INSTRUCTIONS TO CANDIDATES

1. This 80 point examination consists of 40 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only. Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Please make your marks dark and fill in the spaces completely. Fill in that it is Spring 2004, and the exam number 3.

Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. (For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row.) Please write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.

For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

5. At the beginning of the examination, check through the exam booklet for any missing or defective pages. The supervisor has additional exams for those candidates who have defective exam booklets. Verify that you have a copy of “Tables for CAS Exam 3” included in your exam packet.

6. Candidates must remain in the examination center until two hours after the start of the examination. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

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7. At the end of the examination, place the short-answer card in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW. Anything written in the examination booklet will not be graded. Only the short-answer card and the answer sheets will be graded.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. (Do not put the self-addressed stamped envelope inside the Examination Envelope.)

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a copy of the examination from the CAS website.

All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society disqualifying the candidate’s paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. An examination survey and postage-paid reply envelope are included with the examination. No postage is necessary for surveys mailed within the United States. Candidates mailing the survey outside the United States should use the courtesy reply envelope distributed by your exam supervisor. This survey is also available on the CAS website in the “Exams” section. Please either complete the survey and leave it with the examination supervisor, take the survey and envelope with you when leaving the examination center, or submit the survey online. Please submit your survey to the CAS Office by May 23, 2004. Please do not enclose the survey in the Examination Envelope.

END OF INSTRUCTIONS
1. A car leasing company leases cars to customers for a three-year period.

- Each year 15% of the vehicles get into accidents.
- Different accident years are independent.
- At the end of the lease, 25% of the customers decide to keep their cars. This decision is made independently of their accident history.
- The company pays a $1,000 bonus for each car returned that has not been in an accident.
- $i = 10\%$

What is the actuarial present value of the bonus payment at the time that the car is leased?

A. Less than $250
B. At least $250, but less than $275
C. At least $275, but less than $300
D. At least $300, but less than $325
E. At least $325
2. Given:

- \( v=0.95 \)
- \( 10 \ p_{25} = 0.87 \)
- \( \ddot{a}_{25} = 9.868 \)
- \( \ddot{a}_{35} = 4.392 \)

Calculate \( A_{25}^1 \).

A. Less than 0.16
B. At least 0.16, but less than 0.32
C. At least 0.32, but less than 0.48
D. At least 0.48, but less than 0.64
E. At least 0.64
3. For a fully discrete 10-year deferred whole life insurance of 1,000 on (40), you are given:

\[ v = 0.95 \]
\[ p_{48} = 0.98077 \]
\[ p_{49} = 0.98039 \]
\[ a_{50} = 0.35076 \]

The annual benefit premium of 23.40 is payable during the deferral period.

Calculate \( V \), the benefit reserve of this insurance at time \( t=8 \), immediately before the premium payment.

A. Less than 250
B. At least 250, but less than 260
C. At least 260, but less than 270
D. At least 270, but less than 280
E. At least 280
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4. For a single premium discrete 15-year term insurance of $100,000 on a person age (45), you are given:

- $i = 5.0\%$
- Mortality follows DeMoivre with $\omega = 100$.

After 5 years, it is discovered that the insurance should have been calculated using $\omega = 120$. At that time, a one-time premium adjustment is made only for the remaining 10 years of the insurance.

Calculate the one-time adjustment.

A. Refund of $7,091$
B. Refund of $4,412$
C. Refund of $2,679$
D. Additional premium of $4,412$
E. Additional premium of $7,091$

CONTINUED ON NEXT PAGE
5. Twins age (30) purchase a fully continuous joint life annuity along with a provision for joint life insurance. Their future lifetimes are independent and identically distributed.

Given:

- $\delta = 0.05$
- $\mu_x(t) = 0.04$ for all $x$ and $t$
- The special annuity (with insurance provision) pays:
  
  1,000 per year while both are alive,
  1,000 at the moment of the first death,
  600 per year after the first death until the second death, and
  800 at the moment of the second death.

Calculate the actuarial present value of this special annuity (with insurance provision).

