INSTRUCTIONS TO CANDIDATES

1. This 85 point examination consists of 43 essay questions. The number of points for each full question or part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors.

Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.

Do not answer more than one question on a single sheet of paper. Write on only the lined side of the paper, and be careful to give the number of the question you are answering on each sheet.

The answer should be concise and confined to the question as posed. When a list of a specific size is requested, do not offer more items in your list than the number requested. For example, if you are requested to list three items, only the first three responses will be graded.

If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as Page 1 of 2 on the first sheet of paper and then Page 2 of 2 on the second sheet of paper.

In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

2. Attached to the examination, after question 43, is a table of the Normal Distribution.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

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5. At the beginning of the examination, check through the exam booklet for any missing or defective pages. The supervisor has additional exams for those candidates who have defective exam booklets.

6. **Candidates must remain in the examination center until two hours after the start of the examination. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.**

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer pages in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. **BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.**

Anything written in the examination booklet will not be graded. **Only the answer sheets will be graded.**

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. (Do not put the self-addressed stamped envelope inside the Examination Envelope.)

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a **copy** of the examination by contacting the CAS Office.

All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. An examination survey and postage-paid reply envelope are included with the examination. No postage is necessary for surveys mailed within the United States. Candidates mailing the survey outside the United States should use the courtesy reply envelope distributed by your exam supervisor. This survey is also available on the CAS website in the “Exams” section. **Please complete the survey and leave it with the examination supervisor, or take the survey and envelope with you when leaving the examination center. Please submit the survey to the CAS Office by May 23, 2004. Please do not enclose the survey in the Examination Envelope.**

END OF INSTRUCTIONS
1. (1.75 points)

A stock index is constructed based on the following three stocks for the period:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Year 2000</th>
<th>Year 2001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Shares</td>
</tr>
<tr>
<td>A</td>
<td>$20.00</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>$30.00</td>
<td>300</td>
</tr>
<tr>
<td>C</td>
<td>$100.00</td>
<td>200</td>
</tr>
</tbody>
</table>


a. (1 point)

Calculate the rate of return on a price-weighted index.

b. (0.75 point)

Calculate the rate of return on a market-value-weighted index.

SHOW ALL WORK.

2. (1 point)

A Treasury bill has a bank discount yield of 2.5% and a maturity of 90 days.

Calculate the following:

a. (0.5 point)

The bond equivalent yield.

b. (0.5 point)

The effective annual yield.

SHOW ALL WORK.
3. (3 points)

You are given the following information:

- Expected return of the risky asset $(E(r)) = 0.13$
- Variance of the risky asset $(\sigma^2) = 0.01$
- Risk-free rate = 0.06
- Coefficient of risk aversion $(A) = 5$
- Utility function: $U = E(r) - 0.01 A \sigma^2$

a. (1 point)

Calculate the expected return and standard deviation of a portfolio that is invested 40.0% in the risky asset and 60.0% in a risk-free asset.

b. (2 points)

Determine the optimal amount to invest in each asset to maximize the utility.

SHOW ALL WORK.
4. (3.5 points)

   a. (1 point)

      Explain the significance of the Separation Property in determining optimal complete
      portfolios for two clients with different degrees of risk aversion.

   b. (1.5 points)

      Assuming an investor cannot borrow, graph, in expected return-standard deviation space,
      the relationship between:

      - Capital Allocation Line
      - Efficient Frontier of Risky Assets
      - Indifference Curve

      Be sure to label the axes, and each line/curve.

   c. (1 point)

      Briefly describe the importance of the following points from the graph in part b. above:

      1. The point where the Efficient Frontier and Capital Allocation Line meet
      2. The point where the Indifference Curve and Capital Allocation Line meet

SHOW ALL WORK.
5. (2 points)

Briefly describe four of the six simplifying assumptions that lead to the basic version of the capital asset pricing model (CAPM).

6. (1 point)

Calculate the expected return for an individual stock using the capital asset pricing model (CAPM) and the following information:

- Risk-free rate is 3.0%.
- Expected market return is 10.0%.
- Standard deviation of the market is 40.0%.
- Standard deviation of the individual stock is 25.0%.
- The correlation of the individual stock with the market is 0.88.

SHOW ALL WORK.
7. (2 points)

The zero-beta model is an extension of the simple capital asset pricing model (CAPM), and is used when one of the major assumptions underlying the model is relaxed.

a. (0.5 point)

Which assumption of the capital asset pricing model is the zero-beta model intended to address?

b. (0.5 point)

Briefly define the zero-beta portfolio.

c. (1 point)

Assume the zero-beta version of the CAPM holds. Calculate the expected return of security A, \( E(r_A) \), given the following information:

- Expected market return, \( E(r_M) \), is 9.0%.
- Expected return on the zero-beta portfolio, \( E(r_{z(b)}) \), is 5.0%.
- Risk-free rate, \( R_f \), is 4.0%.
- Standard deviation of the market portfolio, \( \sigma_M \), is 10.0%.
- Beta of security A, \( \beta_A \), is 0.75.

SHOW ALL WORK.
8. (2.25 points)

You have the following information about two stocks:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Expected Return</th>
<th>( \beta_i )</th>
<th>Firm-specific Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15.0%</td>
<td>1.2</td>
<td>40.0%</td>
</tr>
<tr>
<td>B</td>
<td>10.0%</td>
<td>0.8</td>
<td>30.0%</td>
</tr>
</tbody>
</table>

- The market index has a standard deviation of 25.0%.
- The risk-free rate is 5.0%.
- Portfolio \( P \) consists of:
  - 50.0% of Stock A
  - 30.0% of Stock B
  - 20.0% of a risk-free security

a. (0.5 point)

Calculate the expected return of the portfolio, \( E(r_p) \).

b. (0.5 point)

Calculate the beta of the portfolio, \( \beta_p \).

c. (0.5 point)

Calculate the systematic component of the variance of the portfolio.

d. (0.5 point)

Calculate the non-systematic component of the variance of the portfolio.

e. (0.25 point)

Calculate the total variance of the portfolio.

SHOW ALL WORK.
9. (2.5 points)

Assume there are two independent economic factors, T and S. All stocks have independent firm-specific components with standard deviations of 40.0%.

You have the following information on well-diversified portfolios:

<table>
<thead>
<tr>
<th>Portfolio 1</th>
<th>Portfolio 2</th>
<th>Portfolio 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{1T} = 2.0$</td>
<td>$\beta_{2T} = 0.8$</td>
<td>$\beta_{3T} = 1.2$</td>
</tr>
<tr>
<td>$\beta_{1S} = -0.5$</td>
<td>$\beta_{2S} = 1.0$</td>
<td>$\beta_{3S} = 1.0$</td>
</tr>
<tr>
<td>$E(r_1) = 13.0%$</td>
<td>$E(r_2) = 10.0%$</td>
<td>?</td>
</tr>
</tbody>
</table>

The risk-free rate is 4.0%.

