Study Note FM-09-05, Problem No. 53

Joe must pay liabilities of 1000 due 6 months from now and another 1000 due one year from now. There are two available investments:

- A 6-month bond with face amount of 1000, an 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually; and

- A one-year bond with face amount of 1000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually.

What is the annual effective yield rate for investment in the bonds required to exactly (absolutely) match the liabilities?

A. 6.5%  B. 6.6%  C. 6.7%  D. 6.8%  E. 6.9%

Solution.

The cost of the bonds required to match the liabilities is

\[
\frac{1000}{1.03} + \frac{1000}{1.035063422^2} \approx 1904.27. 
\]

Therefore, an investment of 1904.27 now produces cash flows of 1000 in six months and 1000 in a year. The yield rate \( i \) is the solution of the equation:

\[
1904.27 \cdot (1 + i) - 1000 \cdot (1 + i)^\frac{1}{2} - 1000 = 0. 
\]

This can be treated as a quadratic equation with the unknown equal to \( x = (1 + i)^\frac{1}{2} \), and it solves to \( i = 6.777201\% \). On the other hand, using the simple interest approximation, we get:

\[
\frac{2000 - 1904.27}{1904.27 - \frac{1000}{2}} \approx \frac{95.73}{1404.27} = 6.817065\%. 
\]

Luckily, this is the same one of the five choices. Answer D.

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