INSTRUCTIONS TO CANDIDATES

1. This 100 point examination consists of 48 problem and essay questions. The number of points for each full question or part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink.

Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.

Do not answer more than one question on a single sheet of paper. Write on only the lined side of the paper, and be careful to give the number of the question you are answering on each sheet.

The answer should be concise and confined to the question as posed. When a list of a specific size is requested, do not offer more items in your list than the number requested. For example, if you are requested to list three items, only the first three responses will be graded.

In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary.

2. Attached to the examination, after question 48, is a table of the Normal Distribution.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

5. At the beginning of the examination, check through the exam booklet for any missing or defective pages. The supervisor has additional exams for those candidates who have defective exam booklets.

6. Candidates must remain in the examination center until two hours after the start of the examination. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS
7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer pages in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

Anything written in the examination booklet will not be graded. Only the answer sheets will be graded.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. (Do not put the self-addressed stamped envelope inside the Examination Envelope.)

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a copy of the examination by contacting the CAS Office.

All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. An examination survey and postage-paid reply envelope are included with the examination. No postage is necessary for surveys mailed within the United States. Candidates mailing the survey outside the United States should use the courtesy reply envelope distributed by your exam supervisor. Please complete the survey and leave it with the examination supervisor, or take the survey and envelope with you when leaving the examination center. Please submit the survey to the CAS Office by May 25, 2001. Please do not enclose the survey in the Examination Envelope.

END OF INSTRUCTIONS
1. (2 points)

Based on the discussion of treasury bills in Bodie, Kane, and Marcus, Investments, answer the following.

a. (½ point)

Briefly describe the difference between competitive and noncompetitive bids in a treasury bill auction.

b. (½ point)

Identify two dangers faced by competitive bidders.

c. (1 point)

Assume a treasury bill has a par value of $100,000, a bank discount yield of 5%, and 92 days to maturity. Calculate the bond equivalent yield of the bill. Show all work.

2. (1½ points)

Based on Fabozzi, The Handbook of Fixed Income Securities, identify three disadvantages of a bond call provision from an investor's perspective.
3. (3 points)

You are considering adding XYZ Corporation common stock to your portfolio. As of December 31, 2001, you are given the following information about XYZ Corporation:

- XYZ consistently uses an 80% plowback ratio.
- Your forecast of XYZ's earnings per share for the next three years are:
  - 2002: $11.00
  - 2003: $12.50
  - 2004: $13.75
- Dividends are paid annually at the end of the calendar year.
- After two years, you forecast that XYZ's return-on-equity is expected to stabilize at 13%.
- You anticipate that investors require a return of 12% on assets with risk equivalent to XYZ.

Based on the discussion of equity valuation models in Bodie, Kane, and Marcus, Investments, answer the following. Show all work.

a. (2 points)

Using a multistage version of the dividend discount model, calculate XYZ's intrinsic value as of December 31, 2001.

b. (½ point)

XYZ's actual stock price on December 31, 2001 is 10% below the intrinsic value calculated in part a. above. Despite this favorable result, why might you hesitate to add XYZ to your portfolio?

c. (½ point)

What additional analysis could you perform to help determine whether or not to purchase XYZ stock?
4. (1 point)

As the Chief Financial Officer of an insurance company, you are trying to convince the board of directors to increase the allocation of foreign investments in the asset portfolio. Your reasoning is that international diversification will further reduce the risk of the portfolio.

One board member suggests that the company should increase investments in U.S. multinationals to achieve international diversification.

Based on *Investments* by Bodie, Kane, and Marcus, identify two arguments against the board member’s suggestion.

5. (2 points)

*Elton and Gruber, Modern Portfolio Theory and Investment Analysis,* provides historical empirical data to demonstrate the benefit of international diversification.

Based on the historical data, the authors claim that a U.S. investor can reduce risk (as measured by the standard deviation of returns) in both the stock and bond markets by investing some proportion of available funds in international markets. However, they state that the risk reduction potential for bonds is significantly less than for stocks.

Identify and briefly explain two reasons why Elton and Gruber’s data demonstrates that less risk reduction is achieved in the bond market.
6. (2 points)

Answer the following based on Elton and Gruber, Modern Portfolio Theory and Investment Analysis. Show all work.

You are given the following information:

<table>
<thead>
<tr>
<th>Number of securities</th>
<th>Expected Portfolio Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.100</td>
</tr>
<tr>
<td>2</td>
<td>21.180</td>
</tr>
<tr>
<td>20</td>
<td>11.352</td>
</tr>
<tr>
<td>500</td>
<td>10.304</td>
</tr>
</tbody>
</table>

- Each security will have equal weight in a given portfolio.
- The variance of all securities is the same.
- The covariance between each pair of securities is the same.

a. (1 point)

Calculate the minimum variance possible for a portfolio constructed from this group of securities.

b. (1 point)

How many securities are needed to achieve an average portfolio variance that is within 5% of this minimum?
7. (2 points)

Assume the single index model described in Elton and Gruber, *Modern Portfolio Theory and Investment Analysis*, perfectly describes portfolio returns.

You are given the following information regarding a portfolio and the market:

- The beta of the portfolio is equal to 1.5.
- The market variance is equal to 4.
- There are equal amounts of money invested in each stock in portfolio.
- The residual variance of each stock in the portfolio are equal and constant.

a. (1 point)

Calculate the number of securities it would take to reduce the residual risk of the portfolio to less than 1% of the residual risk associated with a one-stock portfolio. Show all work.

b. (1 point)

As the number of securities in the portfolio above approaches infinity, what does the variance of the portfolio approach? Show all work.

CONTINUED ON NEXT PAGE
8. (4 points)

You are given the following information regarding an optimal portfolio developed using the single index model with no short sales.

<table>
<thead>
<tr>
<th>Security</th>
<th>Mean Return</th>
<th>Beta</th>
<th>Unsystematic Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>1.0</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>1.0</td>
<td>40</td>
</tr>
<tr>
<td>C</td>
<td>18</td>
<td>2.0</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>11</td>
<td>1.0</td>
<td>25</td>
</tr>
<tr>
<td>E</td>
<td>17</td>
<td>2.0</td>
<td>30</td>
</tr>
</tbody>
</table>

You are also given that the risk-free rate is 9.

Based on Elton and Gruber, *Modern Portfolio Theory and Investment Analysis*, determine the weight given to each security in the optimal portfolio. Show all work.

9. (1 point)

Recent empirical evidence indicates that there is a negative correlation between real rates of return on stocks and inflation levels.

Based on Bodie, Kane, and Marcus, *Investments*, identify and explain two reasons why this negative correlation may exist.
10. (2 points)

Based on Elton and Gruber, Modern Portfolio Theory and Investment Analysis, answer the
following.

a. (1 point)

Draw a graph that shows the relationship between the capital market line and the
efficient frontier for risky assets. Define all items in the graph.

b. (1 point)

State the formula for the capital market line and define all terms.
11. (2 points)

a. (1½ points)

Bodie, Kane, and Marcus, Investments, demonstrate that the conventional Capital Asset Pricing Model (CAPM) is not supported by empirical evidence.

Produce a graph that supports the contention that CAPM performs poorly. Label all items in the graph and explain why the graph indicates poor performance.

b. (½ point)

According to Bodie, Kane, and Marcus in Investments, Jaganathan and Wang identified variables, that when separately added to the CAPM equation, improve the performance dramatically.

Identify two of these variables.

12. (1½ points)

Based on Elton and Gruber, Modern Portfolio Theory and Investment Analysis, the simple form of the capital asset pricing model ignores the presence of taxes in arriving at an equilibrium solution.

a. (1 point)

State the formula for the capital asset model that reflects the differential taxes on income and capital gains. Define all terms.

b. (½ point)

Explain the relationship between the expected return on a U.S. stock and its dividend yield. In your answer, consider the differential between market tax rates on dividends and market tax rates on capital gains.
13. (2 points)

You are provided with an industry multi-index model with the following form:

\[ R_i = a_i + b_m I_m + b_j I_j + b_k I_k + c_i \]

where \( I_m \), \( I_j \), and \( I_k \) are indices for the market, Industry j, and Industry k, respectively.

You are given the following:

- The indices are uncorrelated.
- Security 1 is in Industry j.
- Security 2 is in Industry k.
- The variance of \( I_m \) is 25.
- The variance of \( I_j \) is 30.
- The variance of \( I_k \) is 42.
- The sensitivities to the indices are given below:

<table>
<thead>
<tr>
<th>Security</th>
<th>( a_i )</th>
<th>( b_{im} )</th>
<th>( b_{ij} )</th>
<th>( b_{ik} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.8</td>
<td>1.3</td>
<td>----</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1.2</td>
<td>----</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

Based on Elton and Gruber, Modern Portfolio Theory and Investment Analysis, answer the following questions. Show all work.

a. (1 point)

If the variances of the residual risks \( c_1 \) and \( c_2 \) are 17 and 21, respectively, calculate the variances of the returns on Securities 1 and 2.

b. (1 point)

Calculate the covariance of Securities 1 and 2.
EXAM 8, SPRING 2001

14. (2 points)

An insurer is considering the purchase of a portfolio of bonds on January 1, 2001. The annual bond-equivalent yields-to-maturity, as of January 1, 2001, are as follows:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Yield-to-Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10%</td>
</tr>
<tr>
<td>B</td>
<td>8%</td>
</tr>
<tr>
<td>C</td>
<td>6%</td>
</tr>
</tbody>
</table>

The entire cash flow returns for the bonds are as follows:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Cash Flow at July 1, 2001</th>
<th>Cash Flow at January 1, 2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1,000.00</td>
<td>$500.00</td>
</tr>
<tr>
<td>B</td>
<td>$500.00</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>C</td>
<td>$500.00</td>
<td>$1,000.00</td>
</tr>
</tbody>
</table>

The insurer will only purchase the portfolio if the annual bond-equivalent yield-to-maturity is 7.8% or greater. Does this portfolio meet the insurer’s criteria? Explain your answer.

15. (2 points)

Based on the discussion of bond swaps in Fabozzi, The Handbook of Fixed Income Securities, answer the following.

a. (1 point)

Briefly describe a situation that would lead a portfolio manager to believe that a substitution swap is appropriate.

b. (1 point)

Briefly describe a situation that would lead a portfolio manager to believe that an intermarket spread swap is appropriate.

CONTINUED ON NEXT PAGE
16. (2 points)

Based on Elton and Gruber, Modern Portfolio Theory and Investment Analysis, answer the following.

You are given the following information about three bonds:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Bond Price on January 1, 2001</th>
<th>Cash Inflow on:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$96</td>
<td>$100</td>
</tr>
<tr>
<td>B</td>
<td>$88</td>
<td>$100</td>
</tr>
<tr>
<td>C</td>
<td>$125</td>
<td>$150</td>
</tr>
</tbody>
</table>

Assuming no arbitrage opportunity exists, calculate the forward rate from July 1, 2002 to January 1, 2003. Show all work.