A. Less than 12,700
B. At least 12,700, but less than 14,200
C. At least 14,200, but less than 15,700
D. At least 15,700, but less than 17,200
E. At least 17,200
6. A biological experiment begins with 100 identical independent tests. Each test can be terminated on any day for the following reasons: (1) human error, (2) successful completion of the experiment, or (3) all other reasons. Given the following triple-decrement mortality table:

<table>
<thead>
<tr>
<th>X</th>
<th>q_x^{(1)}</th>
<th>q_x^{(2)}</th>
<th>q_x^{(3)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.08</td>
<td>0.02</td>
<td>0.10</td>
</tr>
<tr>
<td>1</td>
<td>0.10</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.65</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Calculate the expected number of tests to reach a successful completion during the third day.

A. Less than 10
B. At least 10, but less than 20
C. At least 20, but less than 30
D. At least 30, but less than 40
E. At least 40
7. A fund is set up to pay a death benefit of 10 to each of 1,000 lives age (65). The fund is required to have an 80% chance that it will be able to pay all of its claims.

- Mortality is from the Illustrative Life Table.
- $i = 6\%$
- Death benefits are payable at the end of the year of death.

Using the normal approximation, calculate the initial amount of the fund.

A. Less than 4,420
B. At least 4,420, but less than 4,430
C. At least 4,430, but less than 4,440
D. At least 4,440, but less than 4,450
E. At least 4,450
8. You are taking over the management of a fund. Exactly 20 years ago, the fund was set up to pay a death benefit of 1 to each of 10,000 lives age (65).

- The initial fund was 4,500.
- Mortality to date has exactly followed the Illustrative Life Table.
- \( i = 6\% \) has been exactly achieved to date.
- Death benefits are payable at the end of the year of death.

What is the current balance in the fund?

A. Less than 2,450  
B. At least 2,450, but less than 2,525  
C. At least 2,525, but less than 2,600  
D. At least 2,600, but less than 2,675  
E. At least 2,675
9. A company offers two special discrete life policies to a life age (30).

The first policy has a death benefit that starts at 5 and increases 5 per year until age (50). It then decreases at 5 per year until age (70). From age (70) onward it is level at 10.

The second policy has a death benefit that starts at 10 and increases 5 per year until age (50). It then decreases at 5 per year until age (70). From age (70) onward it is level at 20.

- Mortality follows the Illustrative Life Table.
- \( i = 6\% \)

Compute the difference in the actuarial present values of these two death benefits at issue.

A. Less than 0.500
B. At least 0.500, but less than 0.625
C. At least 0.625, but less than 0.750
D. At least 0.750, but less than 0.825
E. At least 0.825
10. 4,000 people age (30) each pay an amount, P, into a fund. Immediately after the 1,000\textsuperscript{th} death, the fund will be dissolved and each of the survivors will be paid $50,000.

- Mortality follows the Illustrative Life Table, using linear interpolation at fractional ages.
- $i = 12\%$

Calculate P.

A. Less than 515
B. At least 515, but less than 525
C. At least 525, but less than 535
D. At least 535, but less than 545
E. At least 545
11. Ten years ago, the employees of XYZ Inc., all age (60), were given a lifetime health plan.

- The only sources of decrement are mortality and retirement.
- Mortality has exactly followed the Illustrative Life Table.
- At age (65) 1,000 of these employees retired and left the plan; there have been no other retirements.
- No new employees were hired.
- Today 2,000 employees remain in the program.

How many people were originally in the program?

A. Less than 3,500
B. At least 3,500, but less than 3,525
C. At least 3,525, but less than 3,550
D. At least 3,550, but less than 3,575
E. At least 3,575
12. You are given the following:

- The probability that a newborn lives to be 25 is 70%
- The probability that a newborn lives to be 35 is 50%
- The following annuities-due each have actuarial present value equal to 60,000:
  - a life annuity-due of 7,500 on (25)
  - a life annuity-due of 12,300 on (35)
  - a life annuity-due of 9,400 on (25) that makes at most ten payments

What is the interest rate?

