INSTRUCTIONS TO CANDIDATES

1. This 95 point examination consists of 41 problem and essay questions. The number of points for each full question or part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use other colors.

Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.

Do not answer more than one question on a single sheet of paper. Write on only the lined side of the paper, and be careful to give the number of the question you are answering on each sheet.

The answer should be concise and confined to the question as posed. When a list of a specific size is requested, do not offer more items in your list than the number requested. For example, if you are requested to list three items, only the first three responses will be graded.

In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

2. Attached to the examination, after question 41, is a table of the Normal Distribution.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

5. At the beginning of the examination, check through the exam booklet for any missing or defective pages. The supervisor has additional exams for those candidates who have defective exam booklets.

6. Candidates must remain in the examination center until two hours after the start of the examination. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

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7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer pages in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. \textbf{BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.}

Anything written in the examination booklet will not be graded. Only the answer sheets will be graded.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. (Do not put the self-addressed stamped envelope inside the Examination Envelope.)

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may \textbf{not} take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a \textbf{copy} of the examination by contacting the CAS Office.

All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. An examination survey and postage-paid reply envelope are included with the examination. No postage is necessary for surveys mailed within the United States. Candidates mailing the survey outside the United States should use the courtesy reply envelope distributed by your exam supervisor. \textbf{Please complete the survey and leave it with the examination supervisor, or take the survey and envelope with you when leaving the examination center. Please submit the survey to the CAS Office by May 28, 2002. Please do not enclose the survey in the Examination Envelope.}

END OF INSTRUCTIONS
1. (1 point)

Based on Modern Portfolio Theory and Investment Analysis by Elton and Gruber, identify and describe the two major risks associated with the holding of foreign investments.

2. (3 points)

In 2002 Pitbull Research Inc. provided the following forecasts for Company XYZ:

- The annual dividend is projected to be $0.80 per share to be paid on December 31, 2004.
- Earnings per share of $6 are projected in 2004.
- A constant dividend growth rate of 10% per year
- The expected rate of return of the market portfolio, E[R_m], is 10%.
- The risk-free rate of return, R_f, is 5%.
- Beta is 2.

a. (2 points)

Based on the constant growth Dividend Discount Model in Investments by Bodie, Kane, and Marcus, what is XYZ's intrinsic value at the beginning of 2003? Show all work.

b. (1 point)

What impact does an increase in the Earnings Retention Ratio have on the firm's growth rate, and when would it increase the firm's price to earnings ratio?
3. (1 point)
   
   a. (0.5 point)

   According to Investments by Bodie, Kane, and Marcus, what government action may shift the demand of funds curve from $D$ to $D'$ as summarized in the graph below?

   ![Graph of Interest Rate vs. Funds]

   b. (0.5 point)

   Assuming that the government acts consistently with your answer to part a., above, what action can the Fed take to maintain the original equilibrium interest rate?

   ![Graph of Interest Rate vs. Funds]
EXAM 8, SPRING 2002

4. (1 point)

Answer the following questions based on Investments by Bodie, Kane, and Marcus:

a. (0.5 point)

Given a nominal interest rate of 9% and an inflation rate of 6%, what is the real interest rate? Show all work. Do not use the approximation rule.

b. (0.5 point)

Further assume that the investor's tax rate is 25%. What is the approximate percentage change in the after-tax real rate of return if both the nominal interest rate and the inflation rate increase by one percentage point? Show all work.

5. (3 points)

Assume that two fixed income investors seek your counsel regarding investing in U.S. treasury bonds with time to maturity of 15 years. The first investor, Investor A, can afford to hold bonds to maturity. The second investor, Investor B, faces the possibility of sale before maturity.

Based on The Handbook of Fixed Income Securities by Fabozzi, describe market, reinvestment, and timing risk and compare how each of the two investors is exposed to these risks.

Assume that both investors are attempting to manage their portfolios with a 15-year investment horizon and that they currently hold identical investments.
6. (1.5 points)

Modern Portfolio Theory and Investment Analysis by Elton and Gruber states that most portfolio literature uses variance or equivalently standard deviation as the measure of dispersion instead of semi-variance.

a. (0.5 point)

What assumption makes this practice reasonable?

The CFO at an insurance company that sells hurricane-exposed homeowners policies has asked you to discuss semi-variance and variance as risk measures.

Answer the following questions based on Elton and Gruber’s discussion of semi-variance.

b. (0.5 point)

Please give one reason why you might prefer semi-variance as a risk measure if you are evaluating returns on a portfolio of insurance policies.

c. (0.5 point)

Please give one reason why you might prefer variance as a risk measure if you are evaluating returns on a portfolio of insurance policies.
7. (1.5 points)

Assume that you are determining how to allocate your company's investments between two investment portfolios, aggressive (Portfolio A) and conservative (Portfolio C).

- Portfolio A has a higher expected rate of return and a higher volatility.
- Portfolio C has a lower expected rate of return and a lower volatility.

a. (1 point)

In Modern Portfolio Theory and Investment Analysis by Elton and Gruber, the authors show the risk-return relationship for possible allocations between two investment portfolios.

Graph the risk-return relationship for all possible allocations between Portfolio A and Portfolio C, if short sales are not allowed and \( \rho = -1 \). Label both axes and points A and C on the graph.

b. (0.5 point)

What is the standard deviation of the minimum variance portfolio when \( \rho = +1 \)?
8. (1.5 points)

Using the procedure described in Elton and Gruber, *Modern Portfolio Theory and Investment Analysis*, answer the following question. Show all work.

Consider the following three securities

<table>
<thead>
<tr>
<th>Company</th>
<th>Expected Return (%)</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>DEF</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>EFG</td>
<td>12</td>
<td>400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Companies Exhibiting Correlation with Each Other</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>DEF</td>
</tr>
<tr>
<td>ABC</td>
<td>EFG</td>
</tr>
<tr>
<td>DEF</td>
<td>EFG</td>
</tr>
</tbody>
</table>

The riskless lending and borrowing rate is 5%.

Set up the simultaneous equations that will be needed to determine the optimum portfolio consisting of these three securities when short sales are allowed with riskless lending and borrowing.
9. (5 points)

In *Modern Portfolio Theory & Investment Analysis* Elton and Gruber discuss methods to construct the optimal portfolio for single-index models.

You are given the following information.

\[ R_f = 5 \]

\[ \sigma_m^2 = 10 \]

<table>
<thead>
<tr>
<th>Security No.</th>
<th>Mean Return</th>
<th>Excess Return</th>
<th>Beta</th>
<th>Unsystematic Risk</th>
<th>Excess Return over Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>0.7</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>1.0</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>25</td>
<td>2.0</td>
<td>40</td>
<td>12.5</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>7</td>
<td>1.2</td>
<td>30</td>
<td>5.8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
<td>1.0</td>
<td>30</td>
<td>5.0</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>3</td>
<td>1.0</td>
<td>30</td>
<td>3.0</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>7</td>
<td>0.9</td>
<td>20</td>
<td>7.8</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>1</td>
<td>0.5</td>
<td>20</td>
<td>2.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Security No.</th>
<th>( C_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.905</td>
</tr>
<tr>
<td>2</td>
<td>8.410</td>
</tr>
<tr>
<td>3</td>
<td>8.066</td>
</tr>
<tr>
<td>4</td>
<td>7.603</td>
</tr>
<tr>
<td>5</td>
<td>7.620</td>
</tr>
<tr>
<td>6</td>
<td>7.452</td>
</tr>
</tbody>
</table>

a. (4 points)

Assuming that short sales are not allowed, determine the weights given to each security in the optimal portfolio. Show all work.

b. (1 point)

Describe how the portfolio would change if short sales were allowed.

CONTINUED ON NEXT PAGE
10. (1.5 points)

In Bodie, Kane, and Marcus, Investments, Samuelson's proof of the major theoretical justification for mean-variance is discussed. Under the conditions of this proof, Samuelson concludes that the mean and variance are equally important to investors and that all moments higher than the variance can be overlooked during portfolio analysis.

a. (1 point)

Explain why the compactness assumption enabled Samuelson to disregard the higher moments in his evaluation of risk-reward.

b. (0.5 point)

Under what circumstances can the rate of return distribution for a portfolio be considered compact?

11. (1 point)

Argue against the following statement based on Modern Portfolio Theory and Investment Analysis by Elton and Gruber.

"I followed the CAPM theory last year and bought high-Beta stocks. Since then the return on the high-Beta stocks was worse than the average return for low-Beta stocks. The CAPM theory does not work!"
12. (5 points)

Answer the following according to Gorvett, “Insurance Securitization: The Development of a New Asset Class.”