17. (1½ points)

Elton and Gruber, Modern Portfolio Theory and Investment Analysis, identify the three most common option features of corporate bonds. Identify these three option features and briefly describe the impact each feature would have on the required yield for a bond.
Assume you are considering the purchase of varying amounts of two option-free bonds for a portfolio. Only these two bonds will be included in the portfolio.

The portfolio will have $3 million in par value of Bond A. Bond A has a five-year term to maturity with 8% coupons paid semi-annually. The par value of Bond A is $100.00. The annual bond-equivalent yield-to-maturity on Bond A is 8%.

The portfolio will have $8 million in par value of Bond B. Bond B has a ten-year term to maturity with 9% coupons paid semi-annually. The par value of Bond B is $100.00. The price of Bond B is $93.77, based on an annual bond-equivalent yield-to-maturity of 10%.

Based on Fabozzi, *The Handbook of Fixed Income Securities*, answer the following. Show all work.

a. (2½ points)

Assuming that there may be a 100 basis point change (either positive or negative) in the yield of all bonds in the portfolio, use Fabozzi’s approximation to calculate the effective duration of the portfolio of bonds.

b. (½ point)

Using the same assumptions as in part a. above, use Fabozzi’s approximation to calculate the effective convexity of Bond A.

c. (1 point)

You now are considering the inclusion of Bond C in your portfolio. This bond’s par value, coupon rate, term to maturity, and yield-to-maturity are identical to Bond A. However, Bond C also is putable after two years. At the current yield to maturity, the effective duration of Bond C is 70% of Bond A’s effective duration.

Assume that immediately after the purchase of the portfolio, the yields to maturity of all bonds in the portfolio decrease by 500 basis points.

Will the effective duration of Bond C move closer to or further away from Bond A’s effective duration? Explain your answer.
19. (2 points)

Elton and Gruber, Modern Portfolio Theory and Investment Analysis, discuss several theories to explain the relationship between term to maturity and yield to maturity. Based on the authors’ discussion of these theories, answer the following.

a. (1 point)

Briefly explain why investors’ behavior under the Segmented Market Theory could be considered as the opposite of investors’ behavior under the Pure Expectations Theory.

b. (1 point)

Assuming the Liquidity Premium Theory holds, briefly explain how a life insurer could earn an “extra return without any extra risk”.

CONTINUED ON NEXT PAGE
20. (4 points)

Based on Fabozzi, The Handbook of Fixed Income Securities, Altman, "Measuring Corporate Bond Mortality and Performance," and Elton and Gruber, Modern Portfolio Theory and Investment Analysis, answer the following.

a. (1½ points)

Based on Fabozzi, identify and describe three types of bond issuers that fall into the speculative grade category.

b. (1½ points)

According to Altman, investors have been more than satisfactorily compensated for investing in high-risk bonds.

Identify and briefly describe three potential reasons why investors have obtained these higher than expected returns.

c. (1 point)

Elton and Gruber state that the historical default experience for low-rated debt understates what can be expected in the future.

Identify one reason why the authors believe this to be true.

21. (1 point)

Based on Fabozzi, The Handbook of Fixed Income Securities, answer the following.

Your company has decided to bid on a 52-week treasury security in the primary market.

Your bid is $94.50 on a bill with a face value of $100.

If the stop-out yield is 5.6%, will the bid be accepted? Explain your answer.
Based on Fabozzi, The Handbook of Fixed Income Securities, answer the following.

a. (½ point)
   
   Briefly describe what a repurchase (repo) agreement is.

b. (1 point)
   
   Identify and briefly describe two ways repurchase agreements may be structured in order to reduce credit risk exposure.

c. (½ point)
   
   Explain why the repo rate is generally below the federal funds rate.
23. (2 points)

Based on Bodie, Kane, and Marcus Investments, answer the following. Show all work.

You currently have $100 million to invest and you would like to institute a contingent
immunization strategy. You are willing to accept an 8.5% effective annual rate of return as a
safety net. Your active strategy is to purchase 8%, 25 year, semiannual bonds at the annual
bond-equivalent yield-to-maturity market rate of 10%. The bonds have a par value of
$1,000. You have a six-year investment horizon.

a. (½ point)

Calculate the minimum acceptable terminal value at the end of the investment
horizon.

b. (½ point)

Calculate the number of bonds that can be purchased under the active strategy.

c. (1 point)

Assume you purchase the bonds calculated in part b. above. If the annual bond-
equivalent yield-to-maturity market rate falls to 8% immediately after purchase, is
immunization necessary? Explain your answer.
John Smith purchased a deferred annuity prior to retirement. John retired on January 1, 1995 and immediately entered the payout phase of the annuity. He selected to receive variable annual payments. You are given the following information:

- The accumulated balance as of January 1, 1995 was $200,000.
- The initial annuity payment was determined using an assumed annual investment return of 5% and a ten-year-certain annuity period.
- The rate of return underlying the portfolio in 1995 was 4%.
- The rate of return underlying the portfolio in 1996 was 6%.
- The rate of return underlying the portfolio in 1997 was 5%.

Based on Bodie, Kane, and Marcus Investments, answer the following. Show all work.

a. (1½ points)

Calculate the benefit paid on December 31, 1996.

b. (1 point)

Calculate the fund balance as of January 1, 1997.
25. (1 point)

Based on Hull, Options, Futures, & Other Derivatives, answer the following.

Hull discusses the convergence of the futures price to the spot price of the underlying asset as the delivery period is reached.

a. (½ point)

   Explain why the futures price cannot remain above the spot price of the underlying asset.

b. (½ point)

   Explain why the futures price cannot remain below the spot price of the underlying asset.
Based on Hull, Options, Futures, & Other Derivatives, answer the following. Show all work.

You are given the following information:

- The two-year risk-free interest rate in Canada is 8% per annum (continuously compounded).
- The two-year risk-free interest rate in the U.S. is 5% per annum (continuously compounded).
- The current U.S. Dollar (USD) to Canadian Dollar (CDN) exchange rate is $0.75 USD for $1.00 CDN.

a. (1 point)

Calculate the two-year forward exchange rate between the Canadian Dollar and the U.S. Dollar so that no arbitrage opportunity exists.

b. (1 1/4 points)

Describe how an investor could make an arbitrage profit if the two-year forward price of $1.00 CDN is $0.85 USD.

c. (1/2 point)

Calculate the risk-free arbitrage profit in CDN per USD borrowed at time = 2 by implementing the arbitrage strategy in part b. above.
27. (2 points)
Based on Hull, Options, Futures, & Other Derivatives, answer the following.

a. (½ point)
Briefly describe a forward-rate agreement (FRA).

b. (1½ points)
You are given the following information:

- The two-year zero-coupon interest rate is 7.5%, compounded continuously.
- The three-year zero-coupon interest rate is 8%, compounded continuously.

You enter into a forward-rate agreement where you receive 10% with annual compounding on a principal of $2,000,000 between the end of year two and the end of year three.

What is the present value of the forward-rate agreement? Show all work.

28. (1½ points)
Based on Hull, Options, Futures, & Other Derivatives, answer the following.

a. (1 point)
Briefly describe how a trader can create a butterfly spread using call and/or put options.

b. (½ point)
Assuming a trader employs a butterfly spread strategy, briefly describe what the trader believes will happen to the stock price.

CONTINUED ON NEXT PAGE
29. (4 points)

Based on Hull, Options, Futures, & Other Derivatives, answer the following. Show all work.

You are given the following information:

- The current price of a share of XYZ Company stock is $100.
- Over each of the next two six-month periods, the price is expected to go up by 10% or down by 10%.
- The risk-free rate of interest with semi-annual compounding is 6.0% per annum.

a. (2 points)

Determine the value of a one-year American put option with a strike price of $115. In calculating this value, clearly demonstrate any points at which early exercise is indicated.

b. (2 points)

For XYZ Company stock, assume you have sold a one-year American put option with a strike price of $115. Determine the number of units of stock that create a risk-less hedge over the first six-month time step for this put option.

In your answer, be certain to state whether the stock is bought or sold to create the risk-less hedge.
30. (2 points)

Based on Hull, Options, Futures, & Other Derivatives, answer the following:

You are given the following information:

- A company’s surplus, measured in millions of dollars, follows a generalized Wiener process.
- For the first six months, the drift rate is 1.0 per month and the variance rate is 4.0 per month.
- For the next six months, the drift rate is 1.5 per month and the variance rate is 10.0 per month.

How high must the company’s initial surplus be to have a 1.0% chance of a negative surplus at the end of one year? Show all work.

31. (3 points)

You are considering the purchase of a three-month European call option on a non-dividend paying stock.

You are given the following information:

- The strike price is $50.
- The current stock price is $52.
- The annual risk-free interest rate is 12%, compounded continuously.
- The volatility of the stock price is 30% per annum.

Using the Black-Scholes methodology presented in Hull, Options, Futures, and Other Derivatives, calculate the price of the option.

Show all work.
32. (3 points)

Suppose that the systematic risk of the market can be described by two uncorrelated macro-economic factors, \( F_1 \) and \( F_2 \), and that you are to use a two-factor APT to model asset returns.

For stocks A, B, and C you are given the following information:

- The risk-free interest rate is 8.
- The variance for \( F_1 \) is 76.
- The variance for \( F_2 \) is 45.
- The expected excess return (risk premium) for \( F_1 \) is 3.25.
- The expected excess return (risk premium) for \( F_2 \) is 2.50.
- The covariance between A and B is 35.53.
- The covariance between A and C is 69.50.
- The covariance between B and C is 12.07.
- The sensitivities of the stock returns to the factors \( F_1 \) and \( F_2 \) are given below.

<table>
<thead>
<tr>
<th>Stock</th>
<th>( b_{1i} )</th>
<th>( b_{2i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.21</td>
<td>0.80</td>
</tr>
<tr>
<td>B</td>
<td>0.68</td>
<td>X</td>
</tr>
<tr>
<td>C</td>
<td>Y</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Based on Elton and Gruber, Modern Portfolio Theory and Investment Analysis, answer the following. Show all work.

a. (1 point)

Determine X and Y.

b. (1 point)

Determine the expected excess returns (risk premiums) for A, B, and C.

c. (1 point)

Assuming that Portfolio P has $20 million invested in stock A, $70 million invested in stock B, and $30 million invested in Stock C.

Determine the sensitivities \( b_{p1} \) and \( b_{p2} \) for this portfolio.

CONTINUED ON NEXT PAGE
33. (1 point)

According to D'Arcy, et al., "Building a Public Access PC-Based DFA Model," there are three basic issues that must be addressed before any DFA application can be constructed. One issue is how should the risk factors be modeled.

a. (½ point)

Identify the other two basic issues that must be addressed before any DFA application can be constructed.

b. (½ point)

In addition to determining how to model a specific risk factor, what other main consideration should be included in the DFA model?