A. 8.0%
B. 8.1%
C. 8.2%
D. 8.3%
E. 8.4%
13. An insurer offers a three-year warranty for a certain piece of equipment.

- A premium payment for the warranty is made at the start of each year.
- All premium payments are the same.
- Once the machine breaks, a benefit payment is made at the end of that year and the policy is cancelled (no further payments, neither premium nor benefit, are made).

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>Probability of breakdown (q_x)</th>
<th>Year-end Benefit Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10%</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>20%</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>20%</td>
<td>50</td>
</tr>
</tbody>
</table>

- \( i = 12\% \)

Calculate the annual premium.

A. Less than 5.00
B. At least 5.00, but less than 7.50
C. At least 7.50, but less than 10.00
D. At least 10.00, but less than 12.50
E. At least 12.50
14. For a 10-state Markov chain, which of the following properties imply that all of the states in the chain communicate?

1. Every state has a finite expected return time.
2. There is a state, X, so that for every state, Y, the expected travel time from X to Y is finite.
3. For every state, X, and every state, Y, the expected travel time from X to Y is finite.
4. For every pair of states, X and Y, either the expected travel time from X to Y is finite or the expected travel time from Y to X is finite.

A. 1 only
B. 2 only
C. 3 only
D. 4 only
E. All four statements are equivalent and imply that all of the states in the chain communicate.
15. You are given:

- The number of broken pipe claims that occur in a short span of time is proportional to the length of time, but the constant of proportionality varies as the temperature varies.
- The number of claims that occur in a given time period is independent of the number occurring in any disjoint time period.
- Broken pipe claims occur one at a time.
- Each broken pipe claim generates 1, 2, or 3 “reports” with equal probability.
- Each “report” corresponds to a loss uniformly distributed between 1 and 50 dollars.

Which of the following random variables could satisfy the definition of a non-homogeneous Poisson random variable?

1. The number of broken pipe claims
2. The number of “reports”
3. The total dollars of loss

A. 1 only
B. 2 only
C. 3 only
D. 1 and 2 only
E. 1, 2, and 3
16. The number of major hurricanes that hit the island nation of Justcoast is given by a Poisson process with 0.100 storms expected per year.

- Justcoast establishes a fund that will pay 100/storm.
- The fund charges an annual premium, payable at the start of each year, of 10.
- At the start of this year (before the premium is paid) the fund has 65.
- Claims are paid immediately when there is a storm.
- If the fund ever runs out of money, it immediately ceases to exist.
- Assume no investment income and no expenses.

What is the probability that the fund is still functioning in 10 years?

A. Less than 60%
B. At least 60%, but less than 61%
C. At least 61%, but less than 62%
D. At least 62%, but less than 63%
E. At least 63%
17. Payfast Auto insures sub-standard drivers.

- Each driver has the same non-zero probability of having an accident.
- Each accident does damage that is exponentially distributed with $\theta = 200$.
- There is a $100 per accident deductible and insureds only "report" claims that are larger than the deductible.
- Next year each individual accident will cost 20\% more.
- Next year Payfast will insure 10\% more drivers.

What will be the percentage increase in the number of "reported" claims next year?

A. Less than 15\%
B. At least 15\%, but less than 20\%
C. At least 20\%, but less than 25\%
D. At least 25\%, but less than 30\%
E. At least 30\%
18. Loans transition through five states (Current, 30, 60, 90, and Foreclosed) based on the following matrix:

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>Foreclosed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>0.80</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>30</td>
<td>0.50</td>
<td>0.00</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>60</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
<td>0.75</td>
<td>0.00</td>
</tr>
<tr>
<td>90</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.90</td>
</tr>
<tr>
<td>Foreclosed</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The transitions happen monthly.

Out of 100,000 Current loans how many are expected to be Foreclosed in six months?

A. Less than 16,500
B. At least 16,500, but less than 16,750
C. At least 16,750, but less than 17,000
D. At least 17,000, but less than 17,250
E. At least 17,250
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19. A company has a machine that occasionally breaks down. An insurer offers a warranty for this machine. The number of breakdowns and their costs are independent.