You have been asked to evaluate a proposal for securitizing terrorism risk.

a. (1 point)

State the two primary reasons cited by Gorvett in favor of securitizing insurance risk and briefly discuss whether each is valid in the case of terrorism securitization.

b. (1.5 points)

Identify and describe the three triggers Gorvett identifies that might be used in an insurance securitization arrangement.

c. (1 point)

Gorvett identifies two risks that may exist depending upon the type of trigger utilized. Identify and briefly describe these risks.

d. (1.5 points)

For each trigger in part b., above, briefly discuss how each of the risks identified in part c., above, may or may not apply for securitizing terrorism risk.

13. (1 point)

In Investments by Bodie, Kane, and Marcus, the authors discuss the fallacy of time diversification.

Describe how the two main return measures identified by the authors are affected by extending the investment horizon.
14. (2.5 points)

In Modern Portfolio Theory and Investment Analysis by Elton and Gruber the authors demonstrate the adjustment to the standard capital asset pricing model that is required to reflect the fact that the world is divided into marketable and nonmarketable assets.

a. (0.5 point)

Identify two of the nonmarketable assets identified by Elton and Gruber.

b. (2 points)

You are given the following information:

- The total value of all nonmarketable assets is $500,000.
- The total value of all marketable assets is $700,000.
- The risk-free rate is 5%.

Let $R_M$, $R_H$, and $R_j$ be the rate of return on all marketable assets, all the nonmarketable assets, and asset $j$, respectively. Also assume that:

- The expected return of all the marketable assets, $E(R_M)$, is 11%.
- The variance of all the marketable assets, $\sigma^2_M$, is 0.2.
- $\text{Cov}(R_M, R_H)$ equals 0.12.
- $\text{Cov}(R_j, R_M)$ equals 0.16.
- $\text{Cov}(R_j, R_H)$ equals 0.20.
- The market risk-return trade-off is defined as $\frac{E(R_M) - R_f}{\sigma^2_M}$.

Compare the market risk-return trade-off under the standard capital asset pricing model with the market risk-return trade-off using the capital asset model that has been adjusted to reflect nonmarketable assets. Show all work.
In Modern Portfolio Theory and Investment Analysis by the Elton and Gruber, the authors discuss the two-index arbitrage pricing model.

You are provided with the following information about four portfolios:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>$b_{11}$</th>
<th>$b_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>0.5</td>
<td>1.2</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>C</td>
<td>11</td>
<td>0.2</td>
<td>1.0</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**a.** (1.5 points)

Find the equation of the plane that must describe equilibrium returns given the four observed portfolios.

**b.** (1 point)

What will happen if there is a portfolio $E$ with $\bar{R}_E = 12$, $b_{E1} = .45$, and $b_{E2} = .9$?
16. (2 points)

Answer the following based on Fabozzi, *The Handbook of Fixed Income Securities*. Show all work.

You purchase a $100 par value default free corporate bond with a coupon rate of 8.0% maturing March 1, 2003. The settlement date of the purchase is October 25th, 2001.

Assume coupon payments are made every six months.

a. (1.5 points)

What would the full price of this corporate bond be if it is priced to yield 5.0%?

b. (0.5 point)

What is the amount of accrued interest at the time of your purchase?
17. (5.5 points)

Answer the following based on Fabozzi, *The Handbook of Fixed Income Securities*. Show all work.

You are the holder of the following bond:

- The term to maturity is 20 years.
- 8% coupon, paid semi-annually
- Yield to maturity is 10%.
- Par Value is $1000.

You want to test this bond’s price sensitivity to a 200 basis point decrease in interest rates.

a. (2.5 points)

Using the duration approximation, calculate the estimated price following the 200 basis point interest rate decline, and determine the error in this approximation. In approximating the duration, use a 10 basis point “shock” in the yield to maturity.

b. (1 point)

Graph the price/yield relationship for this bond, and illustrate the duration approximation error calculated in part a., above.

c. (2 points)

Use the convexity adjustment to calculate a more accurate estimate of the price following the 200 basis point interest rate drop, and determine the error in this new approximation. In calculating the convexity measure, again use a 10 basis point “shock” in yield to maturity.
18. (1 point)

Answer the following based on Fabozzi, *The Handbook of Fixed Income Securities*. Show all work.

a. (0.5 point)

Calculate the yield on a bank discount basis of a 26-week Treasury bill sold at a price of $99 per $100 of face value.

b. (0.5 point)

Fabozzi identifies two ways that the yield on a discount basis differs from more standard return measures. Identify these two differences.

19. (1 point)

Answer the following based on Fabozzi, *The Handbook of Fixed Income Securities*.

Fabozzi suggests that commercial paper rates are higher than those on Treasury bills for three reasons. The investor’s preference for liquidity is cited as one reason.

a. (0.5 point)

Explain why the liquidity preference effect on commercial paper yield is relatively small.

b. (0.5 point)

Identify the other two reasons cited by Fabozzi for higher yields on commercial paper.
20. (2 points)

Answer the following according to Fabozzi, The Handbook of Fixed Income Securities.

a. (1 point)

Briefly describe how principal repayments are divided among classes of investors for each of the following two types of mortgage backed securities:

1. Mortgage Pass Through securities
2. Collateralized Mortgage Obligations (assume three tranches)

b. (1 point)

Briefly explain the effect that a drop in mortgage rates has on the prices of Interest Only Securities and Principal Only securities.

21. (1.5 points)

Answer the following according to Bodie, Kane, and Marcus, Investments. Show all work.

Assume you are managing a portfolio currently worth $10 million, and that current interest rates are 10%. Also assume this portfolio is required to fund an obligation of $11 million due in five years.

a. (1 point)

Briefly describe how you could use contingent immunization to pursue active management without jeopardizing the ability to meet the obligation.

b. (0.5 point)

Assuming interest rates do not change, what is the trigger point for this immunization strategy after two years.

CONTINUED ON NEXT PAGE
22. (2 points)

Answer the following based on Hull, *Options, Futures and Other Derivatives*. Show all work.

You are given the following:

- $\Delta S$ Change in spot price, $S$, during a period of time equal to the life of the hedge
- $\Delta F$ Change in futures price, $F$, during a period of time equal to the life of the hedge
- $\sigma_s$ Standard deviation of the change in the spot price
- $\sigma_f$ Standard deviation of the change in the futures price
- $h$ Hedge ratio
- $\rho$ Coefficient of correlation between $\Delta F$ and $\Delta S$

a. (0.5 point)

If the trader is long the asset and short the futures contract, what is the expression for the change in value of the hedged position over the life of the hedge?

b. (1.5 points)

Define and solve for the optimal hedge ratio.
23. (2 points)

Answer the following based on Hull, *Options, Futures, and Other Derivatives*. Show all work.

You are given the following:

- A stock is currently selling for $20 per share.
- The stock is not expected to pay any dividends over the next 3 years.
- The annual risk-free rate with continuous compounding is 5%.

a. (1 point)

If the three-year forward price is $23, describe the strategy you could employ today to obtain a riskless arbitrage profit.

b. (0.5 point)

What would be your profit per share be in 3 years based on the strategy employed in part a., above?

c. (0.5 point)

Assume it is not possible to short sell a particular asset. Explain why an arbitrage opportunity resulting from an underpriced forward contract could not persist.
24. (2.5 points)

Answer the following based on Hull, Options, Futures and Other Derivatives. Show all work.

You are given the following Bond Price data:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Principal ($)</th>
<th>Time to Maturity (yrs)</th>
<th>Annual Coupon ($) *</th>
<th>Bond Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$100</td>
<td>0.25</td>
<td>0</td>
<td>$98.50</td>
</tr>
<tr>
<td>B</td>
<td>$100</td>
<td>0.50</td>
<td>0</td>
<td>$95.90</td>
</tr>
<tr>
<td>C</td>
<td>$100</td>
<td>1.00</td>
<td>0</td>
<td>$91.00</td>
</tr>
<tr>
<td>D</td>
<td>$100</td>
<td>1.50</td>
<td>8</td>
<td>$97.00</td>
</tr>
<tr>
<td>E</td>
<td>$100</td>
<td>2.00</td>
<td>12</td>
<td>$102.60</td>
</tr>
</tbody>
</table>

* Half the stated coupon is assumed to be paid every six months.

a. (0.5 point)

Assuming continuous compounding, calculate the 0.50-year zero rate.

b. (1 point)

Assuming continuous compounding, calculate the 1.50-year zero rate.

c. (1 point)

Assuming continuous compounding, calculate the forward rate for the period between time 1.50 and 2.00.
25. (2 points)

Answer the following based on Bodie, Kane and Marcus, Investments. Show all work.