34. (1½ points)

Based on CAS Committee on Dynamic Financial Analysis, "Dynamic Financial Models of Property-Casualty Insurers," answer the following.

a. (½ point)

Define "Feedback Loops."

b. (1 point)

Identify the assumption that is often made when feedback loops are included in a DFA Model, and describe how this assumption may understate risk.
35. (1½ points)

Based on D’Arcy, et al., “Using the Public Access DFA Model: A Case Study,” answer the following.

a. (½ point)

Define the “aging phenomenon.”

b. (1 point)

Identify the data needed to incorporate the “aging phenomenon” into a DFA Model.

36. (1½ points)

Based on D’Arcy, et al., “Using the Public Access DFA Model: A Case Study,” answer the following.

a. (1 point)

Identify and briefly describe the four insurance market conditions that are represented in the Public Access DFA Model.

b. (½ point)

Describe briefly how the insurance market evolves from one stage to another through a simulation in the Public Access DFA Model.
37. (1 point)

Based on Kirschner and Scheel, "Specifying the Functional Parameters of a Corporate Financial Model for Dynamic Financial Analysis," identify and briefly describe the two ways in which their model interrelates assets and liabilities.

38. (2 points)

Based on Kirschner and Scheel, "Specifying the Functional Parameters of a Corporate Financial Model for Dynamic Financial Analysis," answer the following.

a. (¼ point)

Describe briefly the three basic factors that are assumed to affect loss payouts.

b. (¼ point)

From the factors identified in part a. above, identify the factor that does not affect the overall amount that will be ultimately paid out.

c. (½ point)

The authors discuss a stress test that is accomplished through the modeling of the factor identified in part b. above. Identify this stress test.

d. (½ point)

Briefly describe the author's assumption regarding the correlation between successive incremental payout percentages.
39. (1 point)


You have the following information about XYZ Insurance Company:

- XYZ management will not allow the risk of a 15% or greater decline in surplus to be more than 2.5%.
- Operating profits have a normal distribution, with a mean of $0 and standard deviation of $10 million.

Based on this information, calculate the minimum surplus required by the company's management. Show all work.
40. (2 points)


a. (1 Point)

Identify and define each of these two components in an insurance context.

b. (½ point)

Briefly explain how one of these two components can be minimized by an insurer.

c. (½ point)

Lowe and Stanard indicate that in defining claim liability risk, cost should be expressed on a present value basis. For what reason, other than interest rate fluctuations, is it important to state these liabilities on a present value basis?

41. (3 points)

You have been asked to develop a DFA model for a multi-line P&C primary insurer. Your model will project results for a three-year time period. Your model will be based on the ideas presented in Correnti, Sonlin, and Isaac. "Applying a DFA Model to Improve Strategic Business Decisions."

Describe how you would simulate claim liabilities in your model. At a minimum, your answer should include discussion of (1) the steps involved in the simulation process, (2) the division of the total claim liability risk category into appropriate subcategories, and (3) the data required for the process.
a. (1 point)

According to Feldblum, "Asset Liability Matching for Property Casualty Insurers," an asset/liability matching strategy for property/casualty insurers would require investment in short-term assets. However, Feldblum indicates that insurers instead invest heavily in long-term bonds.

What are three reasons for the insurance industry's investment pattern?

b. (½ point)

According to Noris, "Asset/Liability Management Strategies for Property & Casualty Companies," what is the ramification if the Property/Casualty industry continues its traditional investment practice of investing in long-term bonds?

c. (½ point)

According to Noris, "Asset/Liability Management Strategies for Property & Casualty Companies," why would an insurance company look to avoid the problem that you've identified in part b. above?

d. (1 point)

According to Feldblum, "Asset Liability Matching for Property Casualty Insurers," asset/liability matching is but one investment concern.

What three traditional investment concerns remain paramount?
43. (1½ points)

You are given the following information about a pension plan:

- The actuarial investment rate assumed in current funding is 5%.
- The investment yield available today is 8%.
- The liability schedule is as follows:

<table>
<thead>
<tr>
<th>Period Ending</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 31, 2000</td>
<td>$ 2,000,000</td>
</tr>
<tr>
<td>December 31, 2001</td>
<td>$10,000,000</td>
</tr>
<tr>
<td>December 31, 2002</td>
<td>$15,000,000</td>
</tr>
<tr>
<td>December 31, 2003</td>
<td>$ 8,000,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$35,000,000</strong></td>
</tr>
</tbody>
</table>

Based on Fabozzi, *The Handbook of Fixed Income Securities*, calculate the expected long-term actuarial gain or loss for the pension plan at December 31, 2000. Show all work.

44. (1 point)

Based on Gorvett, "Insurance Securitization: The Development of a New Asset Class," answer the following.

a. (½ point)

Why would investors be interested in adding an insurance derivative to their portfolios?

b. (½ point)

Capacity is generally given as a primary reason for securitizing insurance risks. Gorvett, however, argues that a capacity shortfall may not exist in the property and casualty industry. Identify two points to support Gorvett's argument.
45. (1 point)

Answer the following questions based on Gorvett, “Insurance Securitization: The Development of a New Asset Class.”

a. (½ point)

Your company has purchased a 20/40 call option. Describe the basis risk associated with this purchase.

b. (½ point)

An investor has purchased a catastrophe bond. Describe the moral hazard risk faced by the investor associated with this purchase.
You are given the following probability distribution function for an insurer's incurred losses:

<table>
<thead>
<tr>
<th>Incurred Loss</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5,000</td>
<td>0.4</td>
</tr>
<tr>
<td>$10,000</td>
<td>0.3</td>
</tr>
<tr>
<td>$15,000</td>
<td>0.3</td>
</tr>
</tbody>
</table>

You are also given the following information:
- Assets are risk-less.
- Losses are booked at their expected value.

Based on Butsic, "Solvency Measurement for Property-Liability Risk Based Capital Applications," answer the following. Show all work.

a. (1 point)

Calculate the ratio of capital to incurred loss required for an EPD Ratio of 0.01.

b. (¾ point)

Calculate the ratio of capital to loss that is required if two perfectly correlated risks with the above distribution are insured.

c. (¾ point)

Calculate the ratio of capital to loss that is required if two independent risks with the above loss distribution are insured.
47. (3 points)

You are given the following information.

- A portfolio consists of long positions in call options on two stocks, A and B.
- The options on stock A have a delta, $\delta_A$, of 1,000.
- The options on stock B have a delta, $\delta_B$, of 10,000.
- The price of stock A is $50 per share.
- The price of stock B is $20 per share.
- The daily volatility of stock A, $\sigma_A$, is 1.5%.
- The daily volatility of stock B, $\sigma_B$, is 1.0%.
- The correlation between daily price changes of stocks A and B is 80%.

Based on Hull, *Options, Futures, and Other Derivatives*, answer the following.

a. (2 points)

Use the linear approximation to estimate the 10-day 99% value at risk for the portfolio. Show all work.

b. (½ point)

Explain why the linear approximation is not exact in this situation.

c. (½ point)

Will the linear approximation tend to understate or overstate the value at risk for this portfolio? Explain your answer.

CONTINUED ON NEXT PAGE
48. (3 points)

You are given the following information for an insurance company as of December 31, 2000:

<table>
<thead>
<tr>
<th>Description</th>
<th>CY 2001</th>
<th>CY 2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Admitted Assets</td>
<td>$1,500</td>
<td></td>
</tr>
<tr>
<td>Book Value of Bonds</td>
<td>$1,000</td>
<td></td>
</tr>
<tr>
<td>Market Value of Bonds</td>
<td>$1,100</td>
<td></td>
</tr>
<tr>
<td>Non-admitted Assets</td>
<td>$150</td>
<td></td>
</tr>
<tr>
<td>Statutory Loss and LAE Reserves</td>
<td>$500</td>
<td></td>
</tr>
<tr>
<td>Statutory Unearned Premium Reserve</td>
<td>$100</td>
<td></td>
</tr>
<tr>
<td>Estimated Loss and LAE Reserve Deficiency</td>
<td>$35</td>
<td></td>
</tr>
<tr>
<td>Unsecured Unauthorized Reinsurance Recoverables (Collectible)</td>
<td>$60</td>
<td></td>
</tr>
<tr>
<td>Uncollectible Authorized Reinsurance Recoverables</td>
<td>$20</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Investment Income on Loss and LAE Reserves</th>
<th>CY 2001</th>
<th>CY 2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment Income on Unearned Premium Reserves</td>
<td>$10</td>
<td>$0</td>
</tr>
<tr>
<td>Investment Income on Surplus</td>
<td>$50</td>
<td>$60</td>
</tr>
</tbody>
</table>

You are also given the following information:

- The expected loss ratio for the December 31, 2000 unearned premium reserve is 70%.
- The expense ratio for unearned premium reserve (2001 only) is 10%.
- The assumed discount rate reflecting buyers assessment of risk and corresponding risk/return requirements (reflecting cost of capital) is 15%.
- No dividends are paid from the insurer.
- All losses are paid by the end of 2002.
- Ignore taxes, new business, and the renewal of existing business.
- Assume that all cash transactions take place at the end of the year.
Based on Miccolis “An investigation of Methods, Assumptions, and Risk Modeling for the valuation of Property/Casualty Insurance Companies,” answer the following. Show all work.

a. (1½ points)

What is the adjusted net worth of the company at December 31, 2000?

b. (1 point)

What is the present value at December 31, 2000 of future earnings?

c. (½ point)

What is the economic value at December 31, 2000?
The Normal Distribution

\[ \Pr(X \leq x) = \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt \]