The number of breakdowns each year is given by the following distribution:

<table>
<thead>
<tr>
<th># of breakdowns</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50%</td>
</tr>
<tr>
<td>1</td>
<td>20%</td>
</tr>
<tr>
<td>2</td>
<td>20%</td>
</tr>
<tr>
<td>3</td>
<td>10%</td>
</tr>
</tbody>
</table>

The cost of each breakdown is given by the following distribution:

<table>
<thead>
<tr>
<th>Cost</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>50%</td>
</tr>
<tr>
<td>2,000</td>
<td>10%</td>
</tr>
<tr>
<td>3,000</td>
<td>10%</td>
</tr>
<tr>
<td>5,000</td>
<td>30%</td>
</tr>
</tbody>
</table>

To reduce costs, the insurer imposes a per claim deductible of 1,000. Compute the standard deviation of the insurer’s losses for this year.

A. 1,359
B. 2,280
C. 2,919
D. 3,092
E. 3,434
20. Losses have an exponential distribution with a mean of 1,000. There is a deductible of 500. The insurer wants to double the loss elimination ratio. Determine the new deductible that achieves this.

A. 219
B. 693
C. 1,046
D. 1,193
E. 1,546
21. Auto liability losses for a group of insureds (Group R) follow a Pareto distribution with \( \alpha = 2 \) and \( \theta = 2,000 \). Losses from a second group (Group S) follow a Pareto distribution with \( \alpha = 2 \) and \( \theta = 3,000 \). Group R has an ordinary deductible of 500, while Group S has a franchise deductible of 200.

Calculate the amount that the expected cost per payment for Group S exceeds that for Group R.

A. Less than 350
B. At least 350, but less than 650
C. At least 650, but less than 950
D. At least 950, but less than 1,250
E. At least 1,250
22. An actuary determines that claim counts follow a negative binomial distribution with unknown $\beta$ and $r$. It is also determined that individual claim amounts are independent and identically distributed with mean 700 and variance 1,300. Aggregate losses have mean 48,000 and variance 80 million.

Calculate the values for $\beta$ and $r$.

A. $\beta = 1.20, r = 57.19$
B. $\beta = 1.38, r = 49.75$
C. $\beta = 2.38, r = 28.83$
D. $\beta = 1,663.81, r = 0.04$
E. $\beta = 1,664.81, r = 0.04$
23. A customer service department receives 0 or 1 complaint each day, depending on the number of complaints on the previous 2 days, as follows:

(i) If there were no complaints the past 2 days, then there will be no complaints today with probability 0.75.
(ii) If there were no complaints 2 days ago but 1 complaint yesterday, then there will be no complaints today with probability 0.40.
(iii) If there was 1 complaint 2 days ago but no complaints yesterday, then there will be no complaints today with probability 0.55.
(iv) If there was 1 complaint on each of the past 2 days, then there will be no complaints today with probability 0.10.

Suppose there were no complaints 2 days ago and 1 complaint yesterday. Calculate the probability that there will be at least 1 complaint over the next 2 days.

A. 0.4375
B. 0.5700
C. 0.6975
D. 0.7800
E. 0.8400
24. The economy in Ameropia at any given time falls into one of four states:

(0) economic peak
(1) contraction (recession)
(2) trough
(3) expansion

The economy moves among these states each quarter according to the following transition matrix:

\[ P = \begin{bmatrix} 0.2 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.7 & 0.3 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.6 \\ 0.4 & 0.0 & 0.0 & 0.6 \end{bmatrix} \]

Long-term, what percentage of the time is the economy in an economic contraction (recession)?

A. Less than 15.5%
B. At least 15.5%, but less than 23.0%
C. At least 23.0%, but less than 30.5%
D. At least 30.5%, but less than 38.0%
E. At least 38.0%
25. A gambler begins with 2 chips. At each play, he/she can

- win 2 chips with probability 0.1
- win 1 chip with probability 0.2
- push (win 0 chips) with probability 0.3
- lose 1 chip with probability 0.3
- lose 2 chips with probability 0.1

Play continues as long as the gambler has exactly 2 or 3 chips. Calculate the expected number of rounds the gambler has 3 chips.