You have a deferred variable annuity account. Assume the following:

- Current account balance = $100,000.
- Assumed investment return = 10%.
- Withdrawals will be made at the end of years six through ten.
- Actual holding period returns are given by the following table:

<table>
<thead>
<tr>
<th>Year</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>20%</td>
</tr>
<tr>
<td>3</td>
<td>0%</td>
</tr>
<tr>
<td>4</td>
<td>5%</td>
</tr>
<tr>
<td>5</td>
<td>10%</td>
</tr>
<tr>
<td>6</td>
<td>10%</td>
</tr>
<tr>
<td>7</td>
<td>15%</td>
</tr>
<tr>
<td>8</td>
<td>10%</td>
</tr>
<tr>
<td>9</td>
<td>20%</td>
</tr>
<tr>
<td>10</td>
<td>15%</td>
</tr>
</tbody>
</table>

a. (1 point)

What is the current assumed constant annual annuity benefit payment?

b. (1 point)

What are the actual annual annuity benefit payments for years 6 and 7?
26. (2.5 points)

Answer the following based on Hull, *Options, Futures and Other Derivatives*. Show all work.

You are an investor desiring to synthetically create the following payoff function, based on the stock price at time $T$:

<table>
<thead>
<tr>
<th>$S_T$</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.00 - $49.99</td>
<td>$0</td>
</tr>
<tr>
<td>$50.00 - $50.50</td>
<td>$S_T - $50</td>
</tr>
<tr>
<td>$50.50 - $51.00</td>
<td>$51 - S_T</td>
</tr>
<tr>
<td>$51.01 - $59.99</td>
<td>$0</td>
</tr>
<tr>
<td>$60.00 - $60.50</td>
<td>$S_T - $60</td>
</tr>
<tr>
<td>$60.50 - $61.00</td>
<td>$61 - S_T</td>
</tr>
<tr>
<td>$61.01 - \infty</td>
<td>$0</td>
</tr>
</tbody>
</table>

a. (1 point)

Graph the desired payoff function.

b. (1 point)

Describe how this payoff can be created using only call options on the stock.

c. (0.5 point)

Hull explains how the process in part b., above, can be used to approximate any payoff function. Describe this approach.
27. (2 points)

Answer the following based on Hull, *Options, Futures, and Other Derivatives*. Show all work.

The current price of a stock is $10 and it is expected to either increase or decrease in price by 10% over each of the next two 6-month periods. The annual risk-free rate with continuous compounding is 6%.

a. (1.5 points)

What is the value of an American put option with a strike price of $10.50 maturing in one year?

b. (0.5 point)

How many shares of stock should be held at T=0 to provide a riskless hedge over the first six months? Assume fractional shares are possible.
28. (3 points)

Answer the following based on Fabozzi, *The Handbook of Fixed Income Securities*. Show all work.

You are evaluating a callable bond using a binomial interest rate tree and are given the following information:

- Par Value is $200.
- Maturity is 2 years.
- 6% coupon, paid annually
- The bond is callable at par after one year.
- The current one-year rate is 4.5%.
- The volatility of one-year rate is 10%.

a. (1 point)

If the lower one-year rate one year forward is 4.979%, what is the higher one-year rate one year forward?

b. (1.5 points)

What is the value of the callable bond?

c. (0.5 point)

What is the value of the embedded call option?
29. (2.5 points)

Answer the following based on Hull, *Options, Futures, and Other Derivatives*. Show all work.

Assume that a stock price follows geometric Brownian motion,

\[ dS = \mu Sdt + \sigma Sdz. \]

If \( \mu = 11\% \) and \( \sigma = 16\% \), what is the probability of exceeding a continuously compounded annual rate of return of 20\% in a two-year period?

30. (3 points)

Answer the following based on the discrete version of geometric Brownian motion in Hull, *Options, Futures, and Other Derivatives*. Show all work.

You have simulated the following random variables from a standard normal distribution and daily change in stock prices:

<table>
<thead>
<tr>
<th>Day</th>
<th>Epsilon</th>
<th>Change in Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.18</td>
<td>0.886</td>
</tr>
<tr>
<td>2</td>
<td>-0.53</td>
<td>-0.367</td>
</tr>
</tbody>
</table>

The current stock price is $40. Assume stock prices follow geometric Brownian motion.

a. (2.5 points)

Calculate the current drift rate of the stock price process.

b. (0.5 point)

Calculate the per annum standard deviation of the continuous compounding return for this stock.
31. (4 points)

Answer the following based on Hull, *Options, Futures, and Other Derivatives*. Show all work.

Assume you are given the opportunity to invest in two options on the stock of Company XYZ:

<table>
<thead>
<tr>
<th></th>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>European Call</td>
<td>European Call</td>
</tr>
<tr>
<td>Current Stock Price</td>
<td>$50</td>
<td>$50</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>5%</td>
<td>5%</td>
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<tr>
<td>Volatility</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>Strike</td>
<td>$65</td>
<td>$55</td>
</tr>
<tr>
<td>Time to Expiry</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>$N(d_1)$</td>
<td>0.7405</td>
<td></td>
</tr>
<tr>
<td>$N'(d_1)$</td>
<td>0.5969</td>
<td></td>
</tr>
<tr>
<td>Dividend Yield</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Delta</td>
<td>0.5931</td>
<td>0.0436</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.0436</td>
<td>0.0436</td>
</tr>
<tr>
<td>Vega</td>
<td>16.36</td>
<td>16.36</td>
</tr>
</tbody>
</table>

a. (1.5 points)

Assuming geometric Brownian motion for the stock prices, calculate delta, gamma, and vega for Option 1.

b. (1.5 points)

Suppose you manage a portfolio of derivatives on Company XYZ stock. The portfolio is currently delta neutral with a gamma of –50 and a vega of –20,000. How much of each of the above options should be added to this portfolio to make it gamma and vega neutral?

c. (1 point)

Show how the portfolio from part b., above, can be made delta, gamma and vega neutral.
32. (2 points)

Answer the following questions based on Lowe & Stanard, "An Integrated Dynamic Financial Analysis and Decision Support System for a Property Catastrophe Reinsurer,"

a. (1 point)

Explain the Asset/Liability Efficient Frontier (ALEF). Include a graphic illustration with your explanation.

b. (1 point)

How does use of the ALEF performance objective contrast with that of the classical efficient frontier objective?

33. (1.5 points)

According to D’Arcy et al., “Using the Public Access DFA Model: A Case Study,” DFA modeling brings specific innovations to the planning process and has several limitations.

a. (0.75 point)

What three innovations does DFA incorporate into the planning process?

b. (0.75 point)

Briefly describe three specific limitations of DFA modeling.
34. (2 points)

You are given the following balance sheet information for an insurer. No other assets or liabilities are material.

<table>
<thead>
<tr>
<th>Description</th>
<th>Statutory Book Value</th>
<th>Market Value</th>
<th>Market Yield</th>
<th>Duration (in years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Market</td>
<td>$7,353</td>
<td>$7,353</td>
<td>3.5%</td>
<td>0.5</td>
</tr>
<tr>
<td>Taxable Bonds</td>
<td>$50,350</td>
<td>$43,765</td>
<td>6.0%</td>
<td>7.0</td>
</tr>
<tr>
<td>Tax-exempt Bonds</td>
<td>$58,822</td>
<td>$52,056</td>
<td>4.5%</td>
<td>10.0</td>
</tr>
<tr>
<td>Loss Reserve</td>
<td>$48,000</td>
<td>$39,028</td>
<td>3.0%</td>
<td>7.0</td>
</tr>
</tbody>
</table>

According to Noris, “Asset/Liability Management Strategies for Property and Casualty Companies,” calculate the duration of this insurer’s market value surplus. Show all work.

35. (2.5 points)

a. (1 point)

Elton and Gruber in Modern Portfolio Theory state that an investor’s return expectations would be higher for bonds with sinking fund options and call options relative to bonds without such options, assuming that all other pricing factors are the same.

Explain why the investor expects higher returns for bonds with sinking fund options and call options.

Include brief discussions of sinking fund and call provisions in your explanation.

b. (0.5 point)

Why might an investor’s return expectations differ between bonds with sinking fund options compared to callable bonds without the sinking fund provision?

c. (1 point)

Fabozzi in The Handbook of Fixed Income Securities cites two advantages and one disadvantage to a sinking fund from the bondholder’s perspective.