Calculate the expected return of Portfolio 3, $E(r_3)$.

SHOW ALL WORK.

10. (2.5 points)

a. (1.5 points)

Briefly describe the three different versions of the efficient market hypothesis.

b. (0.25 point)

Briefly describe the concept of technical analysis.

c. (0.25 point)

Briefly explain what the efficient market hypothesis implies about the merit of technical analysis.

d. (0.5 point)

Provide two reasons portfolio management still has a role in an efficient market.
11. (1 point)

Fama and MacBeth expanded the estimation of the Security Market Line using the following equation:

\[ r_i = \gamma_0 + \gamma_1 \beta_i + \gamma_2 \beta_i^2 + \gamma_3 \sigma(e_i) \]

a. (0.5 point)

What does the term \( \gamma_3 \) measure?

b. (0.5 point)

What would you conclude if \( \gamma_3 \) were positive after performing the Fama and MacBeth tests?

12. (2 points)

Altman, in "Measuring Corporate Bond Mortality and Performance," provides a methodology to calculate the cumulative mortality rate for a cohort of bonds.

You have the following information:

The value of a cohort of 100 bonds was $500,000 as of January 1, 2001. You are given the following information by year about this cohort of 100 bonds:

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of bonds defaulting</th>
<th>Number of bonds redeemed, maturing, or called</th>
<th>Value of defaulted bonds</th>
<th>Loss on defaulted bonds</th>
<th>Value of bonds redeemed, maturing, or called</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>1</td>
<td>2</td>
<td>$30,000</td>
<td>$12,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>2002</td>
<td>2</td>
<td>3</td>
<td>$40,000</td>
<td>$16,000</td>
<td>$20,000</td>
</tr>
<tr>
<td>2003</td>
<td>5</td>
<td>7</td>
<td>$50,000</td>
<td>$20,000</td>
<td>$28,000</td>
</tr>
</tbody>
</table>

Calculate the cumulative mortality rate for this cohort of bonds as of December 31, 2003.

SHOW ALL WORK.
13. (1 point)

A bond has a current yield of 6.0% and a yield to maturity of 7.0%.

a. (0.5 point)

Is the bond selling above or below par value? Briefly explain why.

b. (0.5 point)

Is the annual coupon rate of the bond higher, lower, or equal to 6.0%? Briefly explain why.

14. (1.25 points)

A 10-year maturity, 7.0% coupon bond paying coupons semiannually is selling at a yield-to-maturity of 5.0%.

a. (0.5 point)

Calculate the current yield.

b. (0.75 point)

Assume that you sell the bond in one year at a yield-to-maturity of 4.0%.

Calculate the holding period return.

SHOW ALL WORK.
15. (4 points)

For $1,000 zero-coupon bonds of varying maturities, determine the missing values (A through H) in the following tables.

<table>
<thead>
<tr>
<th>Year</th>
<th>Effective rate in Year i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>8.0%</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>11.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Zero-Coupon Bond Maturity (n, in years)</th>
<th>Price of n-year Zero-Coupon Bond</th>
<th>n-Year Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$943.40</td>
<td>E</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>F</td>
</tr>
<tr>
<td>3</td>
<td>$805.08</td>
<td>G</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>H</td>
</tr>
</tbody>
</table>

SHOW ALL WORK.

16. (3 points)

a. (1.5 points)

Briefly explain why bonds of different maturities have different yields in terms of the:

1. Expectations hypothesis
2. Liquidity Preference hypothesis
3. Market Segmentation hypothesis

b. (1.5 points)

Briefly describe the implication of each of these hypotheses when the yield curve is downward sloping.
17. (1 point)

The following is a list of prices for $1,000 par value zero-coupon bonds of various maturities.

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$950</td>
</tr>
<tr>
<td>2</td>
<td>$850</td>
</tr>
</tbody>
</table>

Calculate the forward rate during the second year.

SHOW ALL WORK.

18. (2 points)

The current price of copper is $0.85 per pound. The storage costs are $0.08 per pound per year, payable quarterly in advance. The current estimated convenience yield is 10.0% of the spot price.

Assume that interest rates are 5% per annum, with continuous compounding, for all maturities.

a. (1 point)

Calculate the futures price of copper for delivery in nine months.

b. (1 point)

Describe the concept of convenience yield and how it reflects the market's expectations concerning the future availability of the commodity.

SHOW ALL WORK.
19. (1.5 points)

The three-month interest rate in Japan is 2.0% per annum and in the United States it is 5.0% per annum, both with continuous compounding.

The spot price of the Japanese Yen is $0.0085. The futures price for a contract deliverable in three months is $0.0086.

a. (0.5 point)

Calculate the theoretical futures price for this contract.

b. (1 point)

Describe the transaction by which an arbitrage profit could be achieved.

SHOW ALL WORK.

20. (3 points)

A stock is expected to pay a dividend of $3 per share in two months and again in eight months.

The stock price is $300 and the annual risk-free rate of interest is 5.0% with continuous compounding for all maturities.

An investor has just taken a long position in a nine-month forward contract on the stock.

a. (1.5 points)

Calculate the forward price.

b. (1.5 points)

Three months later, the price of the stock is $310 and the risk-free rate of interest is still 5% per annum. Calculate the value of the long position.

SHOW ALL WORK.
21. (1.5 points)

Answer the following questions on hedging strategies.

a. (0.5 points)

Describe a perfect hedge.

b. (1 point)

Does a perfect hedge always lead to a better outcome than an imperfect hedge? Explain your answer.

22. (1.5 points)

A company owns a portfolio of stocks worth $10 million with a beta of 1.50.

The company would like to use futures contracts on the S&P 500 index to change the beta of the portfolio to 2.00.

The S&P 500 index is currently $1,000 and the contract size is 250 times the index.

a. (0.25 point)

Should you take a long or a short position in the index futures contract for the company to achieve its goal?

b. (1.25 points)

Given your answer in part a. above, calculate the commensurate number of index futures contracts needed for the company to achieve its goal.

SHOW ALL WORK.
23. (1 point)

Explain why the Black-Scholes-Merton equation results in a risk-neutral valuation of derivatives.

24. (1 point)

Explain how a put option can be viewed as providing insurance when it is held in conjunction with a stock.