A. 0.200
B. 0.465
C. 0.698
D. 1.427
E. 1.628
26. On Time Shuttle Service has one plane that travels from Appleton to Zebrashire and back each day. Flights are delayed at a Poisson rate of two per month. Each passenger on a delayed flight is compensated $100. The numbers of passengers on each flight are independent and distributed with mean 30 and standard deviation 50. (You may assume that all months are 30 days long and that years are 360 days long.)

Calculate the standard deviation of the annual compensation for delayed flights.

A. Less than $25,000
B. At least $25,000, but less than $50,000
C. At least $50,000, but less than $75,000
D. At least $75,000, but less than $100,000
E. At least $100,000
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27. Each day, traffic passing through the Washington Tunnel increases during the morning and afternoon rush hours, and decreases at other times as follows:

(i) From 12 a.m. to 8 a.m., the numbers of cars follows a Poisson distribution with an increasing hourly rate of $\lambda(t) = 12 + 3.5t$ for $0 \leq t \leq 8$.

(ii) From 8 a.m. to 12 p.m., the numbers of cars follows a Poisson distribution with a decreasing hourly rate of $\lambda(t) = 60 - 2.5t$ for $8 \leq t \leq 12$.

(iii) From 12 p.m. to 6 p.m., the numbers of cars follows a Poisson distribution with an increasing hourly rate of $\lambda(t) = -30 + 5t$ for $12 \leq t \leq 18$.

(iv) From 6 p.m. to 12 a.m., the numbers of cars follows a Poisson distribution with a decreasing hourly rate of $\lambda(t) = 204 - 8t$ for $18 \leq t \leq 24$.

What is the probability that exactly 25 cars pass through the tunnel between 11:30 a.m. and 12:30 p.m.?

A. 0.0187
B. 0.0273
C. 0.0357
D. 0.0432
E. 0.0511

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27
28. A pizza delivery company has purchased an automobile liability policy for its delivery drivers from the same insurance company for the past five years. The number of claims filed by the pizza delivery company as the result of at-fault accidents caused by its drivers is shown below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>4</td>
</tr>
<tr>
<td>2001</td>
<td>1</td>
</tr>
<tr>
<td>2000</td>
<td>3</td>
</tr>
<tr>
<td>1999</td>
<td>2</td>
</tr>
<tr>
<td>1998</td>
<td>15</td>
</tr>
</tbody>
</table>

Calculate the skewness of the empirical distribution of the number of claims per year.

A. Less than 0.50  
B. At least 0.50, but less than 0.75  
C. At least 0.75, but less than 1.00  
D. At least 1.00, but less than 1.25  
E. At least 1.25
29. Claim sizes this year are described by a 2-parameter Pareto distribution with parameters $\theta = 1,500$ and $\alpha = 4$. What is the expected claim size per loss next year after 20% inflation and the introduction of a $100 deductible?

A. Less than $490$
B. At least $490$, but less than $500$
C. At least $500$, but less than $510$
D. At least $510$, but less than $520$
E. At least $520$
30. A scientist performs experiments, each with a 60% success rate. Let $X$ represent the number of trials until the first success. Use the inverse transform method to simulate the random variable $X$, and the following random numbers (where low numbers correspond to a high number of trials): 0.15, 0.62, 0.37, 0.78.

Generate the total number of trials until three successes result.

A. 3  
B. 4  
C. 5  
D. 6  
E. 7
31. Coins are tossed into a fountain according to a Poisson process with a rate of one every three minutes. The coin denominations are independently distributed as follows:

<table>
<thead>
<tr>
<th>Coin</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny</td>
<td>0.5</td>
</tr>
<tr>
<td>Nickel</td>
<td>0.2</td>
</tr>
<tr>
<td>Dime</td>
<td>0.2</td>
</tr>
<tr>
<td>Quarter</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Calculate the probability that the fourth dime is tossed into the fountain in the first two hours.

A. Less than 0.89
B. At least 0.89, but less than 0.92
C. At least 0.92, but less than 0.95
D. At least 0.95, but less than 0.98
E. At least 0.98
32. Which of the following statements are true about the sums of discrete, independent random variables?

