A company deposits 1000 at the beginning of the first year and 150 at the beginning of each subsequent year into perpetuity. In return the company receives payments at the end of each year forever. The first payment is 100. Each subsequent payment increases by 5%. Calculate the company’s yield rate for this transaction.

A. 4.7%  
B. 5.7%  
C. 6.7%  
D. 7.7%  
E. 8.7%

Solution.

Let \( j \) be the annual yield rate sought. We have

\[
\frac{1000}{j} + \frac{150}{j} = \frac{100}{1 + j} + \frac{100 \cdot 1.05}{(1 + j)^2} + \frac{100 \cdot 1.05^2}{(1 + j)^3} + \ldots = \\
\frac{100}{1 + j} \sum_{n=0}^{\infty} \left( \frac{1.05}{1 + j} \right)^n = \frac{100}{1 + j} \cdot \frac{1}{1 - \frac{1.05}{1 + j}} = \frac{100}{j - 0.05}.
\]

Therefore,

\[
\frac{1000}{j} + \frac{150}{j} = \frac{100}{j - 0.05},
\]

or

\[
1000j^2 + 150j - 50j - 7.5 = 100j,
\]

or \( 100j^2 - 7.5 = 0 \), so that \( j = \sqrt{\frac{7.5}{100}} = \sqrt{0.075} = 8.660254\% \).

Answer E.