Explain the advantage not discussed in part b., above, as well as the disadvantage.
36. (2.5 points)

Fabozzi, *The Handbook of Fixed Income Securities*, describes the most straightforward approach to funding a single-period liability as purchasing a zero-coupon security with a maturation date simultaneous with the liability payment date.

a. (0.75 point)

If zero-coupon bonds have insufficient yield, what three conditions must be met for a portfolio of coupon-bearing Treasury, agency, and corporate bonds to provide immunization for the same single-period liability?

b. (0.75 point)

The three conditions required to create a single-period immunized portfolio can be extended to create an immunized portfolio to satisfy the funding requirements of multi-period liabilities. Fabozzi identifies a complication associated with determining the duration of multi-period liabilities. Briefly describe this complication.

c. (1 point)

Describe the iterative process suggested in order to construct an immunized portfolio in the multi-period situation.
37. (4 points)

You are trying to duration match an asset to fund an estimated liability payment of $5,000,000 held as of December 31, 2001 stemming from an accident that occurred in 2001. You have the following financial information:

- Risk-Free Rate = 5.0%
- New Investment Rate = 8.0%
- Loss reserve payout pattern:

<table>
<thead>
<tr>
<th>Development Year</th>
<th>Accident Year 2001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>50.0%</td>
</tr>
</tbody>
</table>

Based on Feldblum, “Asset Liability Matching for Property/Casualty Insurers,” answer the following:

a. (1 point)

Assuming mid-year payments, calculate the duration of the $5,000,000 liability. Show all work.

b. (1 point)

The apparent conclusion is that to duration match a liability portfolio you should invest in a medium term bond with an average duration equal to that of the liability reserves.

Explain why Feldblum states that this conclusion is misleading.

c. (2 points)

Feldblum suggests two types of investments that are better suited than bonds for matching insurance liability losses. He provides two specific examples of each of these two types of better-suited investments.

Which among these four specific investments does Feldblum conclude is the apparent investment of choice?

Support your answer by citing Feldblum’s concerns with the other three investments.
38. (3.5 points)

Based on Hull, Options, Futures, and Other Derivatives, use the following information to answer the questions below:

- The 10-day 99% Value at Risk (VaR) for a portfolio consisting of a $10 million position in stock A is $736,811.
- The 10-day 99% Value at Risk (VaR) for a portfolio consisting of a $10 million position in stock B is $1,105,216.
- The daily changes of stocks A and B over the 10-day period are assumed to be independent, normally distributed with means of zero.

a. (0.5 point)

What is the daily volatility of stock A? Show all work.

b. (1.5 points)

Assuming the returns of stocks A and B have a bivariate normal distribution and are perfectly correlated, what is the 10-day 99% VaR for a $10 million portfolio consisting of a $5 million position in stock A and a $5 million position in stock B? Show all work.

c. (1.5 points)

From the perspective of the 10-day 99% VaR, at what coefficient of correlation between stocks A and B would an investor be indifferent between a $10 million position in stock A versus a portfolio consisting of a $5 million position in stock A and a $5 million position in stock B? Show all work.
39. (3 points)

You are given the following information for a company valuation as of December 31, 2001:

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Value of Bonds</td>
<td>$2,000</td>
</tr>
<tr>
<td>Book Value of Bonds</td>
<td>$1,900</td>
</tr>
<tr>
<td>Carried Loss and ALAE Reserves</td>
<td>$1,000</td>
</tr>
<tr>
<td>Estimated Reserve Deficiency</td>
<td>$ 200</td>
</tr>
<tr>
<td>Annual Net Written Premium</td>
<td>$3,000</td>
</tr>
<tr>
<td>2002 Estimated Earnings</td>
<td>$ 300</td>
</tr>
<tr>
<td>2002 Projected Risk Free Investment Return</td>
<td>4%</td>
</tr>
<tr>
<td>2002 Projected Return on Insurer’s Investments</td>
<td>5%</td>
</tr>
<tr>
<td>2002 Projected Return for Benchmark Portfolio</td>
<td>6%</td>
</tr>
<tr>
<td>Required Return of Buyer</td>
<td>10%</td>
</tr>
</tbody>
</table>

Assume that the company has no other assets or liabilities, will only have one year of operations and that earnings occur at the end of the year. Furthermore, ignore the impact of taxes and assume that the risk associated with the benchmark portfolio is the same as that of the insurer’s portfolio.

According to Miccolis in “An Investigation of Methods, Assumption and Risk Modeling for the Valuation of Property-Casualty Insurance Companies”, answer the following:

a. (0.5 point)

What is the adjusted net worth of the Company at December 31, 2001 using Sturgis’s method as outlined by Miccolis? Show all work.

d. (0.5 point)

What is the economic value of the company at December 31, 2001 using Miccolis’s cost of capital? Show all work.
40. (1.5 points)

Answer the following according to Butsic, "Solvency Measurement for Property-Liability Risk-Based Capital Applications."

a. (0.5 point)

Butsic states that the usual measure of risk with respect to insurance solvency is the Probability of Ruin. For what reason does Butsic state this measure is inadequate for public policy?

b. (0.5 point)

Identify and briefly describe the alternative measure Butsic suggests is a more reasonable measure appropriate for public policy.

c. (0.5 point)

Give two reasons why Butsic prefers this alternative measure.
41. (2 points)

Answer the following according to Butsic, “Solvency Measurement for Property-Liability Risk-Based Capital Applications.”

You are given the following information for an insurer:

- Asset value as of January 1, 2002 is $15,000.
- There are no outstanding loss payments as of January 1, 2002 and no future loss payments will occur until December 31, 2002.
- The potential asset value as of December 31, 2002 is distributed according to the following discrete distribution:

<table>
<thead>
<tr>
<th>Asset Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10,000</td>
<td>0.25</td>
</tr>
<tr>
<td>15,000</td>
<td>0.50</td>
</tr>
<tr>
<td>20,000</td>
<td>0.25</td>
</tr>
</tbody>
</table>

- The potential loss payment that will need to be paid on December 31, 2002 is distributed according to the following discrete distribution:

<table>
<thead>
<tr>
<th>Loss Payment Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7,500</td>
<td>0.10</td>
</tr>
<tr>
<td>10,000</td>
<td>0.50</td>
</tr>
<tr>
<td>11,000</td>
<td>0.20</td>
</tr>
<tr>
<td>15,000</td>
<td>0.20</td>
</tr>
</tbody>
</table>

- Assets and liabilities are uncorrelated.
- The current Expected Policyholder Deficit (EPD) ratio is 2.7%.

a. (1 point)

If additional capital is invested in risk-free assets with an expected annual rate of return of 5%, how much additional capital must be provided on January 1, 2002 to reduce the EPD ratio to 0.01?

b. (1 point)

Rather than risk-free assets, now assume additional capital is invested in the same risky assets and therefore has the identical distribution as the other assets with perfect correlation. What is the expected value of the additional capital that must be provided to reduce the EPD ratio to 0.01?
The Normal Distribution

\[ \Pr(X \leq x) = \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt \]

[bundledths]

<table>
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<td>.9948</td>
<td>.9949</td>
<td>.9949</td>
<td>.9951</td>
</tr>
</tbody>
</table>
Question #1

Exchange Risk – is the variability of the exchange rate between two currencies.
Domestic Risk - relates to the standard deviation of return when returns are calculated in the indexes’ own currency.

(Note: Other answers were given full credit if risks identified by other authors were appropriately identified and defined.)

Question #2

a. 
\[ P_0 = \frac{D_1}{k-g} = $13.91 \]

or

\[ P(2003) = \frac{D(2003)}{1+k} + \frac{P(2004)}{1+k} = $14.55 \]

b. 
\( g \) increases with plowback ratio
\( P/E \) increases when \( ROE > k \)

Question #3

a. The government can increase the demand for funds by increasing its own level of borrowing.
b. It can decrease the federal funds rate.

Question #4

a) \[ 1 + \text{Real rate} = \frac{(1 + \text{Nominal rate})}{(1 + \text{Inflation rate})} = \frac{1.09}{1.06} - 1 = 2.830\% \]

b) 
\[ \text{nominal return} = 10\% \]
\[ \text{inflation} = 7\% \]
\[ \text{after-tax return} = (1 - .25) \times 10\% = 7.5\% \]
\[ \text{real after-tax return} = \frac{1.075}{1.07} - 1 = 0.467\% \]

Old way
\[ \text{Nominal} = 9\% \]
\[ \text{Inflation} = 6\% \]
\[ \text{After-tax return} = (.75) (9\%) = 6.75\% \]
\[ \text{Real after-tax} = (1.0675/1.06) - 1 = 0.708\% \]

\[ \text{Change} = 1 - \frac{(0.467\%)}{0.708\%} = -34\% \]
Question #5

Market Risk: Risk that an increase in interest rate will decrease the value of a bond.

Reinvestment Risk: Risk that interim cash flows are invested at a lower rate.

Timing Risk: Risk that the bond is called before the maturity. This happens usually when interest rate drops below coupon rate.

Note: Market risk and reinvestment risk have offsetting forces. The point in time at which they exactly offset each other is the duration.

Investor A: If this investor holds the bonds to maturity, they will not face market risk. However, they face reinvestment risk (i.e. reinvesting coupons at a lower rate) as well as timing risk (if bonds are callable).

Investor B: If Investor B faces the possibility of selling the bonds before maturity, he/she is exposed to market risk. Investor B is also exposed to timing and reinvestment risks as explained above.