1. The sum of two Poisson random variables is always a Poisson random variable.

2. The sum of two negative binomial random variables with parameters \((r, \beta)\) and \((r', \beta')\) is a negative binomial random variable if \(r = r'\).

3. The sum of two binomial random variables with parameters \((m, q)\) and \((m', q')\) is a binomial random variable if \(q = q'\).

A. None of 1, 2, or 3 is true.
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. 1, 2, and 3

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33. \( F(x) \) is the cumulative distribution function for the size-of-loss variable, \( X \).

\( P, Q, R, S, T, \) and \( U \) represent the areas of the respective regions.

What is the expected value of the savings to the insurance company of implementing a franchise deductible of "DED" and a limit of "LIM" to a policy that previously had no deductible and no limit? (For clarity, that is a policy that pays its first dollar of loss for a loss of \( \text{DED} + 1 \) and its last dollar of loss for a loss of \( \text{LIM} \).)

A. \( S \)
B. \( S + P \)
C. \( S + Q + P \)
D. \( S + P + R + U \)
E. \( S + T + U + P \)
34. Claim severities are modeled using a continuous distribution and inflation impacts claims uniformly at an annual rate of $i$. Which of the following are true statements regarding the distribution of claim severities after the effect of inflation?

1. An Exponential distribution will have scale parameter $(1+i)\theta$
2. A 2-parameter Pareto distribution will have scale parameters $(1+i)\alpha$ and $(1+i)\theta$
3. A Paralogistic distribution will have scale parameter $\theta/(1+i)$

A. 1 only
B. 3 only
C. 1 and 2 only
D. 2 and 3 only
E. 1, 2, and 3
35. The XYZ Insurance Company sells property insurance policies with a deductible of $5,000, policy limit of $500,000, and a coinsurance factor of 80%. Let $X_i$ be the individual loss amount of the $i^{th}$ claim and $Y_i$ be the claim payment of the $i^{th}$ claim.

Which of the following represents the relationship between $X_i$ and $Y_i$?

A. $Y_i = \begin{cases} 
0 & X_i \leq 5,000 \\
0.80(X_i - 5,000) & 5,000 < X_i \leq 625,000 \\
500,000 & X_i > 625,000 
\end{cases}$

B. $Y_i = \begin{cases} 
0 & X_i \leq 4,000 \\
0.80(X_i - 4,000) & 4,000 < X_i \leq 500,000 \\
500,000 & X_i > 500,000 
\end{cases}$

C. $Y_i = \begin{cases} 
0 & X_i \leq 5,000 \\
0.80(X_i - 5,000) & 5,000 < X_i \leq 630,000 \\
500,000 & X_i > 630,000 
\end{cases}$

D. $Y_i = \begin{cases} 
0 & X_i \leq 6,250 \\
0.80(X_i - 6,250) & 6,250 < X_i \leq 631,250 \\
500,000 & X_i > 631,250 
\end{cases}$

E. $Y_i = \begin{cases} 
0 & X_i \leq 5,000 \\
0.80(X_i - 5,000) & 5,000 < X_i \leq 505,000 \\
500,000 & X_i > 505,000 
\end{cases}$
36. For a surplus process, define:

P to be the continuous-time, infinite horizon ruin probability;
Q to be the continuous-time, finite horizon, ruin probability;
R to be the discrete-time, infinite horizon ruin probability; and
S to be the discrete-time, finite horizon ruin probability.

Which of the following must always be true?

1. $R \leq Q \leq P$
2. $S \leq Q \leq P$
3. $S \leq Q \leq R$

A. 1 only
B. 2 only
C. 3 only
D. 1 and 2 only
E. 2 and 3 only

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36
37. An insurance portfolio produces \( N \) claims with the following distribution:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( P(N = n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Individual claim amounts have the following distribution:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f_x(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7</td>
</tr>
<tr>
<td>10</td>
<td>0.2</td>
</tr>
<tr>
<td>20</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Individual claim amounts and claim counts are independent.

Calculate the probability that the ratio of aggregate claim amounts to expected aggregate claim amounts will exceed 4.

