Study Note FM-09-05, Problem No. 51

Joe must pay liabilities of 1000 due 6 months from now and another 1000 due one year from now. There are two available investments:

- A 6-month bond with face amount of 1000, an 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually; and
- A one-year bond with face amount of 1000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually.

How much of each bond should Joe purchase in order to exactly (absolutely) match the liabilities?

A. Bond I: 1, Bond II: 0.97561
B. Bond I: 0.93809, Bond II: 1
C. Bond I: 0.97561, Bond II: 0.94293
D. Bond I: 0.93809, Bond II: 0.97561
E. Bond I: 0.98345, Bond II: 0.97561

Solution.
The liabilities require a cash flow of 1000 in one year. Bond I does not provide any cash flows in one year, so that 1000 cash flow must be provided fully by Bond II. But the cash flow of Bond II in one year is 1025, therefore we need

\[
\frac{1000}{1025} = \frac{40}{41} \approx 0.97561
\]

units of Bond II. This number (0.97561) of units of Bond II in six months will provide a cash flow of

\[
\frac{1000}{1025} \cdot 25 \approx 24.39024,
\]

while the liabilities need a total cash flow of 1000, so that the rest, i.e.,

\[
1000 - 24.39024 \approx 975.61
\]

must be provided by Bond I, whose unit cash flow in six months is 1040. Therefore, the following number of units of Bond I is needed:

\[
\frac{975.61}{1040} \approx 0.93809.
\]

Answer D.