25. (6 points)

You are given the following information for a European call option on a stock:

- Time to maturity is six months.
- The stock has a current price of $75.
- The option has a strike price of $72.
- The volatility of the stock’s price is 25.0%.
- The risk-free rate is 6.0% compounded continuously.
- The stock pays dividends of $0.75 and $1.50 after two and five months respectively.

a. (2 points)

If $d_1 = 0.3202$, use the Black-Scholes model to calculate the price of the call option.

b. (4 points)

If the call option is American rather than European, use Black’s approximation method to calculate the value of the call option.

SHOW ALL WORK.
26. (2.5 points)

The Black-Scholes formula to derive the price of a call option is dependent on several key assumptions. Two of these assumptions are listed below:

- Assumption #1: The volatility of the underlying stock’s price is known and constant over the life of the option.
- Assumption #2: The stock price changes smoothly and does not experience large up or down jumps in price in a short time.

According to Black in “How to Use the Holes in Black-Scholes” and given information on the following two options on a stock, answer the questions below.

- Option A: A one-year European call option with current stock price of $50 and a strike price of $50.
- Option B: A one-year European call option with current stock price of $50 and a strike price of $60.

a. (0.75 points)

If Assumption #1 is relaxed, which option’s value will be most distorted by using the Black-Scholes formula? Explain your answer.

b. (1 point)

If Assumption #2 is relaxed and the jumps in stock price are assumed to be symmetric, describe the impact to the Black-Scholes formula value for each option. Explain your answer.

c. (0.75 points)

If an investor believed that upward jumps in stock price are more likely to occur than reflected in the market, how would the investor trade Option B to take advantage of this belief? Explain your answer.
27. (1 point)

A stock price is currently $50.00. It is known that at the end of two months, it will be either $54.00 or $46.00. The risk-free interest rate is 9.0% per annum with continuous compounding.

Calculate the value of a two-month European call option with a strike price of $48 on this stock.

SHOW ALL WORK.

28. (1 point)

Consider a stock with a current price of $45.00. At the end of six months, it can go either up 6.0% or down 6.0%.

You have sold 1,000 six-month European call options on this stock with a strike price of $46.00.

Calculate the number of shares of stock that you would need to purchase in order to create a riskless hedge.

SHOW ALL WORK.

29. (2 points)

Assume the current one-year spot rate is 5.0% and that the one-year spot rate one year from now will be either 3.0% or 7.0% with equal probability.

a. (1 point)

Determine the value of a two-year bond with a face value of $800 and an annual coupon (paid at the end of each year) of 6.0% using a binomial interest rate tree.

b. (0.75 point)

Assume an identical bond has an embedded call option with a strike price at 100% of par exercisable at time \( t=1 \). Determine the value of this bond.

c. (0.25 point)

Calculate the value of the embedded call option.

SHOW ALL WORK.
30. (3 points)

You have the following information:

- A stock will go up in value by 12.0% or down in value by 8.0% in a given year.
- The risk-free rate of interest is 5.0% per annum and the price of the stock is now $60.
- There exist options with an exercise price of $70 and maturity of two years.
- There are no dividends.

Assuming two years until expiration, determine the value of an American call on the stock.

SHOW ALL WORK.

31. (2 points)

Explain two characteristics of inverse floaters that may cause their value to decline as interest rates rise.

32. (1 point)

A bond has a Macaulay duration of 10.0 and a convexity of 400.

Using annual compounding, the yield-to-maturity for this bond is 8.0%.

Using modified duration and convexity, estimate the percentage price change of this bond for a 75 basis point rise in yield-to-maturity.

SHOW ALL WORK.
33. (2 points)

The loss and allocated loss adjustment expense payout pattern for an insurance company is as follows:

<table>
<thead>
<tr>
<th>Payout Year</th>
<th>Cumulative Percent Paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>20%</td>
</tr>
<tr>
<td>3</td>
<td>50%</td>
</tr>
<tr>
<td>4</td>
<td>75%</td>
</tr>
<tr>
<td>5</td>
<td>100%</td>
</tr>
</tbody>
</table>

Assuming that all loss and allocated loss adjustment expense payments are made at mid-year, calculate the Macaulay duration of liabilities using a 5.0% interest rate.

SHOW ALL WORK.

34. (1.5 points)

On June 1 a portfolio manager has a bond portfolio worth $25 million. The duration of the portfolio in September will be 5.5 years. The November Treasury-bond futures price is currently 94-15 and the cheapest-to-deliver bond will have duration of 7.0 years at maturity.

Calculate the number of bond futures contracts that should be shorted to immunize the portfolio against changes in interest rates over the next three months.

SHOW ALL WORK.

35. (2 points)

If financial risk management can theoretically add value to a firm, it must do so through “violations” of one or more of Modigliani’s and Miller’s assumptions in the formulation of their irrelevance propositions.

Explain two of the three assumptions that might be “violated” and, for each, how financial risk management could add value to a firm.
36. (2.5 points)

You are about to finish your first half-year as CEO of a company. Your bonus is worth $1,000,000 and is directly tied to the company stock price, which is currently $40.00.

You want to protect your current bonus against a fall in stock price of 10.0% in the next six months and you want to buy insurance against this event. The risk-free rate is 3.0% per year, dividend yield is 2.0% and volatility is 40.0% per annum.

Assume you can only buy traded European put options.

Calculate the amount you would expect to pay for this financial peace of mind.

SHOW ALL WORK.

37. (2 points)

a. (0.5 point)

Briefly explain what the gamma of a portfolio measures.

b. (0.5 point)

If you owned a portfolio of European call options on a non-dividend paying stock, explain what would happen to the gamma of the portfolio if you added a position in the underlying stock to the portfolio.

c. (0.5 point)

If the value of a portfolio’s vega is high (in absolute terms), what does this say about the portfolio’s value?

d. (0.5 point)

Consider a European call option on a dividend-paying stock. In absolute terms, how would you expect the value of the option’s vega to change as the price of the stock moves from out of the money to in the money?
38. (2.25 points)

You are given the following information regarding two stocks:

- Stock A: Share Price = $100, Expected Annual Return = 10.0%, Annual Volatility = 48%
- Stock B: Share Price = $50, Expected Annual Return = 5.0%, Annual Volatility = 24%

Assume that you have a position of $2,000,000 in Stock A and $1,000,000 in Stock B.

Also assume there are 252 trading-days in a year.

You would like to determine a loss amount that you are 99% certain will not be exceeded during the next 10 days.

Assuming that the returns on the two shares have a bivariate normal distribution with a correlation of 0.4, calculate this loss amount.

SHOW ALL WORK.