Question #6

a. Returns are symmetrical so semi-variance is proportional to variance, and thus semi-variance is not necessary.

b. Semi-variance is a measure of the squared deviations below the mean. We may prefer to use it if we believe that the distribution is skewed and we want to take notice of those returns below the mean. This may be applicable for hurricane policies since they have infrequent but potentially very damaging returns below the mean.

c. Variance of a portfolio is easy to calculate in terms of individual security variances/covariances. The semi-variance does not combine in such a nice way => computationally impractical.
Question #7

a.

\[ \overline{R}_p \]

\[ \sigma_p \]

\[ A \]

\[ \text{b. Minimum variance portfolio: If short sales are not allowed and } \rho \text{ = +1,} \]

there is no benefit to diversification. Invest all in portfolio C. Thus the
standard deviation is the standard deviation of portfolio C.

Question #8

The equations are in the form:

\[ r_i - \overline{r}_p = Z_1 \sigma_1 + Z_2 \sigma_2 + Z_3 \sigma_3 \]

\[
15 - 5 = Z_{ABC} (4) + Z_{DEF} (\sqrt{4})(\sqrt{100})(1) + Z_{EFG} (\sqrt{4})(\sqrt{400})(0.6)
\]

(1) \[ 10 = 4Z_{ABC} + 20Z_{DEF} + 24Z_{EFG} \]

(2) \[ 5 = 20Z_{ABC} + 100Z_{DEF} + 60Z_{EFG} \]

(3) \[ 7 = 24Z_{ABC} + 60Z_{DEF} + 400Z_{EFG} \]

Solve equations (1), (2), (3) to obtain \( Z_{ABC}, Z_{DEF}, Z_{EFG} \). The proportion in the
portfolio will be

\[ X_k = Z_k / \sum Z_i \text{ where } i = 1 \ldots n \]
Question #9

a.

(Note: We also accepted the answer that security 4 was required on the basis that the securities should have been reordered. According to the rule that we keep all securities where the excess return to beta is higher than $C_i$.)

\[ X_i = \frac{Z_i}{\sum Z_i} \quad \text{where} \quad Z_i = \frac{B_i \cdot \left( \overline{R}_i - R_E - C^* \right)}{\sigma_{ei}^2} \]

\[ Z_1 = \frac{0.7}{30} (22.9 - 8.905) = 0.3266, \]
\[ Z_2 = \frac{1.0}{30} (18 - 8.905) = 0.3032 \quad \text{and} \]
\[ Z_3 = \frac{2.0}{40} (12.5 - 8.905) = 0.1798 \]
\[ X_1 = \frac{0.3266}{0.3266 + 0.3032 + 0.1798} = \frac{0.3266}{0.8096} = 40.34\% \]

\[ X_2 = \frac{0.3032}{0.3266 + 0.3032 + 0.1798} = \frac{0.3032}{0.8096} = 37.45\% \]

\[ X_3 = \frac{0.1798}{0.3266 + 0.3032 + 0.1798} = \frac{0.1798}{0.8096} = 22.21\% \]

b.

All stocks are now included in the optimal portfolio. Or \( c^* = 7.452 \)

The value of \( C^* \) will be different.

Stocks that have an excess return to beta above \( C^* \) are held long (as before) but stocks with an excess return to beta below \( C^* \) are now sold short.

Question #10

a. Because compactness corresponds to continuity of price changes. If price changes is continuous \( \Rightarrow \) there can not be an instanteous “jump” \( \Rightarrow \) so by reducing time interval of holding security we, eliminate risk as change in \( t \) (time) goes to 0. That corresponds to the ability of investor to control the risk \( \Rightarrow \) higher moment can be disregarded.

b. If the investor can control the risk of the portfolio.

Question #11

If high beta stocks continuously, year after year, provide returns greater than low Beta stocks than they would be less risky; thus making them low beta stocks.

High Beta stocks are riskier and offer higher returns on average, greater than low beta (lower risk) stocks. Thus in some years, returns may be lower than lower Beta stocks, but in the long run, high Beta stocks will have higher returns.

(Note: Other responses were given full on this question that appropriately argued against the assertion in the question.)
Question #12

a) 1. Capital: The Capital Market can provide additional capital to insure risks. This is valid in the case of terrorism securitization.
   2. Provides Diversification: Insurance losses are not correlated with Capital Market losses. This will not work well in all cases as the terrorism act of September 11 impacted both Insurance and Capital Markets. (Economy in general).

b) Direct Trigger: trigger based on the company’s loss experience.
   Industry Trigger: trigger based on the industrywide loss experience.
   Event Trigger: trigger based on the occurrence of an event.

c) 1. Basis Risk: the risk that the defined trigger is not very closely related to the insurance company’s exposure.
   2. Moral Hazard: the risk that increased losses for a company could result in debt relief.

d) 1. Direct Trigger
   Basis Risk: if terrorism hits one building and only one company insures it, then trigger will be hit and there is little basis risk
   Moral Hazard: if terrorism hits one building, company will want to pad losses so trigger is activated – large moral hazard
   2. Industry Trigger
   Basis Risk: if terrorism hit one building insured by one company, then little industry loss so trigger is not activated – large basis risk
   Moral Hazard: hardly any
   3. Event Trigger
   Basis Risk: terrorism event but doesn’t activate trigger, can cause loss if affects one company more than others
   Moral Hazard: none

Question #13

The volatility of the average annual return decreases with time \((t^{1/2})\). This can be easily recognized since the annual return of a one year investment is very volatile but the average annual return of a multi-year investment is less volatile.

The volatility of the dollar return increases with the time \((t^{1/2})\). The volatility of the dollar return is said to compound over time as a wider-and-wider range of possible dollar returns become possible as horizon increases.
Question #14

a. Human capital
   Social security payments or future payments from a private retirement program

b. Standard market risk return trade-off

\[ \frac{E(R_M) - R_f}{\sigma_M^2} = \frac{.11 - .05}{.20} = .30 \]

Market risk return trade-off adjusted to reflect non-marketable assets

\[ = \frac{E(R_M) - R_f}{\sigma_M^2 + \frac{P_H}{P_M} \cdot \text{cov}(R_M, R_H)} = \frac{.11 - .05}{.20 + \frac{500,000}{700,000} \cdot .12} = .21 \]

The adjusted market risk return trade-off (.21) is lower than the standard market risk return trade-off (.30).
Question #15

a.

\[ R_i = a_i + b_{1i} I_1 + b_{2i} I_2 + e_i \]

Equation A: \( 15 = a + .5I_1 + 1.2I_2 \)
Equation B: \( 12 = a + .4I_1 + .8I_2 \)
Equation C: \( 11 = a + .2I_1 + 1.0I_2 \)
Equation D: \( 4 = a \)

Use \( a=4 \) and subtract Equation B from \( 2 \times \) Equation C:

\[
\begin{align*}
21 & = 8 + .4I_1 + 2.0I_2 \\
12 & = 4 + .4I_1 + .8I_2 \\
10 & = 4 + 1.2I_2 \Rightarrow I_2 = 5
\end{align*}
\]

Plug known values into equation C to solve for \( I_1 \)

\[ 11 = 4 + .2I_1 + 1.0 \times 5 \Rightarrow I_1 = 10 \]

Equation of Plane: \( R_i = 4 + 10b_{1i} + 5b_{2i} \)

b.

\[ R_E = 4 + 10 \times .45 + 5 \times .9 = 13 \]

but, \( \overline{R_E} = 12 \)

Since E does not lie on the plane, there is an opportunity for riskless arbitrage. In this case, \( R_E = 13 > 12 = \overline{R_E} \), so the investor will sell E short and buy a combination of portfolios A, B, C, and D. This will drive down the price of E until its return is 13.
Question #16

a. First, calculate \( w \).

Number of days between settlement and next coupon payment: \( 126 \)
Number of days in the coupon period: \( 180 \)
\( w \) is therefore \((126/180)\) or \( 0.700 \)

Next, calculate the remaining cash flows.
The number of remaining coupon payments is 3.
The semi-annual interest rate is 2.5%.
The coupon is worth $4.00 per $100 of par.

