A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay $X$ at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment. Immediately after the 10th payment of the 25-year annuity, the annuity will be exchanged for a perpetuity-immediate paying $Y$ per year. The annual effective rate of interest is 8%. Calculate $Y$.

A. 110  B. 120  C. 130  D. 140  E. 150

Solution.
The value of the perpetuity is
\[
\frac{100}{0.08} = 1250.
\]
The value of the 25-year annuity-immediate it is exchanged for is:
\[
1250 = X \cdot \left( \frac{1}{1.08} + \frac{1.08}{1.08^2} + \cdots + \frac{1.08^{24}}{1.08^{25}} \right) = \frac{X}{1.08} \left( 1 + \frac{1.08}{1.08} + \frac{1.08^2}{1.08^2} + \cdots + \frac{1.08^{24}}{1.08^{24}} \right) = 25Xv = 25\frac{X}{1.08}.
\]
This implies that
\[
X = \frac{1.08 \cdot 1250}{25} = 1.08 \cdot 50 = 54.
\]
When the second exchange happens, there are only 15 payments remaining of the 25-year annuity, and nine increases of its payments have already happened, so that the equation of value is:
\[
\frac{Y}{0.08} = 54 \cdot \left( \frac{1.08^{10}}{1.08} + \frac{1.08^{11}}{1.08^2} + \cdots + \frac{1.08^{24}}{1.08^{15}} \right) = 54 \left( \frac{1.08^{10}}{1.08} + \frac{1.08^{11}}{1.08^2} + \cdots + \frac{1.08^{24}}{1.08^{15}} \right) = 54 \cdot 1.08^9 \cdot 1.5.
This gives
\[ Y = 0.08 \times 54 \times 1.08^9 \times 15 \approx 129.5. \]
Answer C.

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