39. (2 points)

You are given the following information:

The value of assets is uncertain and is distributed as follows:

<table>
<thead>
<tr>
<th>Asset Amount (in millions)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30</td>
<td>30%</td>
</tr>
<tr>
<td>$20</td>
<td>40%</td>
</tr>
<tr>
<td>$15</td>
<td>30%</td>
</tr>
</tbody>
</table>

Liabilities are uniformly distributed between $10 million and $20 million, and assets and liabilities are independent.


SHOW ALL WORK.
40. (2.5 points)

Consider an insurance company that faces the following discrete claims cost distribution for all of its lines of business combined over a one-year time horizon:

<table>
<thead>
<tr>
<th>Probability Mass</th>
<th>Present Value Loss ($ Millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0</td>
</tr>
<tr>
<td>0.25</td>
<td>1.0</td>
</tr>
<tr>
<td>0.10</td>
<td>1.5</td>
</tr>
<tr>
<td>0.04</td>
<td>3.0</td>
</tr>
<tr>
<td>0.01</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Assume this is the only risk the insurance company faces.

a. (0.5 point)

Use “P&C RAROC: A Catalyst for Improved Capital Management in the Property and Casualty Insurance Industry” for the definition of required economic capital.

Calculate the required economic capital, assuming that the company targets a one-year 98% solvency standard.

b. (0.5 point)

During the next year, the company expects to earn $1.1 million in premium and to incur, in addition to losses, $200,000 in expenses. The company also expects to earn $160,000 on the capital that it holds. These amounts are all on a present value basis. The corporate income tax rate is 35 percent. Using your answer from part a. above, calculate the company’s after-tax expected risk-adjusted return on capital (RAROC).

c. (0.5 point)

Assuming a 10% hurdle return on capital, can the company expect to add shareholder value through its activities? Explain briefly.

d. (0.5 point)

The company holds statutory surplus of $2.3 million and carries $700,000 in reserves on an undiscounted basis. It has $100,000 of unrealized capital gains on bonds. There is no real estate appreciation. Calculate available equity capital.

e. (0.5 point)

Given that the company is targeting a 98% solvency standard, is its available equity capital adequate? Explain briefly.

SHOW ALL WORK.
41. (1 point)

Based on the Casualty Actuarial Society *Statement of Principles Regarding Property and Casualty Valuations*:

a. (0.5 point)

Explain how a reinsurance program can decrease valuation risk.

b. (0.5 points)

Explain how a reinsurance program can increase valuation risk.

42. (1 point)

a. (0.5 point)

Briefly discuss how the ratio of price to operating cash flow may be a more useful ratio in assessing the value of a firm than the price to earnings (P/E) ratio.

b. (0.5 point)

Briefly discuss under what circumstances the price to sales ratio could be a more useful ratio than the price to earnings ratio (P/E) for evaluating a company.
43. (1.5 points)

Consider the following information for Company A and Company B

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The market risk premium is 5.0%.

The risk-free rate is 5.0%.

a. (1 point)

Calculate the price to earnings (P/E) ratio for each company using the dividend discount model.

b. (0.5 point)

Which company is a better value?

SHOW ALL WORK.

END OF EXAMINATION

23
### The Normal Distribution

\[
\Pr(X \leq x) = \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} \, dt
\]

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Exam 8
Question 1

a. \[
\frac{2000}{20 + 30 + 100} = 50
\]
\[
50 = \frac{20 + 30 + 50}{x} \Rightarrow x = 2
\]
New divisor due to split

\[
\frac{2001}{30 + 20 + 60} = 55
\]

Rate of return = \(\frac{55}{50} - 1 = 10\%\)

b. \[
\text{MV} = (20 \times 100) + (30 \times 300) + (100 \times 200) = 31,000
\]
\[
\text{MV} = (30 \times 100) + (20 \times 300) + (60 \times 400) = 33,000
\]

Rate of return = \(\frac{33,000}{31,000} - 1 = 6.45\%\)
Exam 8
Question 2

a. \[ r_{BEY} = \frac{\text{Par} - \text{Price}}{\text{Price}} \times \frac{365}{n} \]
\[ = \frac{100 - 99.375}{99.375} \times \frac{365}{90} \]
\[ = 2.55\% \]

b. \[ r_{BD} = \frac{\text{Par} - \text{Price}}{\text{Price}} \times \frac{360}{n} = 2.5\% \]

\[ r_{EFF} = \left( \frac{100}{\text{Price}} \right)^{\frac{365}{n}} - 1 \]
\[ = \left( \frac{100}{99.375} \right)^{\frac{365}{90}} - 1 \]
\[ = 2.575\% \]
Exam 8  
Question 3

a. Expected return = (40% * 13%) + (60% * 6%)  
= 8.8%

Standard deviation = 40% * \sqrt{0.01}  
= 0.04

b. Utility of portfolio with portion y invested in the risky asset and 1-y in the risk-free asset is:

\[ U = y * 13 + (1-y)6 - 0.01 * 5 * y^2 \times 10^2 \]
\[ = 6 + 7y - 5y^2 \]

which is maximized when

\[ 0 = \frac{dU}{dy} = 7 - 10y \]

So \[ y = 70\% \]

Thus to maximize utility 70% of the portfolio should be invested in the risky asset and 30% in the risk-free asset.
Exam 8
Question 4

a. Separation Property:
   Determining complete portfolio divided into 2 steps
   
   **Step 1:** Determine the optimal risk portfolio
   => this will be the same for both investors despite their different degrees of risk aversion

   **Step 2:** Determine the proportion to invest in the risky portfolio & the proportion to invest in the risk-free asset
   => Different proportions depending on degree of risk aversion

b. 

\[ \text{E}(r) \]

\[ \text{Indifference curve } \rightarrow \]

\[ \text{Efficient frontier} \]

\[ \text{Capital Allocation Line} \]

\[ r_f \]

\[ s \]

c. 1. Pt. p is the optimal risky portfolio
   2. Pt. c is the investor’s complete portfolio & depends on the amount invested at the risk-free rate & the amount invested in the risky portfolio.
1. Many investors, each with wealth that is small compared to the total wealth of all investors. Investors are price takers, in that they act as if their trades have no affect on stock prices.

2. All investors have the same identical holding period.

3. Investors are limited to the universe of publicly traded financial assets and risk-free borrowing and lending arrangements.