<table>
<thead>
<tr>
<th>x</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.5000</td>
<td>.5040</td>
<td>.5080</td>
<td>.5120</td>
<td>.5160</td>
<td>.5199</td>
<td>.5239</td>
<td>.5279</td>
<td>.5319</td>
<td>.5359</td>
</tr>
<tr>
<td></td>
<td>.5398</td>
<td>.5438</td>
<td>.5478</td>
<td>.5517</td>
<td>.5557</td>
<td>.5596</td>
<td>.5636</td>
<td>.5675</td>
<td>.5714</td>
<td>.5753</td>
</tr>
<tr>
<td></td>
<td>.5793</td>
<td>.5832</td>
<td>.5871</td>
<td>.5910</td>
<td>.5948</td>
<td>.5987</td>
<td>.6026</td>
<td>.6064</td>
<td>.6103</td>
<td>.6141</td>
</tr>
<tr>
<td></td>
<td>.6179</td>
<td>.6217</td>
<td>.6255</td>
<td>.6293</td>
<td>.6331</td>
<td>.6368</td>
<td>.6406</td>
<td>.6443</td>
<td>.6480</td>
<td>.6517</td>
</tr>
<tr>
<td></td>
<td>.6554</td>
<td>.6591</td>
<td>.6628</td>
<td>.6664</td>
<td>.6700</td>
<td>.6736</td>
<td>.6772</td>
<td>.6808</td>
<td>.6844</td>
<td>.6879</td>
</tr>
<tr>
<td></td>
<td>.6915</td>
<td>.6950</td>
<td>.6985</td>
<td>.7019</td>
<td>.7054</td>
<td>.7088</td>
<td>.7123</td>
<td>.7157</td>
<td>.7190</td>
<td>.7224</td>
</tr>
<tr>
<td></td>
<td>.7257</td>
<td>.7291</td>
<td>.7324</td>
<td>.7357</td>
<td>.7389</td>
<td>.7422</td>
<td>.7454</td>
<td>.7486</td>
<td>.7517</td>
<td>.7549</td>
</tr>
<tr>
<td></td>
<td>.7580</td>
<td>.7611</td>
<td>.7642</td>
<td>.7673</td>
<td>.7704</td>
<td>.7734</td>
<td>.7764</td>
<td>.7794</td>
<td>.7823</td>
<td>.7852</td>
</tr>
<tr>
<td></td>
<td>.7881</td>
<td>.7910</td>
<td>.7939</td>
<td>.7967</td>
<td>.7995</td>
<td>.8023</td>
<td>.8051</td>
<td>.8079</td>
<td>.8106</td>
<td>.8133</td>
</tr>
<tr>
<td></td>
<td>.8159</td>
<td>.8186</td>
<td>.8212</td>
<td>.8238</td>
<td>.8264</td>
<td>.8289</td>
<td>.8315</td>
<td>.8340</td>
<td>.8365</td>
<td>.8389</td>
</tr>
<tr>
<td></td>
<td>.8413</td>
<td>.8438</td>
<td>.8461</td>
<td>.8485</td>
<td>.8508</td>
<td>.8531</td>
<td>.8554</td>
<td>.8577</td>
<td>.8599</td>
<td>.8621</td>
</tr>
<tr>
<td></td>
<td>.8643</td>
<td>.8665</td>
<td>.8686</td>
<td>.8708</td>
<td>.8729</td>
<td>.8749</td>
<td>.8770</td>
<td>.8790</td>
<td>.8810</td>
<td>.8830</td>
</tr>
<tr>
<td></td>
<td>.8849</td>
<td>.8869</td>
<td>.8888</td>
<td>.8907</td>
<td>.8925</td>
<td>.8944</td>
<td>.8962</td>
<td>.8980</td>
<td>.8997</td>
<td>.9015</td>
</tr>
<tr>
<td></td>
<td>.9032</td>
<td>.9049</td>
<td>.9066</td>
<td>.9082</td>
<td>.9099</td>
<td>.9115</td>
<td>.9131</td>
<td>.9147</td>
<td>.9162</td>
<td>.9177</td>
</tr>
<tr>
<td></td>
<td>.9192</td>
<td>.9207</td>
<td>.9222</td>
<td>.9236</td>
<td>.9251</td>
<td>.9265</td>
<td>.9279</td>
<td>.9292</td>
<td>.9306</td>
<td>.9319</td>
</tr>
<tr>
<td></td>
<td>.9332</td>
<td>.9345</td>
<td>.9357</td>
<td>.9370</td>
<td>.9382</td>
<td>.9394</td>
<td>.9406</td>
<td>.9418</td>
<td>.9429</td>
<td>.9441</td>
</tr>
<tr>
<td></td>
<td>.9452</td>
<td>.9463</td>
<td>.9474</td>
<td>.9484</td>
<td>.9495</td>
<td>.9505</td>
<td>.9515</td>
<td>.9525</td>
<td>.9535</td>
<td>.9545</td>
</tr>
<tr>
<td></td>
<td>.9554</td>
<td>.9564</td>
<td>.9573</td>
<td>.9582</td>
<td>.9591</td>
<td>.9599</td>
<td>.9608</td>
<td>.9616</td>
<td>.9625</td>
<td>.9633</td>
</tr>
<tr>
<td></td>
<td>.9641</td>
<td>.9649</td>
<td>.9656</td>
<td>.9664</td>
<td>.9671</td>
<td>.9678</td>
<td>.9686</td>
<td>.9693</td>
<td>.9699</td>
<td>.9706</td>
</tr>
<tr>
<td></td>
<td>.9713</td>
<td>.9719</td>
<td>.9726</td>
<td>.9732</td>
<td>.9738</td>
<td>.9744</td>
<td>.9750</td>
<td>.9756</td>
<td>.9761</td>
<td>.9767</td>
</tr>
<tr>
<td></td>
<td>.9773</td>
<td>.9778</td>
<td>.9783</td>
<td>.9788</td>
<td>.9793</td>
<td>.9798</td>
<td>.9803</td>
<td>.9808</td>
<td>.9812</td>
<td>.9817</td>
</tr>
<tr>
<td></td>
<td>.9821</td>
<td>.9826</td>
<td>.9830</td>
<td>.9834</td>
<td>.9838</td>
<td>.9842</td>
<td>.9846</td>
<td>.9850</td>
<td>.9854</td>
<td>.9857</td>
</tr>
<tr>
<td></td>
<td>.9861</td>
<td>.9864</td>
<td>.9868</td>
<td>.9871</td>
<td>.9875</td>
<td>.9878</td>
<td>.9881</td>
<td>.9884</td>
<td>.9887</td>
<td>.9890</td>
</tr>
<tr>
<td></td>
<td>.9893</td>
<td>.9896</td>
<td>.9898</td>
<td>.9901</td>
<td>.9904</td>
<td>.9906</td>
<td>.9909</td>
<td>.9911</td>
<td>.9913</td>
<td>.9916</td>
</tr>
<tr>
<td></td>
<td>.9918</td>
<td>.9920</td>
<td>.9922</td>
<td>.9925</td>
<td>.9927</td>
<td>.9929</td>
<td>.9931</td>
<td>.9932</td>
<td>.9934</td>
<td>.9936</td>
</tr>
<tr>
<td></td>
<td>.9938</td>
<td>.9940</td>
<td>.9941</td>
<td>.9943</td>
<td>.9945</td>
<td>.9946</td>
<td>.9948</td>
<td>.9949</td>
<td>.9951</td>
<td>.9952</td>
</tr>
<tr>
<td></td>
<td>.9953</td>
<td>.9955</td>
<td>.9956</td>
<td>.9957</td>
<td>.9959</td>
<td>.9960</td>
<td>.9961</td>
<td>.9962</td>
<td>.9963</td>
<td>.9964</td>
</tr>
<tr>
<td></td>
<td>.9965</td>
<td>.9966</td>
<td>.9967</td>
<td>.9968</td>
<td>.9969</td>
<td>.9970</td>
<td>.9971</td>
<td>.9972</td>
<td>.9973</td>
<td>.9974</td>
</tr>
<tr>
<td></td>
<td>.9974</td>
<td>.9975</td>
<td>.9976</td>
<td>.9977</td>
<td>.9977</td>
<td>.9978</td>
<td>.9979</td>
<td>.9979</td>
<td>.9980</td>
<td>.9981</td>
</tr>
<tr>
<td></td>
<td>.9981</td>
<td>.9982</td>
<td>.9982</td>
<td>.9983</td>
<td>.9984</td>
<td>.9984</td>
<td>.9985</td>
<td>.9985</td>
<td>.9986</td>
<td>.9986</td>
</tr>
<tr>
<td></td>
<td>.9987</td>
<td>.9987</td>
<td>.9987</td>
<td>.9988</td>
<td>.9988</td>
<td>.9989</td>
<td>.9989</td>
<td>.9989</td>
<td>.9990</td>
<td>.9990</td>
</tr>
</tbody>
</table>
1. (2 points)
   
a. A competitive bid is an order for a given quantity of bills at a specific offer price.

   A noncompetitive bid is an unconditional offer to purchase bills at the average price of the successful competitive bid.

b. They may bid too high and over pay

   They may bid too low and be shut out of the auction

c. \[100,000 \times [1 - 0.5 \times (92 / 360)] = 98,722.22\]
   \[\left(\frac{100,000 - 98,722.22}{98,722.22}\right) \times (\frac{365}{92}) = .0514 \text{ or } 5.14\%\]

   or
   \[
   \frac{365 \times r_{BD}}{360 \times r_{BD} \times n} = \frac{365 \times 5\%}{360 - 5\% \times 92} = 5.14\%
   \]

2. (1½ points)

   1. The cash flow of a callable bond is not known with certainty.

   2. Because the issuer may call the bonds when interest rates have dropped, the investor is exposed to reinvestment rate risk. The investor will have to reinvest the proceeds at a lower interest rate.

   3. The capital appreciation potential of a bond will be reduced because the price of a callable bond may not rise much above the price at which the issuer may call the bond.
3. (3 points)

a) Under dividend discount models, the intrinsic value is the present value of all expected future dividends into perpetuity. The dividends are discounted at the market capitalization rate.

1. Forecasted dividends for the first 3 years:
   
   \[
   \text{Dividend} = (\text{Earnings per Share}) \times (1-\text{plowback ratio}) \\
   D_{2002} = 11.00 \times .2 = 2.20 \\
   D_{2003} = 12.50 \times .2 = 2.50 \\
   D_{2004} = 13.75 \times .2 = 2.75
   \]

2. \( g = \text{Dividend growth rate beyond 3rd year} = (\text{ROE}) \times (\text{plowback}) \) 
   \[
   = 13\% \times 0.80 = 10.4\% 
   \]

3. The market capitalization rate, \( k \), is 12% 

4. Under constant growth DDM, the intrinsic value as of 12/31/2004 is:
   
   \[
   V_{2004} = \frac{D_{2002} \times (1+g)}{k-g} \\
   = \frac{2.75 \times 1.104}{.12 - .104} \\
   = 189.75
   \]

5. The intrinsic value as of 12/31/2001 is:
   
   \[
   V_{2001} = \frac{D_{2002}}{(1+k)^1} + \frac{D_{2003}}{(1+k)^2} + \frac{D_{2004}}{(1+k)^3} + \frac{V_{2004}}{(1+k)^3} \\
   = \frac{2.20}{1.12} + \frac{2.50}{1.12^2} + \frac{2.75}{1.12^3} + \frac{189.75}{1.12^3} \\
   = 140.97
   \]

   [ In addition to this 3 period solution, there is also a similar 2 period solution and an ‘in perpetuity’ solution. ]

b) The estimate of the intrinsic value relies heavily on forecasts of future dividends, the ROE and the capitalization rate. Small changes in any of these factors can have a significant impact on the calculated intrinsic value.

c) The authors recommend performing sensitivity analysis on the input assumptions. You could calculate a range of intrinsic values under varying dividend levels, ROEs and capitalization rates. You could then make your purchase decision based on how the actual price compares to the range of intrinsic values.
4. (1 point)

1. U.S. multinationals tend to be owned by U.S. investors, who will be swayed by the ups and downs of the U.S. market.

2. While these multinationals may derive substantial revenue from abroad, most of their costs (especially labor costs) will be incurred in the U.S.

3. "The factor that drives their (multinationals) performance is their home market."

