Exam 9
Advanced Ratemaking, Rate of Return,
And Individual Risk Rating Plans

INSTRUCTIONS TO CANDIDATES

1. This 100 point examination consists of 48 questions divided into two sections. Section I contains 20 multiple choice questions worth 1 point each. Section II contains 28 problem and essay questions worth a total of 80 points.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil. Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Please make your marks dark and fill in the spaces completely. Fill in that it is Fall 2003, and the exam number 9.

Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. (For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row.) Please write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.

For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. For the problem and essay questions, the number of points for each full question or part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink.

Write your Candidate ID number and the examination number, 9, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.

Do not answer more than one question on a single sheet of paper. Write on only the lined side of the paper, and be careful to give the number of the question you are answering on each sheet.

The answer should be concise and confined to the question as posed. When a list of a specific size is requested, do not offer more items in your list than the number requested. For example, if you are requested to list three items, only the first three responses will be graded.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS
In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

4. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. At the beginning of the examination, check through the exam booklet for any missing or defective pages. The supervisor has additional exams for those candidates who have defective exam booklets.

7. Verify that you have received the reference materials:

8. Candidates must remain in the examination center until two hours after the start of the examination. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

9. At the end of the examination place the included reference materials, the short-answer card and all answer sheets in the Examination Envelope. Please insert your answer pages in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

Anything written in the examination booklet will not be graded. Only the short-answer card and the answer sheets will be graded.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS
10. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. (Do not put the self-addressed stamped envelope inside the Examination Envelope.)

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a copy of the examination from the CAS website.

All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

11. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

12. An examination survey and postage-paid reply envelope are included with the examination. No postage is necessary for surveys mailed within the United States. Candidates mailing the survey outside the United States should use the courtesy reply envelope distributed by your exam supervisor. This survey is also available on the CAS website in the “Exams” section. Please either complete the survey and leave it with the examination supervisor, take the survey and envelope with you when leaving the examination center, or submit the survey online. Please submit your survey to the CAS Office by November 17, 2003. Please do not enclose the survey in the Examination Envelope.

END OF INSTRUCTIONS
1. According to the American Academy of Actuaries Committee on Risk Classification's "Risk Classification Statement of Principles," which of the following statements are intentions of risk classification?

1. to identify good and bad risks
2. to predict the experience for an individual risk
3. to group individual risks having reasonably similar expectations of loss

A. 1 only
B. 2 only
C. 3 only
D. 1 and 3 only
E. 2 and 3 only
2. Which of the following statements is false for private passenger auto experience rating?

A. Credibility assigned to an individual risk within a highly refined classification rating plan would be higher than the credibility assigned in a less refined rating plan.
B. Credibility for experience rating depends on the amount of variation of individual hazard within the class.
C. Credibility for experience rating is significant and measurable when based on data from one car for one year.
D. Credibility for classification rating increases in proportion to the square root of the volume of data.
E. Credibility within a highly refined private passenger classification rating system would be larger where a wide range of hazard is encompassed within a classification.
3. You are determining the price for a workers compensation policy effective January 1, 2004 for an insured doing business entirely in South Carolina. The insured has one factory that manufactures widgets and no other operations. Assume the insured is eligible for both experience and schedule rating.

Which of the following independent scenarios should result in the application of a schedule rating credit?

A. Insured stopped conducting drug testing on new hires in 2002 due to a shortage of qualified workers.
B. Insured has had a full-time doctor and nurse on site for the past ten years.
C. Insured’s only full-time risk manager for the factory left the job in June 2002 and insured has no plans to replace him.
D. Insured installed new safety devices in 2003 on two machines that had been the cause of many accidents over the previous three years.
E. Insured has experienced a 20% increase in widget demand in each of the past three years.
4. Given the following information, calculate the general liability experience modification credit or debit for this insured.