<table>
<thead>
<tr>
<th>nominal cash flow</th>
<th>time in ( \frac{1}{2} ) yrs</th>
<th>Semi-annl yield</th>
<th>PV of $1 at 2.5%</th>
<th>pv dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.00</td>
<td>0.7</td>
<td>2.5%</td>
<td>0.982864</td>
<td>$3.93</td>
</tr>
<tr>
<td>$4.00</td>
<td>1.7</td>
<td>2.5%</td>
<td>0.958891</td>
<td>$3.84</td>
</tr>
<tr>
<td>$104.00</td>
<td>2.7</td>
<td>2.5%</td>
<td>0.935504</td>
<td>$97.29</td>
</tr>
</tbody>
</table>

\$105.06 is the full price of the bond.

b. Calculate accrued interest,

Number of days from last coupon payment to settlement date: \( 54 \)
Number of days in coupon period: \( 180 \)
Semiannual coupon payment: \( 4.00 \)
\( AI \) is then: \((54/180) \times 4\) \( = 1.20 \)
Question #17

a. First need \( V_0 \), the current price of the bond:

\[
V_0 = c \left[ 1 - \frac{1}{(1 + i)^n} \right] + \frac{M}{(1 + i)^n}
\]

where

\[
c = \$1000 * \frac{8\%}{2} = \$40
\]
\[i = 5\%
\]
\[M = \$1000
\]
\[n = 40
\]

So, \( V_0 = \$828.41 \)

\[
V_- = \$40 \left[ 1 - \frac{1}{(1 + .0495)^{40}} \right] + \frac{\$1000}{(1 + .0495)^{40}} = \$835.87
\]
\[
V_+ = \$40 \left[ 1 - \frac{1}{(1 + .0505)^{40}} \right] + \frac{\$1000}{(1 + .0505)^{40}} = \$821.06
\]

\[
d = \frac{V_- - V_+}{2(V_0)\Delta y}
\]

with \( \Delta y = .001 \)

\[
d = \frac{\$835.87 - \$821.06}{2 * \$828.41 * .001} = 8.94
\]
\[
\Delta p = -d \cdot \Delta y \cdot 100 \\
\Delta p = -8.94 \cdot (-0.02) \cdot 100 = 17.88\% 
\]

So, the approximate new price of the bond is

\[
$828.41 \cdot (1.1788) = $976.53 
\]

Finally, when yields decline from 10% to 8%, we know the price should equal $1000 (since yields now equal the coupon rate).

Thus, the dollar approximation error is $1000 – $976.53 = $23.47

b) The price/yield relationship of this bond, including the duration approximation, are graphed below.

(DRAWN BY HAND)

c) The convexity measure is given by:

\[
CM = \frac{V_v + V_\Delta - 2V_o}{2V_o(\Delta y)^2} 
\]
\[ CM = \frac{821.06 + 835.87 - 2 \times 828.41}{2 \times 828.41 \times (0.001)^2} = 66.39 \]

Now, the convexity adjustment for a 200 basis point drop is

\[ CA = CM \times (\Delta y)^2 \times 100 = 66.39 \times (0.02)^2 \times 100 = 2.66 \]

The new approximate price change for 200 basis points decrease is:

\[ TotalChange = 17.88\% + 2.66\% = 20.54\% \]

The (more accurate) estimated price is now

\[ p = 828.41 \times 1.2054 = 998.57 \]

So the new dollar approximation error is $1000 - 998.57 = $1.43

*Note that Part C is very sensitive to rounding of the bond prices. This sample solution uses two-decimal place rounding. The candidate could have rounded to 3 or 4 or more decimal places and obtained different answers than what is shown in the sample solution (for example, 3 decimal places yields a convexity measure of 62.77, a convexity adjustment of 2.511 and a dollar approximation error of $2.69).*

**Question #18**

a. \[ Yd = \frac{\text{[(F-P) / F]} \times (360/t)}{100} \]

\[ \frac{(100 - 99)}{100} \times \frac{360}{182} = 1.98\% \]

b. The discount rate differs from more standard return measures for two reasons:

First, the measure compares the dollar return to the face value rather than to the price.

Second, the return is annualized based on a 360-day year rather than a 365-day year.
Question #19

a. The liquidity premium demanded is probably small because investors typically follow a buy-and-hold strategy with commercial paper and so are less concerned with liquidity.

b. First, the investor in commercial paper is exposed to credit risk

   Second, interest earned from investing in Treasury bills is exempt from state and local income taxes. As a result, commercial paper has to offer a higher yield to offset this tax advantage.

Question #20

a. For mortgage pass-throughs, all investors receive a proportionate share of all principal and interest payments.

   For CMOs, the principal payments are paid “tranch by tranch”. Class A receives all principal until it is paid off, then Class B, then Class C.

b. P/Os receive only principal payments. As rates fall, refinancings increase, resulting in higher principal payments. This will increase the value of P/Os. In addition, as rates fall, the discount rate is less also causing the P/O to rise in value.

   I/Os receive only interest payments. As rates fall, refinancings increase, cutting into interest payments. This causes I/Os to fall. However, the discount rate is less, causing them to rise. The net effect is typically a FALL in price.

Question #21

a. I would manage the portfolio actively, constantly monitoring the value. Once the value reaches the trigger, I would switch my investment to the risk free rate and be guaranteed $11M by the end of the period.

b. I would still have 3 years left to invest at 10% rate therefore the trigger would be

\[
\frac{11M}{1.1^{3}} = 8.26M
\]
Question #22

a. If the trader is long the asset and short futures, the change in value is

$$\Delta S - h^* \Delta F$$

b. The optimal hedge ratio is the position in the hedge such that the variance of the hedged position is minimal. This occurs where the derivative of the variance of the hedged position as a function of the hedge ratio equals zero.

Given the value of the hedged position as discussed in a., the variance of the hedged position is

$$v = \sigma_s^2 + h^2 \sigma_F^2 - 2h \rho \sigma_s \sigma_F$$

So, you need

$$\frac{\partial v}{\partial h} = 2h \sigma_F^2 - 2 \rho \sigma_s \sigma_F = 0$$

$$h = \frac{\rho \sigma_s}{\sigma_F}$$

Question #23

a. - Short one share of stock.
   - Invest $20 at risk-free rate for 3 years.
   - Enter forward contract to buy share back for $23 in 3 years

b. Profit = \( S_0 e^{rt} - F_0 = 20 e^{(.05)3} - 23 \) = .24 per share.

c. If the forward price is low relative to the spot price, investors who hold the asset will sell it, invest the proceeds risk free, and enter into a contract to buy it back. This (spot) selling pressure will cause the spot price to fall until it is back in balance with the forward price.
Question #24

a. $95.90 * e^{0.50r} = $100
\[ \ln\left( \frac{100}{95.90} \right) = 0.50r \]
\[ r = 2 \ln\left( \frac{100}{95.90} \right) = 8.37\% \]

b. First need the 1 year zero rate:
\[ $91.00 * e^{1.00r} = $100 \]
\[ \ln\left( \frac{100}{91.00} \right) = 1.00r \]
\[ r = \ln\left( \frac{100}{91.00} \right) = 9.43\% \]

Now, Bond D pays $4 at T=0.50, $4 at T=1.00, and $104 at T=1.50
\[ \begin{align*}
 97.00 &= 4.00e^{-0.837*0.50} + 4.00e^{-0.943*1.00} + 104.00e^{-r*1.50} \\
e^{-r*1.50} &= 0.860808 \\
r &= 9.99\% 
\end{align*} \]

c. Compare the timing of the bond blows for D and E:

<table>
<thead>
<tr>
<th></th>
<th>time= 0.50</th>
<th>time= 1.00</th>
<th>time= 1.50</th>
<th>time= 2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond D</td>
<td>4</td>
<td>4</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>Bond E</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>106</td>
</tr>
</tbody>
</table>

Use the 0.50 and 1.00 zero rates to calculate the PVs of those first two cash flows:
\[ \begin{align*}
  PV(\text{first 2D}) &= 4.00e^{-0.837*0.50} + 4.00e^{0.0943*1.00} = 7.48 \\
  PV(\text{first 2E}) &= 6.00e^{-0.837*0.50} + 6.00e^{0.0943*1.00} = 11.21 
\end{align*} \]

Calculate the remainder of the bond price for D and E.
\[ \begin{align*}
  Re\ m(D) &= 97.00 - 7.48 = 89.52 \\
  Re\ m(E) &= 102.60 - 11.21 = 91.39 
\end{align*} \]

These prices are for the flows:

<table>
<thead>
<tr>
<th></th>
<th>time= 1.50</th>
<th>time= 2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond D</td>
<td>104</td>
<td></td>
</tr>
</tbody>
</table>
Now, 1.50 zero rate = 9.99% (already solved above)

So, the zero rate for time=2.00 is found as follows:

\[ 6.00e^{-0.999*1.50} + 106.00e^{-2.00*10} = 91.39 \]

\[ r = 10.32\% \]

Finally, the forward rate is:

\[ f = \frac{1.032*2 - 0.999*1.5}{2-1.5} = 11.33\% \]

Question #25

a. The current assumed annual benefit is calculated by using the estimated returns, here 10% per year:

