Joe must pay liabilities of 1000 due 6 months from now and another 1000 due one year from now. There are two available investments:

- A 6-month bond with face amount of 1000, an 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually; and

- A one-year bond with face amount of 1000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually.

What is Joe’s total cost of purchasing the bonds required to exactly (absolutely) match the liabilities?

A. 1894  B. 1904  C. 1914  D. 1924  E. 1934

Solution.

There are two ways to do this:

- Find the total price of 0.93809 units of Bond I at its yield of 3% per half year and 0.97561 units of Bond II at its yield of 3.5% per each half year, using the solution of Problem No. 2:

\[
0.93809 \cdot \frac{1040}{1.03} + 0.97561 \left( \frac{25}{1.035} + \frac{1025}{1.035^2} \right) \approx 1904.27.
\]

- Find the spot rates for half a year and a year, and use them to discount the liabilities cash flows. The spot rate for half a year (semi-annual spot rate) is 3.0%. The semi-annual spot rate for the full year \( s_2 \) has to be derived from the pricing of the one-year bond:

\[
\frac{25}{1.035} + \frac{1025}{1.035^2} = \frac{25}{1.03} + \frac{1025}{(1 + s_2)^2}.
\]

This can be solved for \( s_2 \):

\[
\frac{1025}{(1 + s_2)^2} = \frac{25}{1.035} + \frac{1025}{1.035^2} - \frac{25}{1.03},
\]

\[
s_2 = 3.506342\%.
\]

The cost of purchasing the bonds required to exactly match the liabilities is the same as the present value of liabilities cash flows at the current market rates. Therefore, the present value of liabilities cash flows is:
\[
\frac{1000}{1.03} + \frac{1000}{1.03506342^2} \approx 1904.27.
\]

Answer B.