4. "Those who choose to hedge their foreign currency raise the correlation with US stocks, and so the diversification benefit won't be nearly as great."
5. (2 points)

1. The correlations between bond indexes of various countries are higher than the correlation between the stock indexes of various countries. The average correlation coefficients between countries are 0.69 for bonds and 0.40 for stocks. A higher correlation produces less reduction of risk in the portfolio.

2. The relativity of non-U.S. bond risk to U.S. bond risk is much higher than the relativity non-U.S. stock risk to U.S. stock risk. Risk is measured as the standard deviation of returns on stock and bond indexes. The greater total risk is due to the more significant impact of exchange risk on bonds.

The relativity of risk of non-U.S. bonds to U.S. bonds is very high. The value-weighted index of non-U.S. bonds had a standard deviation of 12.875 while the U.S. bond index had a standard deviation of 6.916. The risk for the composite non-U.S. bond index is 86% higher.

The relativity of risk (standard deviation of returns) of non-U.S. stocks to U.S. stocks is much lower. The value-weighted index of non-U.S. stocks had a standard deviation of 19.00 while the U.S. stock index had a standard deviation of 15.79. The risk for the composite non-U.S. stock index is only 20% higher.

This higher risk relativity of non-U.S. bonds leads to a lower amount of risk reduction for a diversified portfolio.
6. (2 points)

a. Use $\sigma_p^2 = (1/N)\sigma_j^2 + [(N-1)/N] \sigma_{jk}$ to find the average covariance, which is the minimum variance.

\[
\frac{0.5}{(32.10)} + \frac{(2-1)/2}{\sigma_{jk}} = 21.18
\]

\[
\sigma_{jk}/2 = 21.18 - 16.05 = 5.13
\]

\[
\sigma_{jk} = 10.26
\]

b. \(10.26 \times 1.05 = 10.773 > (1/N)(32.1) + [(N-1)/1](10.26)\)

\[10.773N > 32.1 + 10.26(N-1)\]

\[0.513N > 21.84\]

\[N > 42.57\]

Therefore need at least 43 securities to be within 5% of minimum variance.
7. (2 points)

a. \[ \sigma_p^2 = \beta_p^2 \sigma_m^2 + (1/N)[\Sigma(1/N) \sigma_{e_j}^2] \]

The last term on the right can be expressed as 1/N times the average residual risk for the portfolio

\[ (1/N)[\Sigma(1/N) \sigma_{e_j}^2] < 0.01 \sigma_{e_j}^2 \]

\[ 1/N < 0.01 \]

\[ N > 100 \]

Therefore need 101 securities

b. \[ \sigma_p^2 = \beta_p^2 \sigma_m^2 + (1/N)[\Sigma(1/N) \sigma_{e_j}^2] \]

As N goes to infinity then residual term goes to zero

\[ \sigma_p^2 = \beta_p^2 \sigma_m^2 = (1.5)^2(4) = 9 \]
Step 1: Calculate revised C's (Pages 184 – 185)

Assume an arbitrary market variance: 10%

\[
\begin{align*}
\text{Security} & & \bar{R}_i - R_f & & \beta_i & & \frac{\beta_i^2}{\sigma^2_{\epsilon_i}} & & \sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma^2_{\epsilon_j}} & & \sum_{j=1}^{i} \frac{\beta_j^2}{\sigma^2_{\epsilon_j}} & & C_i = \frac{\sigma^2_M (\text{Col5})}{[1 + \sigma^2_M (\text{col6})]} \\
A & & 6.0 & & 0.1500 & & 0.0250 & & 0.1500 & & 0.0250 & & 1.200 \\
C & & 4.5 & & 1.8000 & & 0.4000 & & 1.9500 & & 0.4250 & & 3.714 \\
E & & 4.0 & & 0.5333 & & 0.1333 & & 2.4833 & & 0.5583 & & 3.772 \\
B & & 3.0 & & 0.0750 & & 0.0250 & & 2.5583 & & 0.5833 & & 3.744 \\
D & & 2.0 & & 0.0800 & & 0.0400 & & 2.6383 & & 0.6233 & & 3.647
\end{align*}
\]

Step 2: Calculate New weights (Pages 188 – 189)

\[
\begin{align*}
\text{Security} & & B_i/\sigma^2_{\epsilon_i} & & (\bar{R}_i - R_f)/B_i & & C^* & & (2)[(3) - (4)] & & (5) / \Sigma(5) \\
A & & 0.0250 & & 6.0 & & 3.772 & & 0.0557 & & 0.257 \\
C & & 0.2000 & & 4.5 & & 3.772 & & 0.1456 & & 0.673 \\
E & & 0.0667 & & 4.0 & & 3.772 & & 0.0152 & & 0.070
\end{align*}
\]
9. (1 point)

1. Economic shocks, such as oil price hikes, can cause a simultaneous increase in the inflation rate and decline of expected real earnings (and dividends). This would result in a negative correlation between inflation and real stock returns.

2. The higher the rate of inflation, the riskier real stock returns are perceived to be. The reasoning is that higher inflation is associated with greater uncertainty about the economy, which tends to induce a higher required rate of return on equity ($k$). The higher $k$ implies a lower level of stock prices.

3. Higher inflation results in lower real dividends because our tax system results in lower after-tax real earnings as the inflation rate rises.

4. There is a view that many investors in the stock market suffer from a form of “money illusion”. Investors mistake the rise in the nominal rate of interest for a rise in the real rate. As a result, they undervalue stocks in a period of higher inflation.
1. Note that M is the intercept at the point of tangency.

2. 

\[
\bar{R}_e = R_F + \frac{\bar{R}_M - R_F}{\sigma_M} \sigma_e
\]

\(\bar{R}_e\) = The return on the efficient portfolio.

\(R_F\) = The risk free rate.

\(\bar{R}_M\) = The expected return on the market.

\(\sigma_e\) = The standard deviation of the market return.

\(\sigma_M\) = The amount of risk in the efficient portfolio.
11. (2 points)

1. Strong performance of CAPM would be demonstrated by clustering of points close to the 45% axis that is drawn, representing that fitted returns are consistent with realized returns. This graph demonstrates that fitted returns are uncorrelated with realized returns.

2. In addition to the standard security betas estimated using the value-weighted stock market index, JW also estimated the beta of assess with respect to labor income growth and they considered the possibility that business cycles affect asset betas.

Therefore, the two variables that they identified were:

- State of the economy and
- Market value of Equity, ME
12. (1½ points)

a. 

\[ E(R_i) = R_F + \beta_i(E(R_M) - R_F) - \tau(\delta_i - R_F) + \tau(\delta_i - R_F) \]

- \( R_F \) is the risk-free rate
- \( R_M \) is the return on the market portfolio
- \( \beta_i \) is the expected change in a security \( i \) given the change in \( R_M \)
- \( \tau \) is a tax factor that is a complex function of the tax rate and wealth
- \( \delta_M \) is the dividend yield of the market portfolio
- \( \delta_i \) is the dividend yield for stock \( i \)

As \( \tau \) is positive, expected return is an increasing function of dividend yields.

b. 

Expected returns are an increasing function of dividend yields. This is due to the fact that dividends are taxed at a higher rate than capital gains.

13. (2 points)

a. \( \sigma_i^2 = \beta_{im}^2 \sigma_m^2 + \beta_{ij}^2 \sigma_j^2 + \beta_{ik}^2 \sigma_k^2 + \text{Var}(\epsilon_i) \)

\[ \sigma_i^2 = (0.8^2)(25) + (1.3^2)(30) + 17 \]

\[ \sigma_i^2 = 83.70 \]

\[ \sigma_i^2 = (1.2^2)(25) + (-0.4^2)(42) + 21 \]

\[ \sigma_i^2 = 63.72 \]

b. For firms in different industries the covariance is

\[ \sigma_{12} = b_{1m}b_{2m}\sigma_m^2 = (0.8)(1.2)(25) \]

\[ \sigma_{12} = 24 \]
14. (2 points)

Calculate the current prices of each bond

Bond A: \( \frac{1000}{1.05} = 952.38 \)
Bond B: \( \frac{500}{1.04} + \frac{500}{(1.04)^2} = 943.05 \)
Bond C: \( \frac{1000}{(1.03)^2} = 942.60 \)

Market value of portfolio = 2838.03

YTM of a portfolio = interest rate that equates cash flows with portfolio’s market value.

\[ 2838.03 = \frac{1500}{1 + \frac{y}{2}} + \frac{1500}{(1 + \frac{y}{2})^2} \]

Use quadratic formula to solve for \( y \), which gives

\[ \frac{y}{2} = 0.0378 \]
\[ y = 0.0756 \text{ or } 7.56\% \]

This yield is less than the 7.8% required by the insurer, so the insurer should not buy the portfolio.

Alternatively, could plug \( y = 7.8\% \ (\frac{y}{2} = 3.9\%) \) into equation above:

\[ 1500/1.039 + 1500/((1.039)^2) = 2833.20 \]

This is less than 2838.03, or the market value of the portfolio which implies the yield is less than 7.8%, so the insurer should not buy the portfolio.
15. (2 points)
   a. Would be appropriate when 2 bonds are exactly the same
      - Due to market inefficiencies/mispricing; however, one of the 2 bonds has a higher yield.
      Therefore it would be good to swap into the bond with the higher yield.
   
   b. When the spreads between 2 different sectors of the market are out of line with historical spreads
      between the sectors. If a sector has a higher yield spread then historically, swap for one of those
      bonds, under the impression that the spread will realign in the future.

16. (2 points)

<table>
<thead>
<tr>
<th></th>
<th>1/1/01</th>
<th>7/1/01</th>
<th>7/1/02</th>
<th>1/1/03</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>96</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>88</td>
<td></td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>125</td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

   \[
   S_{01} = \left( \frac{100}{96} \right)^{\frac{1}{1}} - 1 \cdot 2 = .083
   
   S_{03} = \left( \frac{100}{88} \right)^{\frac{1}{3}} - 1 \cdot 2 = .087
   
   S_{04} = \left( \frac{150}{125} \right)^{\frac{1}{4}} - 1 \cdot 2 = .093
   
   F_{34} = \left\{ \frac{\left[ \frac{1+(S_{03}/2)}{1+(S_{03}/2)} \right]^4}{1+(S_{03}/2)} - 1 \right\} \cdot 2 = .112
   
   or
   
   F_{34} = \left\{ \frac{(150/125)}{(100/88)} - 1 \right\} \cdot 2 = .112
17. (2 points)

1. Callability - Issuer has the right to call the bond for a price specified in the bond agreement. The bond must have a higher yield to compensate for the risk that the bond is called in unfavorable conditions (lower market interest rates).

2. Convertibility - Bondholder has the option to convert bond into a number of shares of company stock (as specified in bond agreement). Because this option is valuable to the holder, the required yield will be lower.

