{137}$. Since the probability of having a loss is 0.05, for the unconditional distribution of $N$ we obtain

\begin{align*}
N = 1 & \quad \text{with probability } 0.05 \cdot \frac{K}{1} = \frac{3}{137}, \\
N = 2 & \quad \text{with probability } 0.05 \cdot \frac{K}{2} = \frac{3}{274}, \\
N = 3 & \quad \text{with probability } 0.05 \cdot \frac{K}{3} = \frac{1}{137}, \\
N = 4 & \quad \text{with probability } 0.05 \cdot \frac{K}{4} = \frac{3}{548}, \\
\end{align*}

and

\begin{align*}
N = 5 & \quad \text{with probability } 0.05 \cdot \frac{K}{5} = \frac{3}{685}.
\end{align*}

But the policy has a deductible of 2, so for $N = 0$ and for $N = 1$, actual payment is 0. Also, this policy pays 0 when there is no loss, which happens with probability 0.95. Let $Y$ be the amount paid after the application of the deductible. Then

\begin{align*}
Y = 0 & \quad \text{with probability } \frac{3}{137} + \frac{3}{274} + 0.95 = \frac{9}{274} + 0.95, \\
Y = 1 & \quad \text{with probability } \Pr(N = 3) = \frac{1}{137}, \\
Y = 2 & \quad \text{with probability } \Pr(N = 4) = \frac{3}{548}, \\
\end{align*}

and

\begin{align*}
Y = 3 & \quad \text{with probability } \Pr(N = 5) = \frac{3}{685}.
\end{align*}

Therefore the net premium for this policy is

\begin{align*}
P = E(Y) = 1 \cdot \frac{1}{137} + 2 \cdot \frac{3}{548} + 3 \cdot \frac{3}{685} \approx 0.0314.
\end{align*}

Answer A.

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