You are given the following term structure of spot interest rates:

<table>
<thead>
<tr>
<th>Term (in years)</th>
<th>Spot interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.00%</td>
</tr>
<tr>
<td>2</td>
<td>5.75%</td>
</tr>
<tr>
<td>3</td>
<td>6.25%</td>
</tr>
<tr>
<td>4</td>
<td>6.50%</td>
</tr>
</tbody>
</table>

A three-year annuity-immediate will be issued a year from now with annual payments of 5000. Using the forward rates, calculate the present value of this annuity a year from now.

A. 13,094  B. 13,153  C. 13,296  D. 13,321  E. 13,401

Solution.
We will denote by \( s_t \) the current spot rate for maturity \( t \) and by \( f_t \) the current one-year forward rate from time \( t - 1 \) to time \( t \). Then \( f_1 = s_1 = 5.00\% \), and

\[
1 + f_2 = \frac{(1 + s_2)^2}{1 + s_1} = \frac{1.0575^2}{1.05},
\]

\[
1 + f_3 = \frac{(1 + s_3)^3}{(1 + s_2)^2} = \frac{1.0625^3}{1.0575^2},
\]

\[
1 + f_4 = \frac{(1 + s_4)^4}{(1 + s_3)^3} = \frac{1.065^4}{1.0625^3}.
\]

The discounted value of the annuity at time 1, based on these forward rates is
\[
\begin{align*}
\frac{5000}{1 + f_2} + \frac{5000}{(1 + f_2)(1 + f_3)} + \frac{5000}{(1 + f_2)(1 + f_3)(1 + f_4)} &= \\
= \frac{5000 \cdot (1 + s_1)}{(1 + s_2)^2} + \frac{5000 \cdot (1 + s_1) \cdot (1 + s_2)^2}{(1 + s_2)^2 \cdot (1 + s_3)^3} + \frac{5000 \cdot (1 + s_1) \cdot (1 + s_2)^2 \cdot (1 + s_3)^3}{(1 + s_2)^2 \cdot (1 + s_3)^3 \cdot (1 + s_4)^4} &= \\
= \frac{5000 \cdot (1 + s_1)}{(1 + s_2)^2} + \frac{5000 \cdot (1 + s_1)}{(1 + s_3)^3} + \frac{5000 \cdot (1 + s_1)}{(1 + s_4)^4} &= \\
= \frac{5000 \cdot 1.05}{1.0575^2} + \frac{5000 \cdot 1.05}{1.0625^3} + \frac{5000 \cdot 1.05}{1.065^4} &= 13,152.50.
\end{align*}
\]

Answer B.

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