3. Sinking Fund - Issuer sets aside money to retire a certain portion of the bond issue each year. Because the investor can have its bond called and retired as part of the sinking fund, a higher yield is required (same reasoning as callability). However, the gradual retirement of debt reduces default risk and thus would somewhat lower the required yield.
18. (4 points)

a. Sample solutions- There are 2 solutions
   Method1 – do weighted average of individual bond durations
   Method2 – calculate portfolio duration

**Method 1 Sample Solution**

1. Calculate the new prices of the bonds for a +100 basis point change:
   
   Bond A: \( 4\left(1 – v^{10}/i\right) + 100v^{10} = \$96.04 \) \hspace{1cm} \text{where } i=4.5\% \hspace{1cm} v = 1/(1 + i) 
   
   Bond B: \( 4.5\left(1 – v^{20}/i\right) + 100v^{20} = \$88.05 \) \hspace{1cm} \text{where } i=5.5\% \hspace{1cm} v = 1/(1 + i) 

2. Calculate the new prices of the bonds for a -100 basis point change:
   
   Bond A: \( 4\left(1 – v^{10}/i\right) + 100v^{10} = \$104.16 \) \hspace{1cm} \text{where } i=3.5\% \hspace{1cm} v = 1/(1 + i) 
   
   Bond B: \( 4.5\left(1 – v^{20}/i\right) + 100v^{20} = \$100.00 \) \hspace{1cm} \text{where } i=4.5\% \hspace{1cm} v = 1/(1 + i) 

3. Calculate durations of individual bonds using formula on page 99
   
   Duration = \[V_0 - V_+]/[2(V_0)(\Delta y)]
   
   Bond A: \([104.16-96.04]/[2(100)(.01)] = 4.06\)
   
   Bond B: \([100.00-88.05]/[2(93.77)(.01)] = 6.37\)

1. Determine the weights to be used to determine the portfolio duration
   
   Market value of Bond = (Amount of par value held) x (Mkt Value)/(Par Value)

   Market value of Bond A = ($3 million) x (100/100) = $3,000,000
   
   Market value of Bond B = ($8 million) x (93.77/100) = $7,501,600

   Weight for Bond A = \(3,000,000 / (3,000,000 + 7,501,600) = 0.286\)
   
   Weight for Bond B = \(1.0 – 0.286 = 0.714\)

2. Calculate duration of portfolio
   
   \[(0.286 \times 4.06) + (0.714 \times 6.37) = 5.709\]
Method 2 Sample Solution

1. Calculate the new prices of the bonds for a +100 basis point change:

   Bond A: $96.04  \quad \text{where } i=4.5\% \quad v = 1/(1 + i)$
   $4[(1 – v^{10})/i] + 100v^{10}$

   Bond B: $88.05  \quad \text{where } i=5.5\% \quad v = 1/(1 + i)$
   $4.5[(1 – v^{20})/i] + 100v^{20}$

2. Calculate the new prices of the bonds for a -100 basis point change:

   Bond A: $104.16  \quad \text{where } i=3.5\% \quad v = 1/(1 + i)$
   $4[(1 – v^{10})/i] + 100v^{10}$

   Bond B: $100.00  \quad \text{where } i=4.5\% \quad v = 1/(1 + i)$
   $4.5[(1 – v^{20})/i] + 100v^{20}$

3. Calculate value of bond portfolio for a +100 basis point change, a -100 basis point change and the market value.

   +100 value = $3,000,000 \times (96.04/100) + 8,000,000 \times (88.05/100) = 9,925,200$

   -100 value = $3,000,000 \times (104.16/100) + 8,000,000 \times (100/100) = 11,124,800$

   market value = $3,000,000 \times (100/100) + 8,000,000 \times (93.77/100) = 10,501,600$

4. Calculate durations of portfolio using formula on page 99

   $\text{Duration} = [V_+ - V_-]/[2(V_0)(\Delta y)]$

   $[11,124,800-9,925,200]/[2(10,501,600)(.01)] = 5.71$

   b. For Bond A, convexity = $[96.04 + 104.16 – 2(100)]/(2(100)(.01)^2) = 10.0$

   c. At very low yield rates, the put option on Bond C is unlikely to be exercised. Without the put option, Bond C will be very similar to Bond A, and therefore the effective durations will move closer together.
a. Under the segmented market theory, investors are risk adverse and they will only invest in their
desired maturity spectrum. No yield differential will induce them to change maturities.

Under pure expectations theory, investors are risk neutral and they will select securities with the
highest return, no matter what the time horizon is. This is the exact opposite of the segmented
market theory.

b. The liquidity premium theory argues that there is a shortage of longer-term investors so that an
extra return (premium) must be offered on long-term bonds to induce investors to hold them.

“An investor with a long-term horizon can hold a bond matching his or her horizon and earn the
liquidity premium. Thus, such an investor earns an extra return without any additional risk”

The authors imply that life insurers have a long-term horizon because their liabilities are long
term in that they are unlikely to require payment for a long time.
a. Original Issuers, Venture Capital situations, Story Bonds – includes young, growing concerns lacking the stronger balance sheet and income statement profile of established corporations

Fallen Angels – companies with investment grade that which have come upon hard times.

Restructurings and leveraged buyouts – companies that have deliberately increased their debt burden with a view towards maximizing shareholder value. Situations where companies have issued debt to retire debt, or issued debt to pay dividends.

b. Market Inefficiencies – the fixed income market has been mis-pricing debt issues

Liquidity Risk – lower rated bonds are less liquid and therefore must be priced to provide a higher yield

Variability in retention values after default

Investor restrictions – certain institutions may be prohibited from investing in low-grade bonds reducing demand

Interest rate risk

Reinvestment risk

c. There has been tremendous growth in new low-rated debt issues over the historical period studied, so the age composition of the outstanding bonds is very young. Assuming that default experience increases the longer a bond is outstanding, then as new issues become a smaller percentage of the outstanding bonds, then the age composition will rise which will lead to a corresponding increase in the default rate.

The low-rated new issue bond market has been dominated by one firm – Drexel Burnham. It has been argued that with relatively few firms in financial difficulty Drexel Burnham profited by losing money on deals that helped failing firms refinance in order to hold down the bankruptcy experience of low-rated debt. Without this protection (and Drexel Burnham is now bankrupt) it has been argued that default rates will rise.
21.  (1 point)

**Method 1**

Calculate the yield on a discounted basis:

\[ Y_d = \left(\frac{F - P}{F}\right) \times \frac{360}{t} = \left(\frac{100 - 94.5}{100}\right) \times \frac{360}{364} = 5.44\% \]

The stop-out yield in the highest yield accepted.

Since your yield is lower than the stop-out yield, the bid will be accepted.

**Method 2**

Calculate the price of the treasury at the stop-out yield on discounted basis:

\[ \text{Price} = F - (F \times Y_d \times \frac{t}{360}) = 100 - (100 \times 0.056 \times \frac{364}{360}) = 94.34 \]

Since your bid price is higher than the stop-out bid price, the bid will be accepted.

22.  (2 points)

a. A repurchase agreement is the sale of a security with a commitment by the seller to buy back the security from the purchaser at a specified price at a later date.

b. (1) Use of a margin – The amount loaned is less than the value of the market value of the security used as collateral.
   (2) Mark the collateral to market – When market values change by a certain percentage, the repo position is adjusted accordingly either through the use of a margin call or by repricing the repo agreement.
   (3) The borrower delivers the collateral to the lender.
   (4) The lender allows the borrower to hold the collateral in a segregated customer account.
   (5) The borrower delivers the collateral to the lender’s custodial account at the borrower’s clearing bank.

c. The repo rate involves collateralized borrowing, whereas a federal funds transaction is unsecured borrowing and has a higher credit risk exposure.
23. (2 points)

a. $(100 \text{ million}) \times (1 + .085)^6 = $163,146,751

b. Price of bond = \((40 \times [(1 - v^{50})/i]) + 1000/[(1 + i)^{50}]\) where \(i=5\%\) \(v = 1/(1 + i)\)
\[= $817.44\]

Number of bonds purchased = \(100,000,000/817.44 = 122,333\)

c. After interest rate change, the bond price increases to $1,000.
Therefore, the portfolio value is \(122,333 \times $1,000 = $122,333,000\)

After the interest rate change, the PV of the new minimum terminal value is:
\[\$163,146,751/(1.04^{12}) = $101,900,979\]

The new safety margin is \($122,333,000 - $101,900,979 = $20,432,021\)

Since the portfolio value is currently worth more than the present value of the minimum terminal value, \textit{don't immunize}.\]
24. (2½ points)

a. \( \text{AIR} = \text{Assumed investment return} = 5\% \)

\( R_t = \text{rate of return underlying portfolio in year } t \)

Calculate \( B_{1996} \), the benefit received at end of 1996

First, solve for \( B_{1994} \), the hypothetical constant benefit at start of payout period

\[
(B_{1994}) \times [(1 - v^10)/i] = 2000,000 \quad \text{where } i=5\% \quad v = 1/(1 + i)
\]

\[
B_{1994} = (2000,000 \times 0.05) / (1 - 1.05^{-10}) = $25,900.91
\]

Then, \( B_{1995} = B_{1994} \times [(1 + R_{1995})/(1 + AIR)] \)

\[
= 25,900.91 \times (1.04/1.05) = $25,654.23
\]

Finally, \( B_{1996} = B_{1995} \times [(1 + R_{1996})/(1 + AIR)] \)

\[
= 25,654.23 \times (1.06/1.05) = $25,898.56
\]

b. Balance after 12/31/95 payment = 200,000(1.04) – 25,654.23 = $182,345.77

Balance after 12/31/96 payment = 182,345.77(1.06) – 25,898.56

\[
= $167,387.96.
\]
25. (1 point)

a. If futures price remained above spot price, investors would short futures contracts and buy underlying asset – this would eventually drive down the futures price.

b. If futures price remained below spot price, investors would all be looking to buy futures contracts – demand would drive up price of futures contracts.