Money at end of 5 years 100,000 x (1.10^5) = 161,051

Annual Benefit = (Annuity Value)/(i(1+i)^n)/(1+i)^n-1)

\[ n=5 \quad i=10.0\% \quad AV=161,051 \]

\[ B=(161,051)(0.1(1.10)^5)/(1.10)^5-1) = 42,484.85 \]

b. After the actual investment performance for 5 years is available, the account only has $152,460 in it. The assumed payment is now calculated as above:

\[ n=5 \quad i=10.0\% \quad AV=152,460 \]

\[ B=(152,460)(0.1(1.10)^5)/(1.10)^5-1) = 40,218.56 \]

\[ B_t=B_{t-1}(1+R)/(1+AIR) \]

\[ B_6=40,218.56(1.10)/(1.10) = 40,218.56 \]

\[ B_7=40,218.56(1.15)/(1.10) = 42,046.68 \]
Question #26

a.

b. Buy call w/ exercise price of $50, Sell two calls w/ exercise price $50.50, Buy call w/ exercise price $51

Buy call w/ exercise price of $60, Sell two calls w/ exercise price $60.50, Buy call w/ exercise price $61

Maturities all the same (Time = T)

We use these butterflys to create peaks and add them up to get what we want. We can make the peaks narrower by making the strike prices closer.
Question #27

a. \( s = $10, r = .06, x = $10.50 \)
\[
p = \frac{e^{rt} - d}{u-d} = \frac{e^{(.06)(.5)} - 0.9}{1.1-0.9} = 0.65
\]

\[
\begin{array}{c}
10 \\
\left[9\right]
\end{array}
\]
\[
10x1.1=11 \rightarrow 11x1.1=12.1
\]
\[
10x0.9=9 \rightarrow 10x0.9=9
\]
\[
11x.9=9.9 \rightarrow 11x.9=9.9
\]
\[
\left[0\right] \rightarrow \left[0\right] \rightarrow \left[0\right] \rightarrow \left[0\right] \rightarrow \left[0\right]
\]
\[
\left[.5\right] \rightarrow \left[.5\right] \rightarrow \left[.5\right] \rightarrow \left[.5\right] \rightarrow \left[.5\right]
\]
\[
\left[.6\right] \rightarrow \left[.6\right] \rightarrow \left[.6\right] \rightarrow \left[.6\right] \rightarrow \left[.6\right]
\]
\[
\left[1.5\right] \rightarrow \left[1.5\right] \rightarrow \left[1.5\right] \rightarrow \left[1.5\right] \rightarrow \left[1.5\right]
\]
\[
\left[2.4\right] \rightarrow \left[2.4\right] \rightarrow \left[2.4\right] \rightarrow \left[2.4\right] \rightarrow \left[2.4\right]
\]

\[
f_u = e^{-.06/2}[0(.65)+(.6)(.35)] = 0.2038
\]
\[
f_u = \max[0,0.2038] = 0.2038
\]

\[
f_d = e^{-.06/2}[(.6)(.65)+(.35)(2.4)]=1.1936
\]
\[
f_d = \max[1.5,1.1936]=1.5
\]

\[
f = e^{-.06/2}[(.65)(.2038)=(.35)(1.1936)]= .533 \text{ (without early exercise)}
\]

\[
f= e^{-.3}[(.65)(.2038)+(.35)(1.5)]=.638 \text{ (with early exercise)}
\]

value of American put is $0.638 with early exercise at t=0,t=1/2

b. \( \triangle = \frac{f_u-f_d}{s_o-u-s_o d} = \frac{0.2038-1.4}{(11-9)} = -.6481. \) Short 0.6482 shares.
Question #28

a. \[ R(l) = 4.979\% \]
   \[ R(h) = R(l) \times \exp(2 \times \sigma) = 4.979\% \times \exp(2 \times 0.1) = 6.081\% \]

b. \[ R(h) = 6.081\% \]
   \[ C = 12 \]
   \[ V(h) = \frac{212}{1.06081} = 199.85 \text{ – Won’t be called} \]
   \[ R(l) = 4.979\% \]
   \[ C = 12 \]
   \[ V(l) = \frac{212}{1.04979} = 201.95 \text{ – Will be called at 200} \]
   \[ V(0) = \frac{((199.85 + 12) + (200 + 12))}{2} \times \frac{1}{1.045} = 202.8 \]
   Value of callable bond = 202.8

c. Value of bond without call option
   \[ V(0) = \frac{((199.85 + 12) + (201.95 + 12))}{2} \times \frac{1}{1.045} = 203.83 \]
   Value of call option = 203.73 – 202.8 = 0.93

Question #29

\[ \mu = 11\% \]
\[ \sigma = 16\% \]
\[ t = 2 \]

\[ Z \sim N(\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{2}}) \]
\[ Z \sim N(0.11 - \frac{0.16^2}{2}, \frac{0.16}{\sqrt{2}}) \]
\[ Z \sim N(0.0972, 0.113) \]

\[ \text{Prob}(Z > .2) = 1 - \text{Prob}(Z < .2) \]
\[ = 1 - \Phi(\frac{.2 - 0.0972}{0.113}) \]
\[ = 1 - \Phi(0.91) \]
\[ = 1 - .8186 \]
\[ = 18.14\% \]
Question #30

a. The formula for calculating the stock change is
\[ \Delta S = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t} \]

For period 1, we have
\[ .886 = \mu \times 40 \times (1/365) + \sigma \times 40 \times 1.18 \times \sqrt{\frac{1}{365}} \]

For period 2, we have
\[ -.367 = \mu \times (40 + .886) \times (1/365) + \sigma \times (40 + .886) \times (-.53) \times \sqrt{\frac{1}{365}} \]

Solving these simultaneous equations leads to
\[ \mu = 0.245, \sigma = 0.348 \]

The current drift rate then equals
\[ \mu S = .245 \times 40 = 9.80 \]

b. The standard deviation of the rate of return is just \( \sigma \), so \( \sigma = .348 \).

Question #31

a. For a call option,
\[ \Delta = N(d_1) = .7405 \quad \Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{t}} = 0.0398 \quad \text{Vega} = S_0 \sqrt{t} N'(d_1) = 29.85 \]

b. For gamma and vega neutrality, the overall gamma and overall vega must = 0.
\[-50 + 0.0398w_1 + 0.0436w_2 = 0 \]
\[-20000 + 29.85w_1 + 16.36w_2 = 0 \]
\[w_2 = 1,070.99, \quad w_1 = 83.03 \]

Therefore, 83.03 units of the first option and 1,070.99 units of the second option should be bought.

c. If the portfolio was originally delta neutral, then the new delta following the addition of these two assets is:
\[ (83.03)(.7405) + (1,070.99)(.5931) = +696.69. \]

Therefore, 696.69 units of the underlying asset must be sold to maintain delta neutrality.
Question #32

a.

Similar to efficient frontier for CAPM. Return can be any measure selected by company. Generally should be consistent with maximizing shareholder/policyholder value. Risk can also be any measure selected by company. It must be consistent. Company will never select actions with results below frontier because can always get more return for same risk if below the frontier.

b. The ALEF performance objective can be any financial or economic measure or combination of such measures, and can be chosen to reflect any time horizon. It should be consistent with the maximization of shareholder value. The classical efficient frontier objective is the expected rate of return on the portfolio.

Question #33

a.

1. Focuses on distribution of results rather than point estimates.
2. Incorporates correlations between variables to better model impact of various risk factors.
3. Through computing advances, we are able to do more thorough analyses with many simulations to build a distribution of results, rather than looking at a few isolated scenarios.

b.

1. Models are generalizations of reality in general
2. Can not account for factors that are not actuarially calculable.
3. Based on past experience and current operational plan, such that if significantly different events occur, results will not be accurate.
Question #34

Calculate Market Value Surplus:

\[ MVS = MV(Assets) - MV(Liabilities) \]

\[ MV(Assets) = 7,353 + 43,765 + 52,056 = 103,174 \]

\[ \text{Duration (Assets)} = \frac{7,353 \times 0.5 + 43,765 \times 7.0 + 52,056 \times 10.0}{7,353 + 43,765 + 52,056} = 8.05 \]

\[ MV(Liabilities) = 39,028 \]

Therefore, \( MVS = 103,174 - 39,028 = 64,146 \)

\[ \text{Duration (Surplus)} \times MVS = \text{Duration (Assets)} \times MV(Assets) - \text{Duration (Liabilities)} \times MV(Liabilities) \]

\[ = > \text{Duration of Market Value Surplus(MVS)} = 8.05 \times 103,174 - 7.0 \times 39,028 = 8.689 \times 64,146 \]

Question #35

a. Call provisions provide the issuer the option to repurchase the bonds from the investors prior to maturity at a special call price. Bonds with sinking fund provisions require the issuer to repurchase the bonds incrementally over time according to a specified schedule. An investor purchasing a bond with one of these provisions bears additional risks such as limited capital appreciation, reinvestment risk, and uncertainty of future cash flows. Since the issuer has the right to exercise options, the investor is faced with potentially disadvantageous calls on his investment and therefore expects higher returns.

b. The presence of a sinking fund lowers the risk of default to the investor since principal is retired gradually, thus preventing the issuer from having to make one large balloon payment at maturity.

c. The additional advantage is that the issuer of the bond may provide price support by repurchasing the bonds on the open market in order to satisfy the sinking fund requirement. This price support will be realized when interest rates are rising and there is downward pressure on bond prices.

