Bill purchases an annuity at a price of 10,000. The annuity makes payments of 500 at the beginning of every 6 months for 20 years. The payments are reinvested in a fund, which earns interest at an annual effective rate $i$. Interest payments are received every 6 months and reinvested at an nominal rate of 6%, convertible semiannually. Bill realizes an overall effective annual yield of 7% on his original investment over the 20-year period. Calculate $i$.

A. 5.90%  B. 6.05%  C. 6.20%  D. 6.35%  E. 6.50%

Solution.
Since Bill realizes an overall effective annual yield of 7% over 20 years, he ends up with $10,000 \cdot 1.07^{20} \approx 38,696.84$.

The pattern of accumulation implies that

$$10,000 \cdot 1.07^{20} = 500 \cdot 40 + \left(500 \cdot \left(\sqrt{1+i} - 1\right)\right) \cdot \left(\frac{\ddot{s}_{40|3\%} - 40}{0.03}\right).$$

Therefore,

$$38,696.84 \approx 20,000 + 500 \cdot \left(\sqrt{1+i} - 1\right) \cdot \left(\frac{\ddot{s}_{40|3\%} - 40}{0.03}\right).$$

This gives

$$\sqrt{1+i} - 1 \approx 2.98\%$$

and

$$i \approx 1.0298^2 - 1 \approx 6.05\%.$$

Answer B.