4. Investors pay no taxes on returns and incur no fees on trades in securities.
Exam 8
Question 6

\[ r_f = 0.03 \quad E(r_m) = 0.10 \quad \sigma_m = 0.40 \quad \sigma_i = 0.25 \quad \rho = 0.88 \]

\[ \beta_i = \frac{\text{cov}(r_i, r_m)}{\sigma_i^2} = \rho \cdot \frac{\sigma_i \sigma_m}{\sigma_m^2} = \frac{0.88(0.40)(0.25)}{(0.40)^2} = 0.55 \]

\[ E(r_i) + r_f + \beta(E(r_m) - r_f) \]

\[ = 0.03 + 0.55(0.10 - 0.03) = 0.0685 \]
Exam 8
Question 7

a. It addresses the assumption that investors can borrow and lend at the risk free rate.

b. The zero beta portfolio is on the inefficient part of the minimum variance frontier. Each efficient portfolio has a corresponding zero beta portfolio that it is uncorrelated with.

c. 
\[ E(r_A) = E(r_a) + \beta_A(E(r_m) - E(r_a)) \]

\[ = 5.0 + 0.75 (9.0 - 5.0) \]

\[ = 8\% \]
σ_n = .25
r_f = .05

50% A
30% B
20% r_f

Beta of risk free = 0

a. \[ E(r_p) = W_A(E(r_A)) + W_B(E(r_B)) + W_f(E(r_f)) \]
   \[ = .5 (.15) + .3 (.10) + .2 (.05) = .115 \]

b. \[ B_p = \text{?} \]
   \[ B_p = .5 (1.2) + .3 (.8) + .2 (0) = .84 \]

c. Syst. component = \( B_p s_m^2 \)
   \[ = (.84)^2 (.25)^2 = .0441 \]

d. Non syst. component = \( s^2(e_p) \)
   \[ s^2(e_p) = \text{?} \]
   \[ s^2(e_p) = .5^2 (.4)^2 + .3^2 (.3)^2 + .2^2 (0)^2 \]
   \[ = .0481 \]

e. Total variance = \( B_p^2 s_m^2 + s^2(e_p) \)
   \[ = .0441 + .0481 = .0922 \]
Portfolio 1

\[.13 = .04 + 2.0(T - .04) - 0.5 (S - .04)\]

\[.13 = .04 + 2T - .08 - 0.5S + .02\]

\[.13 = -2T - 0.5S - 0.2\]

\[.15 = 2T - .5S\]

Portfolio 2

\[.10 = .04 + .8(T - 0.4) + 1(S - 0.4)\]

\[.06 = .8T - .032 + S - 0.4\]

\[.132 = .8T + S\]

Let

\[T = E(r_T)\]

\[S = E(r_s)\]

2 \times Portfolio 1

\[.3 = 4T - S\]

\[.132 = .8T + S\]

\[.432 = 4.8T\]

\[.09 = T\]

\[.132 = (.8)(.09) + S\]

\[S = .06\]

\[E(r_3) = r_f + \beta_{3T} (E(r_T) - r_f) + \beta_{3S} (E(r_s) - r_f)\]

\[= .04 + 1.2 (.09 - .04) + 1 (.06 -.04)\]

\[= .12 \text{ or } 12\%\]
Exam 8  
Question 10  

a. Weak form- stock prices reflect all information that can be gathered by studying past trading data, such as prices & volume.  
   Semi-strong form- stock prices reflect all fundamental information about a company that is publicly available, such as information about their product line, earnings forecasts, & quality of company management.  
   Strong form- all relevant information is reflected in a stock’s price, even that information that is only available to company insiders.  

b. Technical analysis is analysis of the history of stock prices in the attempt to identify trends that can be used to predict future stock prices.  

c. The efficient market hypothesis indicates that there is no value to technical analysis.  Even the weak-form indicates that stock prices already incorporate any relevant information from the path of past stock prices.  

d. 1. Investors need to make sure that they have established a well-diversified portfolio that coincides w/the level of risk they are willing to accept.  
   2. Investors also need to make sure that their portfolios are established appropriately considering their time horizon of investment.
a. $\gamma_3$ measures the significance of the nonsystematic or firm specific components of the return.

b. You would conclude that the Capital Asset Pricing Model is not valid (b/c it doesn’t include a firm-specific component) and that you could earn additional return through investing in firm-specific risk.
Exam 8
Question 12

CMR_t = 1 - ? survival rate,

SR= Survival rate = 1 – marginal mortality rate
MMR = Marginal mortality rate = Amount of defaulted issues in year
    Bond population at start of the year

MMR_{2001} = \frac{30,000}{500,000} = 6\%
SR = 1 – MMR = 94\%

MMR_{2002} = \frac{40,000}{500,000 - 30,000 - 10,000} = \frac{40,000}{460,000} = 8.7\%
SR = 91.3\%

MMR_{2003} = \frac{50,000}{460,000 - 40,000 - 20,000} = \frac{50}{400} = 12.5\%
SR = 87.5\%

CMR_t = 1 – (0.94)(0.913)(0.875) = 24.9\%
a. Current yld < YTM means that the bond is selling at a discount; i.e. below par. This is because the investor won’t buy the bond at par if the yield is lower than market rates. He’ll only buy it if he can get the coupon yield + capital appreciation, which means he’ll only buy it if the current price < par.

b. Annual coupon rate < current yld in this case since the denominator of coupon rate = par and the denominator of current yld = current price. As described above, current price < par; so current yld > coupon rate.
a. Calculate price with calculator:
\( n = 20 \)
\( FV = 100 \quad PV = 115.59 \)
\( PMT = 3.5 \)
\( i = 2.5 \)

\[
\text{current yield} = \frac{\text{coupon}}{\text{Price}} = \frac{7}{115.59} = 6.056\%
\]

b. Need to calculate price in 1 year:
\( n = 18 \)
\( FV = 100 \quad PV = 122.49 \)
\( PMT = 3.5 \)
\( i = 2.0 \)

\[
\text{HPR} = \frac{\text{Dividend} + (P_t - P_o)}{P_o}
\]

Note: See BKM p. 131
HPR ignores reinvestment of dividends

\[
= \frac{3.5 + 3.5 + (122.49 - 115.59)}{115.59}
= 12.03\%
\]

Also given full credit:

a. Assume par = 1000

\[
\text{Price} = \sum_{i=1}^{20} \frac{35}{(1.025)^i} + \frac{1035}{(1.025)^{20}} = 1155.89
\]

\[
\text{Current yield} = \frac{\text{Annual coupon}}{\text{Price}} = \frac{70}{1155.89} = 6.06\%
\]

b. \[
\text{Price} = \sum_{i=1}^{18} \frac{35}{(1.02)^i} + \frac{1035}{(1.02)^{18}} = 1224.88
\]

\[
\text{HPR} = \frac{35(1.025) + 1035 + 1224.88 - 1155.89}{1155.89}
\]

Assume that 1st coupon of $35 can be reinvested for 6-months at 2.57 rate

\[
= 12.17
\]
a. \( \frac{1000}{943.40} - 1 = 6\% \)

b. \( \frac{1000}{805.08} \) in parentheses \((1.06)(1.08)\) \(- 1 = 8.5\% \)

c. \( \frac{1000}{(1.06)(1.08)} = $873.52 \)

d. \( \frac{1000}{(1.06)(1.08)(1.085)(1.11)} = $725.3 \)

e. Same as a) = 6\%

f. \( [(1.06)(1.08)]^{1/2} - 1 = 7\% \)

g. \( \left( \frac{1000}{805.08} \right)^{1/3} - 1 = 7.5\% \)

h. \( [(1.06)(1.08)(1.085)(1.11)]^{1/4} - 1 = 8.36\% \)
Exam 8
Question 16

a.  
1. Expectation hypothesis
   Expected yields on bonds of different maturities reflect market expectations of future short rates.