26. (3 points)

a. \[ F_0 = S_0 e^{(r-d)T} \]

\[ = .75 e^{(.05-.08)2} \]
\[ F_2 = .706 \]

b. You want to borrow American dollars, buy Canadian dollars, and enter a short forward contract on Canadian dollars.

c. Borrow \(0.75 e^{-0.08(2)} = 0.639\) US Dollars; buy \(e^{-0.08(2)}\) of Canadian Dollars spot; at end of two years, the investment is worth 1 Canadian dollar which you would use to close out the forward contract. You would receive 0.85 and pay back the loan for \((0.639)(e^{0.05}) = 0.706\) for an arbitrage profit of \(0.85 - 0.706 = 0.143\).
27. (3 points)

a. FRA: an agreement to pay a specified interest rate at a future time period on a specified principal amount.

b. \[ V = L (R_F - R_K) (T_2 - T_1) e^{-R_F T_2} \]

\[ R_F = .10 \]

\[ (1 + R_K) = \frac{e^{.08(3)}}{e^{.075(2)}} \Rightarrow R_K = 9.42 \]

\[ V = 2,000,000 (0.10 - 0.0942) (3 - 2) e^{-0.08(3)} = 9,124.88 \]

28. (1½ points)

a. buy a call at \( X_1 \)

buy a call at \( X_2 \)

sell 2 calls at \( X_3 \)

\[ X_2 = X_1 + X_3 \]

\( X_2 \) is close to the stock price, \( S_0 \) at purchase.

b. The stock price will not move very much (see graph) from \( S_0 \), which is close to \( X_2 \).
29. (4 points)

a. First, determine the “tree” of values and probability of up move:

\[
\begin{align*}
&110 \times 1.1 = 121 \\
&100 \times 1.1 = 110 \\
&100 \times .90 = 90 \\
&90 \times .90 = 81 \\
\end{align*}
\]

\[
p = \frac{e^{rT} - d}{u - d}
\]

where

\[
\begin{align*}
u & = 1.1 \\
d & = .90 \\
e^{rT} & = 1.03
\end{align*}
\]

(note: we are using semi-annual compounding)

so, \( p = .65 \), and \( 1 - p = .35 \)

Next, work backward through the tree, assuming European exercise:

\[
\begin{align*}
F_{uu} & = Max(115 - 121,0) = 0 \\
F_{ud} & = Max(115 - 99,0) = 16 \\
F_{dd} & = Max(115 - 81,0) = 34 \\
F_u & = \frac{1}{1.03} \times [.65 \times 0 + .35 \times 16] = 5.44 \\
F_d & = \frac{1}{1.03} \times [.65 \times 16 + .35 \times 34] = 21.65 \\
F & = \frac{1}{1.03} \times [.65 \times 5.44 + .35 \times 21.65] = 10.79
\end{align*}
\]
SAMPLE ANSWER FOR QUESTION 29 continued:

Finally, substitute the immediate exercise payoff for the calculated value above wherever it is greater, and recalculate the values:

\[ F_u = \max(115 - 110, 5.44) = 5.44 \]
\[ F_d = \max(115 - 90, 21.65) = 25.00 \]
\[ F = \max(115 - 100, \frac{1}{1.03} \times [0.65 \times 5.44 + 0.35 \times 25.00]) = \max(15, 11.65) = 15.00 \]

Answer: Value = $15.00

Early exercise indicated at t=0, and also if the move is down at t= 6 months

b. From the answer to a, we have:

\[ F_u = \max(115 - 110, 5.44) = 5.44 \]
\[ F_d = \max(115 - 90, 21.65) = 25.00 \]

Now, the delta over the first step is:

\[ \Delta = \frac{F_u - F_d}{S_u - S_d} \]

So,

\[ \Delta = \frac{5.44 - 25.00}{110 - 90} = -0.978 \]

To create a hedge over the first time step against a single short American put option, you would short (sell) 0.978 shares of the underlying stock.
30. (2 points)

Surplus movements for the first six months are governed by:

\[ dS = dt + 2dz \]

\[ a_1 = 1.0 \]
\[ b_1 = 2.0 \]

Surplus movements over the second six months are governed by:

\[ dS = 1.5dt + \sqrt{10}dz \]

\[ a_2 = 1.5 \]
\[ b_2 = \sqrt{10} \]

In both cases, time is in units of month

Since month-to-month movements are independent by virtue of the Weiner process, we have:

Expected move over 12 months = sum of expected moves for each month

Variance over 12 months = sum of variances for each month

\[ E[\Delta S] = 6 \times 1.0 + 6 \times 1.5 = 15.0 \]
\[ Var[\Delta S] = 6 \times 4 + 6 \times 10 = 84 \]
\[ SD[\Delta S] = \sqrt{84} \]

Furthermore, by virtue of the Weiner process, \( \Delta S \) is normally distributed.

Now, we need

\[ P[S_0 + \Delta S > 0] = .99 \]
\[ P[\Delta S > -S_0] = .99 \]
\[ P\left[ \frac{\Delta S - 15}{\sqrt{84}} > \frac{-S_0 - 15}{\sqrt{84}} \right] = .99 \]
\[ P\left[ \frac{\Delta S - 15}{\sqrt{84}} < \frac{-S_0 - 15}{\sqrt{84}} \right] = .01 \]
\[ \Phi \left[ \frac{-S_0 - 15}{\sqrt{84}} \right] = .01 \]
\[ \frac{-S_0 - 15}{\sqrt{84}} = -2.3263 \]
\[ S_0 = 6.321 \]
31. (3 points)

\[ d_1 = \frac{\ln(S_0/X) + r\tau_2 + \sigma^2\tau_1/2}{\sigma\sqrt{\tau_1}} = \frac{\ln(52/50) + (0.12 + 0.3^2/2)*0.25}{0.30\sqrt{0.25}} = 0.5365 \]

\[ d_2 = \frac{\ln(S_0/X) + r\tau_2 - \sigma^2\tau_1/2}{\sigma\sqrt{\tau_1}} = d_1 - \sigma\sqrt{\tau_1} = 0.5365 - 0.30\sqrt{0.25} = 0.3865 \]

\[ c = S_0N(d_1) - Xe^{-r\tau_2}N(d_2) = 52N(0.5365) - 50e^{-0.12*0.25}N(0.3865) \]

\[ = 52 * 0.7042 - 50e^{-0.03*0.6504} \]

\[ = $5.06 \]
32. (3 points)

a. The two-factor APT model

\[ R_i = a_i + b_{i1} F_1 + b_{i2} F_2 + e_i \]

\[ R_i = R_F + b_{i1}(F_1 - R_F) + b_{i2}(F_2 - R_F) + e_i \]

As the economic factors are uncorrelated and all \( e_i \)'s have mean zero and are uncorrelated with the economic factors, the formulas for the covariances are as follows:

\[ \sigma_{AB} = b_{A1}b_{B1} \sigma_1^2 + b_{A2}b_{B2} \sigma_2^2 \]
\[ \sigma_{AC} = b_{A1}b_{C1} \sigma_1^2 + b_{A2}b_{C2} \sigma_2^2 \]

Solving the two equations:

\[ 35.53 = (1.21)(0.68)(76)+(0.8)(X)(45) \]
\[ \rightarrow X = -0.75 \]

\[ 69.50 = (1.21)(Y)(76)+(0.80)(0.50)(45) \]
\[ \rightarrow Y = 0.56 \]

b. To find the risk premiums requires taking the expectation on both sides of the APT model.

\[ E(R_i) - R_F = b_{i1}[E(F_1) - R_F] + b_{i2}[E(F_2) - R_F] \]

Substituting into the formula above:

\[ E(R_A) - R_F = (1.21)(3.25)+(0.80)(2.50) \]
\[ \rightarrow \text{Risk premium for } A = 5.93 \]

\[ E(R_B) - R_F = (0.68)(3.25)+(-0.75)(2.50) \]
\[ \rightarrow \text{Risk premium for } B = 0.34 \]

\[ E(R_C) - R_F = (0.56)(3.25)+(0.50)(2.50) \]
\[ \rightarrow \text{Risk premium for } C = 3.07 \]

c. The proportions of the stocks A, B, and C in the portfolio are:
\( \omega_A = 20/120 \)
\( \omega_B = 70/120 \)
\( \omega_C = 30/120 \)

Hence the sensitivities to \( F_1 \) and \( F_2 \) are:

\[
\begin{align*}
\beta_{p1} &= \omega_A b_{A1} + \omega_B b_{B1} + \omega_C b_{C1} \\
&= (1.21)(20/120) + (0.68)(70/120) + (0.56)(30/120) \\
&\rightarrow \beta_{p1} = 0.738
\end{align*}
\]

\[
\begin{align*}
\beta_{p2} &= \omega_A b_{A2} + \omega_B b_{B2} + \omega_C b_{C2} \\
&= (0.80)(20/120) + (-0.75)(70/120) + (0.50)(30/120) \\
&\rightarrow \beta_{p2} = -0.179
\end{align*}
\]
33. (1 point)

a. How can the exposure be quantified? OR What data is available?
   What risk exposures will be modeled

b. The interaction between risk exposures/factors.
   OR

   How complex should the model be?
   OR

   Should the model incorporate any level of management intervention / feedback loops?
   OR

   How should the data be incorporated that is external to the individual company being modeled?

34. (1½ points)

a. Feedback loops are automated conditional decisions, which are algorithms that make calculations
   for each modeled time period dependent on values calculated for earlier time periods.

b. The models may thus be over-determined, showing little risk regardless of the scenario because of
   the assumption of the appropriate management responses.

   In reality management may not take or be able take appropriate actions.
35.  (1½ Points)

   a. Under the aging phenomenon, loss ratios gradually decline with the length of time the policies have been in force with the company.

   b. Exposure/premium data

   Loss information

   Renewal retention ratios

   All of the above broken down by age of business

36.  (1½ Points)

   a. Mature hard market: price increases can be taken with minimal effect on market share
      Mature soft market: prices increases significantly reduce market share
      Immature hard market: the market is starting to harden
      Immature soft market: the market is starting to soften.

   b. For each of the four cycle conditions, the probability of moving to another condition in the cycle is specified as an input. This probability distribution represents the chance of remaining in that stage or of moving to another stage for the next year.

      Start in the mature hard market move to immature soft to mature soft to immature hard to mature hard.
37. (1 Point)

First, the amount of money available for reinvestment at any point in time is directly related to the underwriting cash flows. A severe underwriting shock such as a catastrophe will force much greater loss outflows than anticipated, with a corresponding need to liquidate assets.

Second, the interest rate and/or inflation rate environment affecting asset market values is linked into the liability cash flow profiles.

38. (2 points)

a. (1/4 pt.) Accept relating to timing of loss payout, settlement pattern. Do not accept loss development or "timing of loss" or "timing", since unclear it applies to payout (some candidates talked about incurred or reported loss development/timing, so it wasn't fair to make the assumption candidate was referring to payments if they didn't say so). Accept "timing" if part b) makes it clear that timing refers to payout.

b. The model is structured so that variability in the timing of loss payments does not affect the overall amount that ultimately will be paid out, but it does affect when the payments occur.

c. Imposing variability on the loss payout pattern stress tests the company’s asset liquidity.

d. The authors generally elected to ignore the possibility of correlations between successive incremental payout percentages.
39. (1 point)

\[ Pr \{ \text{operating profit} < -15\% \text{ of Surplus} \} = 2.5\% \]

Operating Profit \( \sim N(\mu, \sigma^2) \)

\( \mu = 0 \)

\( \sigma = 10 \text{ million} \)

\( Z_{0.025} = -1.96 \)

2.5\% chance of operating profits \( < -1.96 \times 10 \text{ million} = -19.6 \text{ million} \)

So, \(-19.6 \text{ million} \) would constitute exactly a 15\% surplus hit at the minimum surplus of:

\[
\frac{-19.6 \text{ million}}{15\%} = 130.67 \text{ million}
\]

40. (2 points)

a. Process risk - the random nature of claim events.
   Parameter risk - the inability to know the claim frequency and severity distributions from which the events are drawn. -and/or-
   Currency Exchange risk – movements in exchange rates that affect conversion costs – ie. the cost of converting liabilities from the local contract currency to the owner’s currency.

b. For most lines of insurance, a company can write sufficient volumes of business to diversify away process risk,

c. Claim liability risk includes the timing of claim cash flows. Changes in cash flow timing cannot be measured by looking at nominal values. The impact of differing payment patterns can be measured by evaluating the corresponding PV amounts.
Liability simulation should consider both the existing reserves and the company's business plan. Like asset categories, existing reserves and new business liabilities can be broken down into homogeneous lines of business to ensure that the unique characteristics of each line are captured. Historical experience and expected future trends need to be reflected in the assumptions to capture how the insurance company's liability structure will develop in the future.

