November 2005 Course FM/2 Examination, Problem No. 7, and Dr. Ostaszewski’s online exercise 262 posted May 22, 2010

A bank offers the following choices for certificates of deposit:

<table>
<thead>
<tr>
<th>Term (in years)</th>
<th>Nominal annual interest rate convertible quarterly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00%</td>
</tr>
<tr>
<td>3</td>
<td>5.00%</td>
</tr>
<tr>
<td>5</td>
<td>5.65%</td>
</tr>
</tbody>
</table>

The certificates mature at the end of the term. The bank does NOT permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates. An investor initially deposits 10,000 in the bank and withdraws both principal and interest at the end of 6 years. Calculate the maximum annual effective rate of interest the investor can earn over the 6-year period.

A. 5.09%  B. 5.22%  C. 5.35%  D. 5.48%  E. 5.61%

Solution.

The investor has the following options:

- Deposit the funds in one-year certificates every year for six years, producing an annual effective rate of return of

\[
\left( 1 + \frac{0.04}{4} \right)^4 \left( 1 + \frac{0.04}{4} \right)^4 \left( 1 + \frac{0.04}{4} \right)^4 \left( 1 + \frac{0.04}{4} \right)^4 \left( 1 + \frac{0.04}{4} \right)^4 \left( 1 + \frac{0.04}{4} \right)^4 \right)^{1/6} - 1 = 1.014^{6} - 1 \approx 4.060401%.
\]

- Deposit the funds for three years at 5% per year convertible quarterly, followed by three one-year deposits at 4% convertible quarterly, resulting in

\[
\left( \left( 1 + \frac{0.05}{4} \right)^3 \left( 1 + \frac{0.04}{4} \right)^4 \right)^{1/6} - 1 \approx 4.576189%.
\]

- Deposit the funds for three years at 5% per year convertible quarterly, followed by the same deposit for another three years, resulting in an annual effective rate of return of...
\[
\left(\left(1 + \frac{0.05 \cdot 4}{4}\right)^3\right)^{\frac{1}{3}} - 1 = 1.01^4 - 1 \approx 5.094534\%.
\]

- Deposit the funds for five years, and then for one year, or deposit funds for one year and then for five years, producing the same annual effective rate of return of

\[
\left(\left(1 + \frac{0.0565}{4}\right)^5\right) \cdot \left(1 + \frac{0.04}{4}\right)^{\frac{1}{6}} - 1 \approx 5.483827\%.
\]

The last one is the best choice. Given the choices, this was to be expected: you could have guessed intuitively that this is the highest effective rate you can get. The most effective way to help develop that guess is to calculate the effective annual rate of return on each of the available certificates:

- One-year certificate: \(\left(1 + \frac{0.04}{4}\right)^4 - 1 \approx 4.060401\%\).

- Three-year certificate: \(\left(1 + \frac{0.05}{4}\right)^4 - 1 \approx 5.094534\%\).

- Five-year certificate: \(\left(1 + \frac{0.0565}{4}\right)^4 - 1 \approx 5.770841\%\).

Clearly, getting roughly 5.77% per year for five years makes up for getting 4.06% for just one year versus the second best of getting 5.09% per year.

Answer D.

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