2. Liquidity preference
   Majority of investors prefer short term bonds, therefore yields on longer maturity bonds include a risk premium.

3. Market segmentation
   Yields of short-long maturity bonds each find their equilibrium yield in their respective market, independently of other maturity market.

b.  
1. Expectation hypothesis
   Expect future short rates to decrease.

2. Liquidity preference
   Since risk premium for longer maturity, it implies future short rates are expected to decrease even more than under expectations hypothesis.

3. It means that longer term bonds have a lower yield determined by its own market than short term bonds as determined by equilibrium in short term bond market.
Exam 8
Question 17

\[ \frac{1000}{950} = (1 + y_1) \quad \text{Zero rate (y_i)} \quad 5.26\% \]

\[ \frac{1000}{850} = (1 + y_2) \quad 8.47\% \]

\[
(1 + .0526)(1 + f_2) = (1.0847)^2 \\
1 + f_2 = 1.1176 \\
f_2 = 11.76\% 
\]
Exam 8  
Question 18

\[ S_0 = 0.85 \quad U = 0.08/yr \text{ only in advance} \quad y = 0.10 \quad r = 0.05 \]

\[ a. \quad F = (S_0 + U)e^{(r-y)T} \quad T = 0.75 \]

\[ U = 0.02 + 0.02e^{-0.05(25)} + 0.02e^{-0.05(5)} = 0.059 \]

\[ F = (0.85 + 0.059)e^{(0.05-0.1)(0.75)} = 0.876 \]

\[ b. \quad \text{For consumption goods the convenience yield reflects the fact that it may be better to hold on to the commodity than to sell it and buy it back later. If } y \text{ is high, this indicates that the market expects there to be future shortages of the commodity. If } y \text{ is low, it reflects that there may be enough of the commodity in the future.} \]
Exam 8  
Question 19

\[ r_f = 2\% \]
\[ r = 5\% \]
\[ T = 0.25 \]
\[ s_o = 0.0085 \]
\[ f_o = 0.0086 \]

a. \[ F_o = s_o e^{(r-r_f)T} \]
\[ = 0.0085 e^{(0.05 - 0.02)(0.25)} \]
\[ = 0.00856 \]

b. Futures contract is overpriced:
- borrow U.S. dollars at domestic risk-free rate
- invest in yen at Japanese rate for 3-months
- enter into a forward contract to deliver Japanese yen at a rate of $0.0086 per year
- at \( T = 0.25 \) → pay off U.S. dollar loan
  → remainder is a risk-free profit
Exam 8  
Question 20

a. \( F_0 = (S_0 - I)e^{rt} \)
   
   \[ I = 3 \cdot e^{0.05 \times \frac{2}{12}} + 3 \cdot e^{0.05 \times \frac{8}{12}} = 2.9751 + 2.9016 = 5.8767 \]
   
   \[ F_0 = (300 - 5.8767)e^{0.05 \times \frac{9}{12}} = 305.56 \]

b. Three months later, \( F_1 = (S_1 - I_1)e^{rt} \)
   
   \[ I_1 = 3 \cdot e^{0.05 \times \frac{5}{12}} = 2.938 \]
   
   \[ F_1 = (310 - 2.938)e^{0.05 \times \frac{6}{12}} = 314.84 \]
   
   Value of the long position = \( (F_1 - k)e^{rt} = (314.84 - 305.36)e^{0.05 \times \frac{6}{12}} = 9.246 \)
a. A perfect hedge is a hedge which completely eliminates the uncertainty, making risk zero.

b. No, perfect hedge does not always lead to a better outcome. Perfect hedge just eliminates the uncertainty. But if the market goes favorably to the investor, investor will not obtain these profits. In this case, imperfect hedge will do better.
a. You want more sensitivity to market movements. You want to go long on the futures.

b. \[ N = (B^* - B)\left(\frac{P}{A}\right) \]

\[ N = (2.0 - 1.5) \left(\frac{10M}{1000 \times 250}\right) \]

Go long on 20 contracts.
Differential equation of Black-Scholes:

\[
    r \cdot f = \frac{\partial f}{\partial t} + r \cdot s \cdot \frac{\partial f}{\partial s} + \frac{s^2 \sigma^2}{2} \cdot \frac{\partial^2 f}{\partial s^2}
\]

1. All variables in the formula: \( r, s, T, \sigma \) are independent of risk preferences.

2. The variable reflecting risk preferences, \( \mu \rightarrow \) return on the stock, is not in the formula.

3. Since the formula is independent of risk preferences, we can assume any risk preference, including risk neutrality.

4. \( \mu = \pi \rightarrow \) Risk free rate on the stock
A put held with a stock gives the holder the option to sell the stock at the strike price. It ensures that the holder’s value of the stock will not fall below the strike price.
Exam 8
Question 25

\[ T = 6 \text{ months} \quad d_1 = .3202 \]

\[ S_0 = 75 \]

\[ K = 72 \]

\[ s = 25\% \]

\[ r = 6\% \]

Dividends .75 at 2 months, 1.5 at 5 months

\[ \text{a.} \quad C = S_0 \text{N}(d_1) - Ke^{-rt} \text{N}(d_2) \]