<table>
<thead>
<tr>
<th>Basic limit losses subject to experience modification</th>
<th>$75,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic limit expected losses</td>
<td>$100,000</td>
</tr>
<tr>
<td>Expected experience ratio for company subject loss cost of $75,000</td>
<td>0.883</td>
</tr>
<tr>
<td>Expected experience ratio for company subject loss cost of $100,000</td>
<td>0.909</td>
</tr>
<tr>
<td>Credibility for company subject loss cost of $75,000</td>
<td>0.460</td>
</tr>
<tr>
<td>Credibility for company subject loss cost of $100,000</td>
<td>0.530</td>
</tr>
</tbody>
</table>

A. -0.175  
B. -0.093  
C. -0.069  
D. +0.069  
E. +0.093
5. An insured has the following products/completed operations losses:

<table>
<thead>
<tr>
<th>Policy Period</th>
<th>Total Limit</th>
<th>Allocated Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1, 2000 – December 31, 2000</td>
<td>$10,000</td>
<td>$0</td>
</tr>
<tr>
<td></td>
<td>$17,500</td>
<td>$2,000</td>
</tr>
<tr>
<td></td>
<td>$50,000</td>
<td>$50,000</td>
</tr>
<tr>
<td>January 1, 2001 – December 31, 2001</td>
<td>$0</td>
<td>$15,000</td>
</tr>
<tr>
<td></td>
<td>$7,000</td>
<td>$500</td>
</tr>
<tr>
<td></td>
<td>$40,000</td>
<td>$25,000</td>
</tr>
<tr>
<td>January 1, 2002 – December 31, 2002</td>
<td>$150,000</td>
<td>$125,000</td>
</tr>
</tbody>
</table>

Maximum Single Loss = $60,000  
Basic Limit = $25,000

What is the total amount of actual losses that would be included in the calculation of the general liability experience modification for a policy effective January 1, 2004?

A. $127,000  
B. $180,000  
C. $222,000  
D. $262,000  
E. $492,000
6. Which of the following is false with respect to workers compensation experience rating?

A. Experience rating can partially correct errors in class relativities.
B. Experience rating can partially correct errors in a rate level that is too low or too high.
C. In the long run, more stability results from experience rating.
D. Indicated rate level increases will be higher when the off-balance moves above unity.
E. The tendency in experience rating is for too high a rate level to result in a credit off-balance.
7. Given the following information, calculate the tax multiplier.

<table>
<thead>
<tr>
<th>Premium Tax Rate</th>
<th>4.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insolvency Fund Loss Based Assessment</td>
<td>1.0%</td>
</tr>
<tr>
<td>Second Injury Fund Loss Based Assessment</td>
<td>1.5%</td>
</tr>
<tr>
<td>Permissible Loss Ratio</td>
<td>60.0%</td>
</tr>
</tbody>
</table>

A. Less than 1.050  
B. At least 1.050, but less than 1.055  
C. At least 1.055, but less than 1.060  
D. At least 1.060, but less than 1.065  
E. At least 1.065
8. You are given the graph with labeled areas P, Q, R, S and T and the information below.

\[ F(y) \]

Where:

- \( A = \) Actual Unlimited Loss of the Risk
- \( E = E[L] = \) Expected Loss
- \( y = \) \( A/E \)
- \( F = \) Cumulative distribution function of \( y \)
- \( X(r) = \) Excess pure premium ratio

\[
L = \begin{cases} 
  r_1E & \text{if } A \leq r_1E \\
  A & \text{if } r_1E < A \leq r_2E \\
  r_2E & \text{if } r_2E < A 
\end{cases}
\]

Which of the following statements is false?

A. \( Q + S + T = 1 \)
B. \( r_2 = 1 - T + R + P \)
C. \( X(r_1) = S + T \)
D. \( E[L]/E[A] = P + Q + S \)
E. \( E[L] = 1 - T + P \)
9. Using Lee's notation, which of the following are true?

1. $\Phi^*(r) = \int_r^{r_G} (y - r) dF^*(y) + k$

2. $\Psi^*(r) = \int_0^r (r - y) dF^*(y)$

3. $E(L^*)/E = 1.00 + \Psi^*(r_H) - \Phi^*(r_G) - k$

A. 1 only
B. 2 only
C. 3 only
D. 1 and 2 only
E. 1, 2, and 3
10. Given the information below for a retrospectively rated policy, calculate the basic premium.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Premium</td>
<td>1,000,000</td>
</tr>
<tr>
<td>Loss Conversion Factor</td>
<td>1.1</td>
</tr>
<tr>
<td>Expected Loss Ratio</td>
<td>0.7</td>
</tr>
<tr>
<td>Total Expenses, excluding taxes, as a ratio of Standard Premium</td>
<td>0.25</td>
</tr>
<tr>
<td>Table M Charge at Maximum</td>
<td>0.057</td>
</tr>
<tr>
<td>Table M Savings at Minimum</td>
<td>0.041</td>
</tr>
</tbody>
</table>

A. Less than $185,000
B. At least $185,000, but less than $190,000
C. At least $190,000, but less than $195,000
D. At least $195,000, but less than $200,000
E. At least $200,000
11. According to Gillam in "Retrospective Rating: Excess Loss Factors," what is the purpose of the flat loading?

