A family buys two policies from the same insurance company. Losses under the two policies are independent and have continuous uniform distributions on the interval from 0 to 10. One policy has a deductible of 1 and the other has a deductible of 2. The family experiences exactly one loss under each policy. Calculate the probability that the total benefit paid to the family does not exceed 5.

A. 0.13  B. 0.25  C. 0.30  D. 0.32  E. 0.42

Solution.
Define $X$ and $Y$ to be loss amounts covered by the policies having deductibles of 1 and 2, respectively. The total benefit paid to the family is

$$
\begin{align*}
0, & \quad \text{if } X \leq 1 \text{ and } Y \leq 2, \\
X - 1, & \quad \text{if } X > 1 \text{ and } Y \leq 2, \\
Y - 2, & \quad \text{if } X \leq 1 \text{ and } Y > 2, \\
(X - 1) + (Y - 2) = X + Y - 3, & \quad \text{if } X > 1 \text{ and } Y > 2.
\end{align*}
$$

The benefit paid is less than 5 if

$$
\begin{align*}
0 < 5, & \quad \text{if } X \leq 1 \text{ and } Y \leq 2, \\
X - 1 < 5, & \quad \text{if } X > 1 \text{ and } Y \leq 2, \\
Y - 2 < 5, & \quad \text{if } X \leq 1 \text{ and } Y > 2, \\
X + Y - 3 < 5, & \quad \text{if } X > 1 \text{ and } Y > 2.
\end{align*}
$$

The first condition, $0 < 5$, is true always, so the total benefit paid is less than 5 always in the region where $X \leq 1$ and $Y \leq 2$. The second condition gives benefit less than 5 when $X < 6$, $X > 1$, and $Y \leq 2$.

The third condition gives benefit less than 5 when $Y < 7$, $X \leq 1$ and $Y > 2$.

Finally, the last, fourth condition gives benefit less than 5 when $Y < -X + 8$, $X > 1$ and $Y > 2$. 

By combining all of these conditions, we obtained the shaded portion of the graph below showing the region over which the total benefit paid to the family does not exceed 5.

![Graph showing the shaded region](image)

Since the joint distribution is uniform on the square $[0,10] \times [0,10]$, we can calculate the probability we are looking for as the ratio of the area of the shaded region and the area of the square $[0,10] \times [0,10]$, and that equals

$$\Pr(\text{Total benefit paid does not exceed 5}) = \frac{1 \cdot 7 + (6 - 1) \cdot 2 + \frac{1}{2} \cdot 5 \cdot 5}{100} = \frac{29.5}{100} = 0.295.$$ 

Answer C.