For a fully discrete whole life insurance of 5000 on (40), you are given the following:

- The probability of death within a year, \( q_x \), is the same for all ages and equal to 3%.
- The annual interest rate is 5%.
- Policy expenses are 10% of contract premium in the first year, and 2% of contract premium in renewal years, as well as 10 per thousand in the first year, and 5 per thousand in all renewal years.
- The contract premium is based on the Equivalence Principle, and is paid as a level annual premium for the entire life of the insured.

Calculate the variance of the prospective loss function at policy duration 10, when the insured is 50.

A. 60312  B. 98125  C. 125000  D. 173077  E. 225000

Solution.

The key observation is that the future lifetime has geometric distribution. Let us write

\[ q_x = q \] (for any age \( x \)) and \( p = 1 - q \).

We know that under the assumptions of this problem

\[ A_x = \frac{q}{q + i} = \frac{0.03}{0.03 + 0.05} = \frac{3}{8}, \]

and

\[ \bar{a}_x = \frac{1 + i}{q + i} = \frac{1.05}{0.03 + 0.05} = \frac{105}{8}. \]

Let \( G \) be the contract premium for this plan of insurance. Then based on the Equivalence Principle, we have

\[ G\bar{a}_{40} = 5000A_{40} + 0.08G + 0.02G\bar{a}_{40} + 25 + 25\bar{a}_{40}. \]

Therefore,

\[ G = \frac{5000A_{40} + 25 + 25\bar{a}_{40}}{0.98\bar{a}_{40} - 0.08} = \frac{5000 \cdot \frac{3}{8} + 25 + 25 \cdot \frac{105}{8}}{0.98 \cdot \frac{105}{8} - 0.08} = \frac{17825}{102.26} \approx 174.31. \]

The prospective loss function at policy duration 10 is

\[ L = (0.98G - 25)\bar{a}_{x_{10}} - 5000v^{x_{10}}. \]

Its variance is

\[ \text{Var}(L) = (20.98G - 4475)^2 \cdot \left( 2A_{50} - A_{50}^2 \right) = \left( 20.98 \cdot \frac{17825}{102.26} - 4475 \right)^2 \cdot \left( \frac{0.03}{0.03 + 0.10} - \frac{9}{64} \right) = 60312.3612. \]

Answer A.