The disadvantage is that the bonds may be called at the special sinking fund price at a time when interest rates are falling, preventing the investor from taking advantage of higher market prices.
Question #36

a.  
1. Duration of assets must equal duration of liabilities
2. Present/market value of the assets must be greater than the present value of the liabilities
3. Dispersion of the assets must be greater than the dispersion of the liabilities.

b. With multi-period liabilities, the liability duration is derived using the assets' IRR as the discount factor. The IRR of the assets cannot be determined unless the precise portfolio composition, its duration, and its dispersion are known.

c.  
1. An initial IRR for the assets is estimated.
2. Duration, dispersion, and present value of the liabilities is determined using the estimated IRR in step 1.
3. An optimal portfolio is created to match the duration and dispersion estimates in step 2.
4. The optimal portfolio IRR is compared to the estimated IRR in step 1. If they differ, a new IRR is estimated and the process is repeated.

Question #37

a.  

<table>
<thead>
<tr>
<th>Year</th>
<th>Pattern</th>
<th>Time (in yrs)</th>
<th>Amount</th>
<th>Discount factor @ 8% (not 5%)</th>
<th>Amt*Discount</th>
<th>Time<em>Amt</em>Dscnt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50%</td>
<td>0.5</td>
<td>2,500,000</td>
<td>0.9623</td>
<td>2,405,750</td>
<td>1,202,875</td>
</tr>
<tr>
<td>2</td>
<td>30%</td>
<td>1.5</td>
<td>1,500,000</td>
<td>0.8910</td>
<td>1,336,500</td>
<td>2,004,750</td>
</tr>
<tr>
<td>3</td>
<td>20%</td>
<td>2.5</td>
<td>1,000,000</td>
<td>0.8250</td>
<td>825,000</td>
<td>2,062,500</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>5,000,000</td>
<td>4,567,250</td>
<td>5,270,125</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Duration = 5,270,125 / 4,567,250 = 1.1539

b. Asset/liability matching mitigates the risk of interest rate changes. But the procedure does not model the true effects of interest rate changes on insurance loss payments. Cash flows from property/casualty insurers are inflation sensitive.
c. If liability losses are sensitive to inflation through the settlement date, then the reserve is equivalent to an asset with a duration of zero years. That is, to eliminate the influence of interest rate changes on net worth, you should invest either in short term securities (e.g., commercial paper and Treasury Bills) or in securities that are also inflation sensitive (e.g., common stocks and real estate. Expected yields on Commercial paper and Treasury bills are too low for large carriers. Real estate and similar investments are limited by regulation and are too risky for all but the most experienced investment managers. Common stocks, therefore, are the apparent investment of choice.

Question #38

a. \[ \text{VaR} = \text{Sigma (day)} \times \text{sqrt}(N) \times Z \times 10M \]
\[ 736,811 = \text{Sigma (day stock A)} \times \text{sqrt}(10) \times (2.33) \times 10M \]
\[ \text{Sigma (day stock A)} = 0.01 \]

b. If perfectly correlated, then no risk reduction

\[ 1,105,216 = \text{Sigma (day stock B)} \times \text{sqrt}(10) \times (2.33) \times 10M \]
\[ \text{Sigma (day stock B)} = 0.015 \]

\[ \text{VaR portfolio} = (0.5 \times 0.01 + 0.5 \times 0.015) \times \text{sqrt}(10) \times (2.33) \times 10M \]
\[ = 921,013 \]

Could have done \[ \text{VaR} = 0.5 \times 736,811 + 0.5 \times 1,105,215 = 921,013 \]

c. \[ \text{Sigma portfolio} = \text{Sigma (stock A)}^2 + \text{Sigma (stock B)}^2 + 2 \times \text{Sigma (stock A)} \times \text{Sigma (stock B)} \times \rho (\text{stock A and stock B}) \]

\[ \text{VaR} = \text{same as 10M in a)} = 736,811 \]
\[ 736,811 = \text{Sigma (day stock A)} \times \text{sqrt}(10) \times (2.33) \times 10M \]
\[ \text{Sigma (day stock A)} = 0.01 \]

\[ 1,105,216 = \text{Sigma (day stock B)} \times \text{sqrt}(10) \times (2.33) \times 10M \]
\[ \text{Sigma (day stock B)} = 0.015 \]

\[ 100,000^2 = \text{Sigma portfolio}^2 = [0.01 \times 5M]^2 + [0.015 \times 5M]^2 + [2 \times 0.01 \times 0.015 \times 5M \times 5M \times \rho (\text{stock A and stock B})] \]

\[ 100,000^2 = 50,000^2 + 75,000^2 + 7,500,000,000 \times \rho (\text{stock A and stock B}) \]

\[ 1,875,000,000 = 7,500,000,000 \times \rho (\text{stock A and stock B}) \]

\[ \rho (\text{stock A and stock B}) = 0.25 \]
Question #39

a. Adjusted Net Worth = (Market Value of Bonds) + (Non-Admitted Assets) +  
   (Adjustment for collectible Liabilities for Unauthorized Reinsurance) –  
   (Adjustment for Uncollectible Reinsurance for Authorized Reinsurance) -  
   (Principal & Interest owed on Surplus Notes or Debentures) - (Adjustments  
   for Investments in Affiliates) – (Loss Reserve adjusted for deficiency) +  
   (Adjustment for Tax Considerations)

   $2,000 – ($1,000 + $200) = $800

b. The cost of capital is a reduction in the value of a company based on the  
   premise that invested capital and surplus would not earn the same rate of  
   return if those funds were freely invested. (Sturgis suggests a theoretical  
   “regulatory statutory surplus,” e.g., 1/3 of net WP.)

   $1000 x (10% - 5%) = $50

c. Unless there is a yield differential at the same level of risk, the opportunity  
   cost of the restricted investment choices for the capital is not risk-neutral.

   COC = $1000 x (6% - 5%) = $10

   Miccolis says zero COC if the risk associated with the investment income on  
   capital & surplus is directly reflected in the valuation. The appropriate  
   discount rate to apply to the annual COC would appear to be the risk-free rate  
   since there is no real risk involved in the cost of capital itself.

   $10/1.04 = $9.62

d. Adjusted Net worth + PV Future Earnings – Cost of Capital

   800 + (300/1.1) - $9.62 = 800 + 272.73 - 9.62 = $1,063.11
Question #40

a. Classical risk theory has ignored the severity of ruin.

b. Expected Policyholder Deficit. A reasonable measure of insolvency risk is the expected value of the difference between the amount the insurer is obligated to pay the claimant and the actual amount paid by the insurer.

c. EPD is more appropriate since it measures the dollar severity of insolvency risk which represents the policyholders’ exposure to loss in the case of insolvency. EPD can consistently measure insolvency risk in such a way that a standard minimum level of protection is applied to all classes of policyholders and insurers. EPD can apply equally to all risk elements, whether assets or liabilities.

Question #41

a. At 1,000 additional assets, EPD = .25 x [.2 x (15,000 – 11,000)] = 200
200/10,950 = 1.8%. Therefore, will need more than 1,000 and can focus solely on liability of 15,000 and assets at minimum of 10,000 + Additional assets

\[ \text{EPD} = .01 \times (10,950) = 109.5 \]
\[ 109.5 = .25 \times .2 \times (15,000 - \text{Assets}) \]
\[ 109.5/(.25 \times .2) = 15,000 - \text{Assets} \]
\[ \text{Assets} = 15,000 - 109.5 / (.25 \times .2) \]
\[ = 15,000 - 2,190 \]
\[ = $12,810 \]

Therefore, additional capital required on December 31, 2002 = 12,810 – 10,000 = $2,810

The additional capital required on January 1, 2002 is $2,810/1.05 = $2,676.

b. Expected value additional capital = EAC

Again, this deficit only occurs when assets are at their minimum value as in b above. At that contingent asset outcome, the additional capital = 10,000/15,000 x their expected value. Therefore,

\[ 109.5 = .25 \times .2 \times [15,000 - 10,000 - \text{EAC} (10,000/15,000)] \]
\[ 2,190 = 5,000 - 2/3 \times \text{(EAC)} \]
\[ \text{EAC} = (5,000 - 2,190) \times 3/2 \]
\[ \text{EAC} = $4,215 \]