A. Less than 3%
B. At least 3%, but less than 7%
C. At least 7%, but less than 11%
D. At least 11%, but less than 15%
E. At least 15%
38. You are asked to price a Workers' Compensation policy for a large employer. The employer wants to buy a policy from your company with an aggregate limit of 150% of total expected loss. You know the distribution for aggregate claims is Lognormal. You are also provided with the following:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of claims</td>
<td>50</td>
<td>12</td>
</tr>
<tr>
<td>Amount of individual loss</td>
<td>4,500</td>
<td>3,000</td>
</tr>
</tbody>
</table>

Calculate the probability that the aggregate loss will exceed the aggregate limit.

A. Less than 3.5%
B. At least 3.5%, but less than 4.5%
C. At least 4.5%, but less than 5.5%
D. At least 5.5%, but less than 6.5%
E. At least 6.5%
39. PQRE Re provides reinsurance to Telecom Insurance Company. PQRE agrees to pay Telecom for all losses resulting from "events", subject to a $500 per event deductible.

For providing this coverage, PQRE receives a premium of $250.

Use a Poisson distribution with mean equal to 0.15 for the frequency of events. Event severity is from the following distribution:

<table>
<thead>
<tr>
<th>Loss</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.10</td>
</tr>
<tr>
<td>500</td>
<td>0.25</td>
</tr>
<tr>
<td>750</td>
<td>0.30</td>
</tr>
<tr>
<td>1,000</td>
<td>0.25</td>
</tr>
<tr>
<td>1,250</td>
<td>0.05</td>
</tr>
<tr>
<td>1,500</td>
<td>0.05</td>
</tr>
</tbody>
</table>

• \( i = 0\% \)

Using the normal approximation to PQRE's annual aggregate losses on this contract, what is the probability that PQRE will pay out more than it receives?

A. Less than 12%
B. At least 12%, but less than 13%
C. At least 13%, but less than 14%
D. At least 14%, but less than 15%
E. 15% or more
XYZ Re provides reinsurance to Bigskew Insurance Company. XYZ agrees to pay Bigskew for all losses resulting from "events", subject to:

- a $500 deductible per event and
- a $100 annual aggregate deductible

For providing this coverage, XYZ receives a premium of $150.

Use a Poisson distribution with mean equal to 0.15 for the frequency of events. Event severity is from the following distribution:

<table>
<thead>
<tr>
<th>Loss</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.10</td>
</tr>
<tr>
<td>500</td>
<td>0.25</td>
</tr>
<tr>
<td>800</td>
<td>0.30</td>
</tr>
<tr>
<td>1,000</td>
<td>0.25</td>
</tr>
<tr>
<td>1,250</td>
<td>0.05</td>
</tr>
<tr>
<td>1,500</td>
<td>0.05</td>
</tr>
</tbody>
</table>

- i = 0%

What is the actual probability that XYZ will pay out more than it receives?

A. 8.9%
B. 9.0%
C. 9.1%
D. 9.2%
E. 9.3%
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>26</td>
</tr>
<tr>
<td>7</td>
<td>E</td>
<td>27</td>
</tr>
<tr>
<td>8</td>
<td>D</td>
<td>28</td>
</tr>
<tr>
<td>9</td>
<td>C</td>
<td>29</td>
</tr>
<tr>
<td>10</td>
<td>D</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>D</td>
<td>31</td>
</tr>
<tr>
<td>12</td>
<td>A</td>
<td>32</td>
</tr>
<tr>
<td>13</td>
<td>D</td>
<td>33</td>
</tr>
<tr>
<td>14</td>
<td>C</td>
<td>34</td>
</tr>
<tr>
<td>15</td>
<td>A</td>
<td>35</td>
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<td>16</td>
<td>D</td>
<td>36</td>
</tr>
<tr>
<td>17</td>
<td>B</td>
<td>37</td>
</tr>
<tr>
<td>18</td>
<td>D</td>
<td>38</td>
</tr>
<tr>
<td>19</td>
<td>B</td>
<td>39</td>
</tr>
<tr>
<td>20</td>
<td>E</td>
<td>40</td>
</tr>
</tbody>
</table>