On January 1, 2004, an investment account is worth one million dollars. On July 1, 2004, $100,000 is withdrawn. No other transactions are performed in the account until January 1, 2006. On January 1, 2006, the account is worth $1,100,000 and it turns out that the dollar-weighed rate of return for 2004 was the same as the time-weighed rate of return for 2005. Calculate the effective rate of return earned by the investment in this account in 2005.

A. 9.50%  B. 9.75%  C. 10.00%  D. 10.25%  E. 10.50%

Solution.
Let $x$ be the unknown effective rate of return, and $z$ be the balance in the account on January 1, 2005. Because there were no transactions in the account in 2005, the effective rate of return in 2005 is the same as the time-weighed rate of return for 2005. Therefore, the rate of return sought is equal to either of the sides of the equation below (the left-hand side is the dollar-weighed rate of return for 2004, and the right-hand side is the time-weighed rate of return for 2005):

$$\frac{z + 100,000 - 1,000,000}{1 \cdot 1,000,000 - \frac{1}{2} \cdot 100,000} = \frac{1,100,000 - z}{z}.$$  

This is a system of two equations that simplifies to a quadratic equation in $z$, and, unfortunately, it works out better to solve it first for $z$, and only after that, find $x$. The equation to solve for $z$ is:

$$z^2 + 50,000z - 1,100,000 \cdot 950,000 = 0.$$  

This gives

$$z = \frac{-50,000 \pm \sqrt{50,000^2 + 4 \cdot 1,100,000 \cdot 950,000}}{2} \approx \begin{cases} 997,558.07, \\ -1,047,558.07. \end{cases}$$

Only the positive answer is feasible, so that

$$x = \frac{1,100,000 - z}{z} = \frac{1,100,000 - 997,558.07}{997,558.07} \approx 10.27%.$$

Answer D.