Since stock has dividends reduce \( S_0 \) by PV of div payments

\[
\text{PV of div} = .75e^{-0.06(2/12)} + 1.5e^{-0.06(5/12)} = 2.206
\]

\[ S_0 - D = 75 - 2.206 = 72.794 \]

\[ d_1 = .3202 \]

\[ d_2 = .3202 - \sigma \sqrt{T} = .3202 - .25\sqrt{5} = .1434 \]

\[
\begin{align*}
N(3.202) &= .6253 + .02(.6293 - .6255) = .6256 \\
N(1.434) &= .5557 + .34(.5596 - .5557) = .5570 \\

c &= 72.794(.6256) - 72e^{-0.06(5)}(.5570) \\
c &= 6.62
\end{align*}
\]

\[ \text{b.} \]

\[ K(1-e^{-r(t_n-t_2)}) = 72(1 - e^{-0.06(6/12 - 5/12)}) = 0.359 < 1.5 = D_2 \rightarrow \text{Exercise immediately before the 2nd ex-dividend.} \]

\[ I = 0.75e^{-0.06(2/12)} = 0.7425 \]

\[ T = 5/12 \]

\[ S_0 = 75 - .7425 = 74.2575 \]

\[ d_1 = \frac{\ln\left( \frac{S_0}{K} \right) + (r + \frac{s^2}{2})T}{s \sqrt{T}} = \frac{\ln(75 - 0.7425/72) + (0.06 + \frac{0.25^2}{2})(5/12)}{0.25\sqrt{5/12}} = 0.4269 \]

\[ d_2 = 0.4269 - 0.25\sqrt{5/12} = 0.2655 \]

\[
\begin{align*}
N(d_1) &= N(0.4269) = .6628 + .69 (.6664 - .6628) = .6653 \\
N(d_2) &= N(0.2655) = .6026 + .55 (.6064 - .6026) = .6047 \\
c &= S_0N(d_1) - Ke^{-rt}N(d_2) \\
&= 74.2575(.6653) - 72e^{-0.06(5/12)}(.6047) = 6.94
\end{align*}
\]

Greater of 6.6215 and 6.94

\[ \therefore \quad \text{American call 6.94} \]

NOTE: Full credit was also given in part b if candidates did not test the demands for exercise opportunities.
Exam 8
Question 26

a. Option B, out-of-the-money options are more sensitive to changes in volatility than at-the-money options.

b. Relaxing assumption #2 as described here increases the value of out-of-the-money call option B and decreases the value of the at-the-money call option A.

c. If upward jumps are more likely to occur than reflected in the market, option B may be underpriced. Therefore investor should buy option B.
Exam 8
Question 27

\[ \Delta = \frac{6 - 0}{54 - 46} = 0.75 \]

\[ S_T = 54 \quad S_T = 56 \]

@ K = 48 call option payoff

\[ \begin{array}{ll}
\text{stock price} & \text{call payoff} \\
6 & 6 \\
0 & 0 \\
\end{array} \]

? = .75 shares

\[ \begin{array}{ll}
\text{Borrow PU(34.5)} & \text{Payoff} \\
-34.5 & 40.5 \\
6 & 34.5 \\
\end{array} \]

Payoffs are the same

Value of call @ Time 0 = \[ \frac{3}{4} S_o - PU(34.5) \]

\[ = \frac{3}{4} (50) - 34.5 e^{-0.05 \frac{3}{12}} = 3.51 \]
Exam 8
Question 28

\[ ? = \frac{f_u - f_d}{S_u - S_d} \]

\[ S_u = 47.7 = 45 \times 1.06 \]
\[ S_d = 42.3 = 45 \times (1 - .06) \]

\[ f_u = 47.7 - 46 = 1.7 \]
\[ f_d = 42.3 - 46 = \Delta \]

\[ \Delta = \frac{1.7 - 0}{47.7 - 42.3} = .3148 \]

Buy \( ? \) shares for every option sold

\[ 1000 \times .3148 = 314.8 \]

Buy 315 shares.
Exam 8
Question 29

\[ r_1 = 0.05 \]

\[ V = \frac{1}{2} \left( \frac{V_H + C}{1 + r_1} + \frac{V_L + C}{1 + r_1} \right) \]

a. \[ V_H = \frac{800 + 48}{1.07} = 792.52 \]
\[ V_L = \frac{800 + 48}{1.03} = 823.30 \]
\[ V = \frac{1}{2} \left( \frac{792.52 + 48}{1.05} + \frac{823.30 + 48}{1.05} \right) = 815.15 \]

b. Bond would be called if interest rate dropped to 3%, so substitute 800 for \( V_L \)
\[ V = \frac{1}{2} \left( \frac{792.52 + 48}{1.05} + \frac{800 + 48}{1.05} \right) = 804.06 \]

c. Value of call = value of optionless – value of callable
\[ = 815.15 - 804.06 = 11.09 \]
Tree:

\[ p = e^{r-t} - d = e^{-0.05 \times 0.92} = 0.6564 \]

\[ u - d = 1.12 - 0.92 \]

Value @ $67.20: Node = e^{-0.05 \times 0.6564} [5.26 + (1 - 0.6564)(0)] = 3.28 \] (Exercised value) = 0

Value @ $60: Node = e^{-0.05 \times 0.6564} [3.28 + (1 - 0.6564)(0)] = 2.05 \] (Exercised value) = 0

Since exercising early is not profitable at any point value is the same as European call

$2.05
1. The coupon paid will decrease because the coupon rate is inversely tied to the interest rate.

2. The discount rate used is higher, so the present value of the cash flows will decrease.
Exam 8
Question 32

\[ D = 10.0 \]
\[ \text{Convexity} = 400 \]
\[ R = .08 \text{ (annual)} \]

\[ D^* = \frac{D}{1.08} = \frac{10}{1.08} = 9.259 \]

\[ \frac{\partial P}{P} = -D^* 84 + \frac{1}{2} (\text{convexity})(84)^2 \]

\[ \frac{\partial P}{P} = (-9.259)(.0075) + \frac{1}{2}(400)(.0075)^2 \]

\[ \frac{\partial P}{P} = -0.05819 \]

If interest rates rise by 75 basis points, the price of the bond will decrease by 5.819\%
Exam 8
Question 33

<table>
<thead>
<tr>
<th>Time</th>
<th>Pmt</th>
<th>PV(Pmt)</th>
<th>T x PV(Pmt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>10%</td>
<td>0.09759</td>
<td>0.048795</td>
</tr>
<tr>
<td>1.5</td>
<td>10%</td>
<td>0.09294</td>
<td>0.13941</td>
</tr>
<tr>
<td>2.5</td>
<td>30%</td>
<td>0.26555</td>
<td>0.663875</td>
</tr>
<tr>
<td>3.5</td>
<td>25%</td>
<td>0.21075</td>
<td>0.737625</td>
</tr>
<tr>
<td>4.5</td>
<td>25%</td>
<td>0.20072</td>
<td>0.90324</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>0.86755</td>
<td>2.492945</td>
</tr>
</tbody>
</table>

\[D = \text{Macaulay duration} = \frac{(4) \text{ total}}{(3) \text{ total}} = 2.87355\]

\[D = \frac{\sum t \times PV(Pmt_t)}{\sum PV(Pmt_t)}\]
\[ N = \frac{D_P}{D_F} \times \frac{N_A}{Q_A} \]

Size of T-Bond futures contract

\[ \frac{9415}{32} = 94.46875\% \text{ of } \$100,000 \text{ face value} \]

\[ = 94468.75 \]

\[ N = \frac{5.5}{7.0} \times \frac{25,000,000}{94468.75} = 207.9 \]

208 contracts should be shorted.
1. Tax rates are progressive, with higher tax rates applied against higher increments of income. Financial risk management (FRM) may help smooth earnings patterns over several years and add value by reducing the average tax rate.

