November 2001 Course 2 Examination, Problem No. 31
You have decided to invest in two bonds. Bond $X$ is an $n$-year bond with semi-annual coupons, while bond $Y$ is an accumulation bond (i.e., a zero-coupon bond) redeemable in $\frac{n}{2}$ years. The desired yield rate is the same for both bonds. You also have the following information:

- Bond $X$ has par value of 1000. Its ratio of the semi-annual bond rate to the desired semi-annual yield rate, $\frac{r}{i}$, is 1.03125. Finally, the present value of its redemption value is 381.50.
- Bond $Y$ has redemption value, which is the same as the redemption value of bond $X$. The price to yield for Bond $Y$ is 647.80.

What is the price of bond $X$?

A. 1019  
B. 1029  
C. 1050  
D. 1055  
E. 1072

Solution.
Let $C$ be the redemption value of bond $X$, which is also the maturity value of bond $Y$. For bond $X$ its base amount is:

$$G = \frac{F}{i} = 1000 \cdot 1.03125 = 1031.25.$$ 

The present value of the redemption value for bond $X$ is $381.50 = C v_{i}^{2/2}$. For bond $Y$, the price is $647.80 = C v_{i}^{n}$. By dividing the present value of the redemption value of bond $X$ by the price of bond $Y$ we get:

$$\frac{381.50}{647.80} = \frac{v_{i}^{2/2}}{v_{i}^{n}} = v_{i}^{n},$$

and this gives $v_{i}^{n} \approx 0.5889163$. Therefore

$$C = \frac{381.50}{v_{i}^{2/2}} = \frac{381.50}{\left(\frac{381.50}{647.50}\right)^{2}} = \frac{647.50^{2}}{381.50} \approx 1098.98.$$ 

Now we can calculate the price of bond $X$

$$P = G + (C - G) v_{i}^{2/2} = 1031.25 + \left(\frac{647.50^{2}}{381.50} - 1031.25\right) \cdot \left(\frac{381.50}{647.50}\right)^{2} \approx 1054.76.$$ 

Answer D.

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