A. To adjust excess ratios from a per-claim to a per-occurrence basis.
B. To compensate the insurer for parameter risk and anti-selection.
C. To account for loss development after the 5th report.
D. To index the final ELF for the effect of inflation.
E. To avoid inconsistencies in the empirical data.
12. Which of the following statements concerning the calculation of the insurance charge for a large deductible workers compensation policy is false?

A. The loss group adjustment factor depends on the aggregate limit on deductible losses.
B. Deductible policies usually have an aggregate limit on the amount of deductible losses the insured will have to pay.
C. The calculation of the entry ratio must be based on expected deductible losses.
D. The insurance charge factor, from the Table of Insurance Charges, is inversely related to the entry ratio for the policy.
E. The final premium for a retrospectively-rated workers compensation policy is typically greater than for a large deductible workers compensation policy.

CONTINUED ON NEXT PAGE
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13. At what limit do these increased limit factors fail the consistency test?

<table>
<thead>
<tr>
<th>Limit</th>
<th>Increased Limit Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000</td>
<td>1.000</td>
</tr>
<tr>
<td>$250,000</td>
<td>1.300</td>
</tr>
<tr>
<td>$450,000</td>
<td>1.650</td>
</tr>
<tr>
<td>$750,000</td>
<td>2.000</td>
</tr>
<tr>
<td>$1,200,000</td>
<td>2.400</td>
</tr>
<tr>
<td>$1,500,000</td>
<td>2.700</td>
</tr>
</tbody>
</table>

A. $250,000  
B. $450,000  
C. $750,000  
D. $1,200,000 
E. $1,500,000
14. A short-tailed reference line and the line under review have the following historical loss payout patterns as a percentage of ultimate losses:

<table>
<thead>
<tr>
<th>Year</th>
<th>Reference Line</th>
<th>Line under Review</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80%</td>
<td>20%</td>
</tr>
<tr>
<td>2</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>3</td>
<td>20%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Losses are paid at the end of the year. An underwriting profit provision of 6% is assumed to be appropriate for the reference line. An interest rate used for discounting is 3%. The permissible loss ratio is 80%.

Using the Calendar Year Investment Income Offset Method, what is the appropriate profit provision for the line under review?

A. Less than 2%
B. At least 2%, but less than 3%
C. At least 3%, but less than 4%
D. At least 4%, but less than 5%
E. At least 5%
15. According to Butsic, which of the following are reasons a hypothetical asset is a better candidate as a basis of determining a discount rate rather than an insurer’s actual assets?

1. Insurers with identical undiscounted loss reserves but different assets would result in different discounted reserves.
2. Insurers with identical undiscounted loss reserves and identical assets would not have necessarily acquired the assets at the same cost.
3. It is unclear how to designate assets of differing yields to pay for certain losses.

A. 1 only
B. 2 only
C. 1 and 3 only
D. 2 and 3 only
E. 1, 2, and 3
16. In "Insurance Profitability," McClenahan discusses the opportunity cost to the policyholder. According to this discussion, which of the following is false?

A. Each cash flow used to calculate the opportunity cost to the policyholder should be discounted at a rate that reflects its relative risk level.
B. The opportunity cost to the policyholder should be independent of the amount of surplus allocated to the policy.
C. The calculation of the opportunity cost to the policyholder should reflect that not all cash flows go through invested assets.
D. The opportunity cost to the policyholder should be independent of the insurer’s expected investment return.
E. The opportunity cost to the policyholder should be calculated based upon the cash flows associated with the line of business.
17. You are given the following information. Assume the tax basis is undiscounted reserves.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss payment (with certainty) at t=1</td>
<td>$105</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>5%</td>
</tr>
<tr>
<td>Income Tax rate</td>
<td>35%</td>
</tr>
</tbody>
</table>

Calculate the effective discount rate required to produce the fair premium, or after-tax discounted reserve.

