An auto insurance company insures an automobile worth 15000 for one year under a policy with a 1000 deductible. During the policy year there is a 0.04 chance of partial damage to the car and a 0.02 chance of a total loss of the car. If there is partial damage to the car, the amount $X$ of damage (in thousands) follows a distribution with a density function

$$f(x) = \begin{cases} 
0.5003e^{-\frac{x^2}{2}}, & \text{for } 0 < x < 15, \\
0, & \text{otherwise.} 
\end{cases}$$

What is the expected claim payment?

A. 320 B. 328 C. 352 D. 380 E. 540

Solution.
Let $Y$ denote the claim payment made by the insurance company, in thousands. As the policy has a deductible of 1 (thousand), the claim payment is

$$Y = \begin{cases} 
0, & \text{when } X < 1, \text{with probability 0.94}, \\
\max(0, X - 1), & \text{when } 0 \leq X < 15, \text{ with probability 0.04}, \\
14, & \text{when } X \geq 15, \text{ with probability 0.02} 
\end{cases}$$

Therefore, the expected value of this random variable is calculated as follows:
\[ E(Y) = 0.94 \cdot 0 + 0.04 \cdot \int_{1}^{15} 0.5003(x - 1)e^{-\frac{x}{2}} \, dx + 0.02 \cdot 14 = \]

Note that for \(0 < x < 1\), \(Y = 0\), so that the integral from 0 to 1 can be ignored here

\[= 0.020012 \cdot \left( \int_{1}^{15} xe^{-\frac{x}{2}} \, dx - \int_{1}^{15} e^{-\frac{x}{2}} \, dx \right) + 0.28 = \]

Integration by parts in the first integral

\[= 0.020012 \cdot \left( -2xe^{-\frac{x}{2}} \bigg|_{1}^{15} + 2 \int_{1}^{15} e^{-\frac{x}{2}} \, dx \right) + 0.28 = \]

\[= 0.020012 \cdot \left( -30e^{-7.5} + 2e^{-0.5} + \int_{1}^{15} e^{-\frac{x}{2}} \, dx \right) + 0.28 = \]

\[= 0.020012 \cdot \left( -30e^{-7.5} + 2e^{-0.5} - \left( -2e^{-\frac{1}{2}} \right) \bigg|_{x=1}^{x=15} \right) + 0.28 = \]

\[= 0.020012 \cdot \left( -30e^{-7.5} + 2e^{-0.5} - 2e^{-\frac{15}{2}} + 2e^{-0.5} \right) + 0.28 \approx 0.32819738. \]

Since this is \(E(Y)\) expressed in thousands, the expected claim payment is approximately 328.

Answer B.