Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of $i$. The accumulated amount in the account at the end of 40 years is $X$, which is 5 times the accumulated amount in the account at the end of 20 years. Calculate $X$.

A. 4695  B. 5070  C. 5445  D. 5820  E. 6195

Solution.
The effective interest rate over a four-year period is $j = (1 + i)^4 - 1$. Using that rate, we have:

$$X = \frac{100}{d_j} = 100 \left(\frac{1 + j}{1 + j} - 1\right) = 5 \cdot 100 \frac{(1 + j)^5 - 1}{d_j}.$$

Therefore (note that the subscript $j$ refers to the interest functions involving the interest rate $j$),

$$5 = \frac{(1 + j)^5 - 1}{(1 + j)^5 - 1} = (1 + j)^5 + 1.$$

Thus $(1 + j)^5 = 4$, and $j = 31.9508\%$. Also

$$d_j = \frac{j}{1 + j} \approx 24.2142\%.$$

Therefore:

$$X = \frac{100}{d_j} = 100 \cdot \frac{(1 + j)^{10} - 1}{d_j} = 100 \cdot \frac{4^2 - 1}{0.242142} \approx 6194.72.$$

Answer E.

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