Projections of the existing loss reserves are generated stochastically by assuming an underlying distribution for the loss reserves and inputting an expected reserve runoff pattern.

It is imperative to model the company's new business plain in order to accurately reflect the company's complete liability structure in the future.

In order to project the new business liability cash flows, assumptions regarding written and earned premium, loss ratios, expected accident year payout patterns, discount/interest rates, IBNR factors and expenses are needed. Loss ratios should be modeled so as to reflect relationships with the underlying economic environment and should be general enough to allow the user to incorporate cycles and reversions.

Loss ratios and cash flows attributed to catastrophes can be generated using one of these simulation models (AIR, EQE, RMS,…) and merged with the non-cat losses described above to produce the company's overall loss ratio distribution.

Once the projected loss ratios are determined, the total liability cash flows are calculated by multiplying the generated loss ratio by the forecasted earned premium and accident year payout pattern. The carried reserves can then be calculated as a function of the ultimate loss reserve, the expected loss reserve and the appropriate IBNR factor. It is important to recognize that since each line of business has its own characteristics, all of the above projections need to be performed on a line-by-line basis before being aggregate to a total company level.
42. (3 points)

a. Acceptable responses
   - **Higher Return**: High quality, long-term corporate bonds usually offer 2 to 5 percentage points more than short-term commercial paper.
   - **Lower Transaction Costs**: Long-term bonds entail the least expenses, especially if the bonds are held to maturity.
   - Property/Casualty insurers do not face the same **disintermediation** problems that life insurers face.
   - Interest rates have a minor influence on insurance cash flows, since premium income does not vary greatly with investment returns for most lines of business.
   - Real estate and similar investments are **limited by regulation**.
   - Long-term Bonds appear less volatile under statutory accounting rules, because they're valued at amortized cost instead of market value. "Less volatile" is unacceptable and incorrect. "Less volatile due to accounting rules" gets partial credit.
   - P&C insurers can fund loss payments from current cashflows, so no need to liquidate long term investments to fund loss payments.

b. “MVS will continue to be highly volatile.” Also accept MVS exposure to interest rate risk, since this is the real problem with MVS volatility.

Volatility of surplus due to interest rate risk in not acceptable, since it refers to statutory surplus. Bond amortization masks interest rate risk in statutory surplus, and the author argues that insurers typically don't have to liquidate assets to pay losses.

c.
   - “MVS is a leading indicator of the future book value of the firm, since book value converges to market value as asset and liability items mature.”
   - “Managing the MVS will thus enable company management to better serve the goals of the company’s owners – that is, higher future stock prices for stockholders or higher future dividends for policyholders as the case may be.”
   - Regulatory concerns.

d. Maximizing expected returns
   1. Ensuring safety of principal
   2. Balancing the risks of each class of securities. (also accept asset allocation, diversification)
43. (1½ points)

Portfolio cost = market value of liabilities at today’s investment yield

<table>
<thead>
<tr>
<th>Date</th>
<th>Amount</th>
<th>Discount @ 8%</th>
<th>Disc Amt @ 8%</th>
<th>Discount @ 5%</th>
<th>Disc Amt @ 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/31/2000</td>
<td>2,000</td>
<td>1.000</td>
<td>2,000</td>
<td>1.000</td>
<td>2,000</td>
</tr>
<tr>
<td>12/31/2001</td>
<td>10,000</td>
<td>0.926</td>
<td>9,259</td>
<td>0.952</td>
<td>9,523</td>
</tr>
<tr>
<td>12/31/2002</td>
<td>15,000</td>
<td>0.857</td>
<td>12,860</td>
<td>0.907</td>
<td>13,605</td>
</tr>
<tr>
<td>12/31/2003</td>
<td>8,000</td>
<td>0.794</td>
<td>6,352</td>
<td>0.864</td>
<td>6,912</td>
</tr>
<tr>
<td>Total</td>
<td>35,000</td>
<td>0.794</td>
<td>30,471</td>
<td>0.864</td>
<td>32,040</td>
</tr>
</tbody>
</table>

Actuarial savings = $32,040 – $30,471 =  $1,569

44. (1 point)

a. Investment in catastrophe exposures (through cat bonds) should be desired by capital markets because it will generally be uncorrelated with existing portfolios.

b.  
i) “One reason the [insurance] industry has built up its net worth is to address future potential catastrophes.”

ii) “…Although a large catastrophe could result in significant loss to property, the proportion of property values actually covered by insurance is low in certain places and with regard to certain coverages, e.g., earthquake and flood insurance is far from universal.” (Underlining added for emphasis.)

iii) “… new capital enters an industry when rates are high.”
45. (1 point)

a. Basis risk is the risk that the underlying asset will change more or less than the asset you are trying to protect. For catastrophe options, this happens if you have an industry trigger or event trigger. In that case, the industry loss may not be as great as your own company’s loss and the value of the option wouldn’t offset your loss enough.

b. The moral hazard is that the insurance company issuing the bond will inflate the claims to trigger the non-payment of the bond coupons (and possibly principal).
46. (2½ points)

a.  \( E (Loss) = .4 \times 5,000 + .3 \times 10,000 + .3 \times 15,000 = 9,500 = \text{Incurred Loss} \)

\[
\text{EPD} = \text{EPD Ratio} \times \text{Incurred Loss} = .01 \times (9,500) = 95
\]

\( y = \text{Assets} \)

\[
(15,000-y) \times .3 = 95
\]

\( \text{Assets} = 14,683 \)

\( \text{Capital} = 14,683 - 9,500 = 5,183 \)

\[
\text{Ratio} = \frac{5,183}{9,500} = 55\%
\]

b. Answer is same 55%, but math is as follows:

<table>
<thead>
<tr>
<th>Loss</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>.4</td>
</tr>
<tr>
<td>20,000</td>
<td>.3</td>
</tr>
<tr>
<td>30,000</td>
<td>.3</td>
</tr>
</tbody>
</table>

\( E (Loss) = 19,000 \)

\[
.01 \times (19,000) = 190
\]

\[
(30,000-y) \times .3 = 190
\]

\[
y = 30,000 - \frac{190}{.3} = 29367
\]

\[
\text{Ratio} = \frac{29,367-19,000}{19,000} = 55\%
\]
Joint Distribution:

<table>
<thead>
<tr>
<th>Loss</th>
<th>Probability</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>.16</td>
<td>(= (.4)(.4))</td>
</tr>
<tr>
<td>15,000</td>
<td>.24</td>
<td>(= 2(.4)(.3))</td>
</tr>
<tr>
<td>20,000</td>
<td>.33</td>
<td>(= 2(.4)(.3)+(.3)(.3))</td>
</tr>
<tr>
<td>25,000</td>
<td>.18</td>
<td>(= 2(.3)(.3))</td>
</tr>
<tr>
<td>30,000</td>
<td>.09</td>
<td>(= (.3)(.3))</td>
</tr>
</tbody>
</table>

\(E(L) = 19,000\)

\[.01 \times (19,000) = 190\]

\[(30,000-y) \times .09 = 190\]

\[y = 27,889\]

\[\text{Capital} = 27,889 - 19,000 = 8,888\]

\[\text{Ratio} = \frac{8,888}{19,000} = 47\%\]
48. (3 points)

a. 10-day 99% VaR = \( Z_{99} \times \sqrt{10} \times \alpha \)

\( Z_{99} = 2.33 \)

\( \alpha^2 = \alpha_A^2 \cdot \alpha_A + \alpha_B^2 \cdot \alpha_B + \frac{1}{2} \alpha_A \alpha_B \cdot \alpha_A - \alpha_B \)

\( \alpha_A^2 = (1,000)(50) = 50,000 \)

\( \alpha_B^2 = (10,000)(200) = 200,000 \)

\( \alpha = 80\% \)

\( \alpha_A = 1.5\% \)

\( \alpha_B = 1.0\% \)

\( \alpha^2 = (50K)^2 \cdot (0.015)^2 + (200K)^2 \cdot (0.01)^2 + 2(0.8)(50K)(200K)(0.015)(0.01) \)

\( \alpha^2 = 6,962,500 \)

\( \alpha = 2,639 \)

So, 10-day 99% VaR = \( 2.33 \times \sqrt{10} \times 2,639 = 19,442 \)

b. Ignoring the rate of change of the distribution of option values. \( \hat{\cdot} \) = rate of change of \( \cdot \).

c. If the rate of an option is positive, then the linear model overestimates the VaR. Since these are long call options, the rate of these options is positive, and thus VaR based on the linear model is too high.
a. Statutory Surplus = Assets – Liabilities = (Total Admitted Assets) – (Loss and LAE Reserve) – (UEP Reserve) = 1,500 - 500 - 100 = 900

Adj. Net worth = 900 statutory surplus
- (1,000-1,100) excess of book value over market value of bonds
+ 150 non-admitted assets
+ 60 collectible unsecured unauthorized recoverables (not included in admitted assets but is allowed for purposes of calculating net worth, hence the addition)
- 20 uncollectible authorized recoverables (included in admitted assets but not allowed for purposes of calculating net worth, hence the subtraction)

Notes:
unauthorized secured recoverables is an admitted asset and so is included in total admitted assets – no adjustment to net worth
unsecured unauthorized uncollectible recoverables is not an admitted asset and is not included in statutory admitted assets nor it is allowed to be included in the calculation of net worth

- 35 loss and LAE reserve deficiency
= 1,155

b. Present Value of Future Earnings = \[
\begin{array}{c}
70 \\
25
\end{array}
\]
\[
\frac{1}{1.15} + \frac{1}{1.15^2} = 79.77
\]
Year 1 Earnings = 40 ii on Reserves
+10 ii on 12/31/00 UEPR
+20 100 x (1-70%-10%) earning on UEPR
70
Year 2 Earnings = 20 ii on Reserves
+ 5 ii on 12/31/00 UEPR
  25

Full credit for correct answer
Partial credit:
¼ point for investment income on loss reserves
¼ point for investment income on UEPR
¼ point for earnings on the UEPR
¼ point for appropriate discounting of the earnings

c) Pages 283, 299

Economic Value = 1,155 + 79.77 = 1,234.77

Economic value = adjusted net worth + discounted value of future earnings – cost of capital.

“There is a zero cost of capital if the risk associated with the investment income on capital and surplus is directly reflected in the valuation.”