2. Firms in financial distress behave in sub-optimal ways. The costs of distress include direct costs (legal fees for bankruptcy filings) and indirect costs (interference with the firm’s business plan). The costs are reflected in the firm’s value (stock price) by reducing the stock price. To the degree FRM can reduce the probability of financial distress the firm’s value should increase by the reduction in the expected costs of financial distress. The maximum possible increase in the firm’s value is the expected unhedged cost of distress \[\text{Pr(distributed if unhedged)} \times \text{(expected cost of distress)}\].
Buy a put on stock at \( K = 40 \times 0.9 = 36 \)

\( t = \frac{1}{2}, S_0 = 40, s = 0.4, r = 0.03, q = 0.02 \)

Will price a call and then use put-call parity

\[
d_1 = \frac{\ln \left( \frac{S_0}{K} \right) + \left( r - q + \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{T}}
\]

\[= 0.532 \quad N(d_1) = 0.7026 \text{ (interpolated)}\]

\[d_2 = d_1 - \sigma \sqrt{T} = 0.249 \quad N(d_2) = 0.5983\]

\[
c = S_0 e^{qT} N(d_1) - K e^{-rT} N(d_2)
\]

\[= 6.61\]

\[c + K e^{-rT} = p + S_0 e^{qT} \Rightarrow p = 2.47 \text{ for each option}\]

Assuming CEO receives \( \frac{1,000,000}{40} = 25,000 \) shares

Total cost = \( 25,000 \times 2.47 = $61,750 \)
a. Gamma measure the rate of change in the portfolio’s delta with respect to the change in the underlying assets value.
\[ \Gamma = \frac{d^2\Pi}{ds^2} \]

b. Unchanged since the gamma of the stock is assumed to be 0.

c. The portfolio’s value changes significantly with changes in the volatility of the underlying asset.

d. Since the value of the derivative portfolio increases, and assuming that the volatility remains unchanged, then the vega should increase.
Var(A + B) = 2m^2 (.030)^2 + 1m^2 (.015)^2 + 2(2m)(1m)(.4)(.030)(.015)  
= 4,545,000,000  
Where m = 1 million

s_{A+B} = 67,416.62

99% 10 Day Var = 2.33\left(\sqrt{10}\right)(67416.62) = 496,732.83

From normal table
Asset Amt | Liability is uniform on (10M, 20M) | 
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$30M</td>
<td>never a deficit</td>
</tr>
<tr>
<td>$20M</td>
<td>never a deficit</td>
</tr>
<tr>
<td>$15M</td>
<td>prob = 30%</td>
</tr>
<tr>
<td>(A-L)P(L)</td>
<td>Avg deficit = 2.5M</td>
</tr>
</tbody>
</table>

EPD = 15% (2.5M) = 375,000
a. (1/2 point) *diagram top of page 4, Nakada.*

Required economic capital at a 98% solvency standard

\[ \text{PV Loss @ 98\% - E(PV Loss)} = 3.0 - (0.25)(1.0) + (0.10)(1.5) + (0.04)(3.0) + (0.01)(5.0) \]

\[ = 3.0 - 0.57 \]

\[ = \$2.43 \text{ million} \]

b. (1/2 point) *second equation page 10, Nakada.*

RAROC = \[ \frac{\text{Premium - PV(Expected Claims) - PV(Expenses) + Interest on Capital}}{\text{PV(Economic Capital)}} \] * (1 – tax rate)/PV(Economic Capital)

\[ = (1.1 - 0.57 - 0.20 + 0.16)* (1 - 0.35)/2.43 \]

\[ = 13.1\% \]

c. (1/2 point) *top of 2nd column page 11, “Activities with RAROC’s above the hurdle rate increase shareholder value, while activities with RAROC’s below the hurdle rate diminish shareholder value.”*

Yes, the company expects to add shareholder value since its RAROC (13\%) exceeds the hurdle rate (10\%)

d. (1/2 point) *second column, page 11.*

Available equity capital = Statutory Surplus
+ Unrealized gains on bonds
+ Discounting adjustment to reserves
+ Adjustment for reserve conservatism
+ Real Estate appreciation

\[ = 2.3 + 0.1 + (0.7 - 0.57) + 0 \]

\[ = \$2.53 \text{ million} \]

e. (1/2 point) *second column, page 11. “Capital adequacy is measured by comparing required capital (Economic Capital) with available equity capital.”*

Yes, the company has \$2.53 million in available equity capital, whereas its required capital is \$2.43 million. At a 98\% solvency standard, the company has \$100,000 excess capital.
Exam 8
Question 41

a. Reinsurance decreases the volatility of net losses and therefore decreases the uncertainty in loss valuations.

b. Reinsurance may introduce credit risk, which will increase the uncertainty of the valuation.
a. Price to earnings (P/E) ratio is impacted by how earnings are calculated. Therefore it is impacted by accounting rules. Cash flow does not have this weakness because cash flow is easily defined, unlike the lack of well-defined definition for earnings in calculating P/E ratio. The price to operating cash flow may be a more useful ratio.

b. Price-to-sales ratio can be useful for new start-up firms who have not broken even in their operations. Therefore, their earnings are negative and P/E cannot be calculated for such companies. In such situations, price-to-sale is used. However, keep in mind that profit margins do vary from company to company. This will impact the sales figure.
E[r_m] - [r_f] = 0.05

R = E(r) = r_f + \beta (E(r_m) - r_f)

r_f = 0.05

R_A = E(r_A) = 0.110

R_B = E(r_B) = 0.105

a. Company A: \( \frac{P}{E} = \frac{1 - b_A}{R_A - g_A} = \frac{1 - 0.25}{0.11 - 0.035} = 10.0 \)

Company B: \( \frac{P}{E} = \frac{1 - b_B}{R_B - g_B} = \frac{1 - 0.75}{0.105 - 0.050} = 4.54545 \)

b. Company B has better value.