A. Less than 4.85%
B. At least 4.85%, but less than 4.95%
C. At least 4.95%, but less than 5.05%
D. At least 5.05%, but less than 5.15%
E. At least 5.15%
18. You have the following information for a given line of business.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate</td>
<td>4%</td>
</tr>
<tr>
<td>Market premium</td>
<td>7%</td>
</tr>
<tr>
<td>Underwriting beta</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Premiums are collected immediately when the policy becomes effective and loss payments are made as follows: 30% at the end of 3 months, and 70% at the end of 9 months.

Using Fairley's CAPM model, determine the underwriting profit provision for the line.

A. -1.50%
B. -1.20%
C. -0.30%
D. 0.50%
E. 1.00%
19. You are given the following information.

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net underwriting gain or loss</td>
<td>$ 5,000</td>
</tr>
<tr>
<td>Net investment income</td>
<td>$30,000</td>
</tr>
<tr>
<td>Net unrealized capital gains or losses</td>
<td>-$15,000</td>
</tr>
<tr>
<td>Dividends to policyholders</td>
<td>$ 8,000</td>
</tr>
<tr>
<td>Dividends to stockholders</td>
<td>$12,000</td>
</tr>
<tr>
<td>Federal taxes</td>
<td>$ 300</td>
</tr>
</tbody>
</table>

Using Roth’s definition in “Analysis of Surplus and Rate of Return without using Leverage Ratios,” calculate the change in surplus.

A. -$ 300
B. $ 0
C. $ 7,700
D. $11,700
E. $26,700
20. A portfolio of identical insurance policies has the following characteristics:

<table>
<thead>
<tr>
<th>Expected Loss</th>
<th>$1,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposure Ratio</td>
<td>5.0%</td>
</tr>
</tbody>
</table>

You are considering adding a new, independent risk to this portfolio. The expected loss of this risk is $250,000.

Determine the maximum exposure ratio you will accept for the new risk in order to keep the aggregate exposure ratio at 5.0%.

A. Less than 5.0%
B. At least 5.0%, but less than 7.5%
C. At least 7.5%, but less than 10.0%
D. At least 10.0%, but less than 12.5%
E. At least 12.5%
21. (1 point)

Briefly describe two methods to test whether risk parameters shift over time.
22. (3 points)

You are given the following data:

<table>
<thead>
<tr>
<th>Class</th>
<th>Years since last accident</th>
<th>Actual Earned Premium at Present B Rates</th>
<th>Earned Car Years</th>
<th>Number of Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3+</td>
<td>375,000</td>
<td>2,500</td>
<td>200</td>
</tr>
<tr>
<td>X</td>
<td>2</td>
<td>15,000</td>
<td>100</td>
<td>12</td>
</tr>
<tr>
<td>Y</td>
<td>1</td>
<td>22,500</td>
<td>150</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>37,500</td>
<td>250</td>
<td>38</td>
</tr>
</tbody>
</table>

Assume that the same rate is charged to all insureds within a class and there have been no rate changes in or since the experience period.

a. (1 point)

What is the credibility of 3 or more accident-free years of experience?

b. (1 point)

What is the credibility of 1 or more accident-free years of experience?

c. (1 point)

Give two possible reasons why the answer in part a. is not 3 times the answer in part b.
23. (2 points)

Using the graph below, answer the following questions.

$Q_L(p)$ and $Q_H(p)$ are the demand curves for low and high risks respectively. $\theta_H$ and $\theta_L$ are the actuarially fair rates for high and low risks respectively. $\theta$ is the average rate for all risks. $Q^*$ is the maximum amount of coverage offered to any one risk. The vertical axis is the price of insurance, per dollar of coverage.

\[ \begin{array}{c}
\text{Quantity Demanded} \\
Q_L \quad Q^* \\
\end{array} \]

\[ \begin{array}{c}
\theta_L \\
\theta \\
\theta_H \\
\end{array} \]

a. (1 point)

Assume companies can identify the high- and low-risk applicants and offer coverage at actuarially fair rates. What quantity will be demanded? Briefly explain your reason.

b. (1 point)

Assume companies are unable to identify the high- and low-risk applicants and charge the average rate to all applicants. Assume low-risk applicants are not risk averse. Briefly explain what will happen to the insurance companies' financial position in this situation.
24. (5 points)

You are given the following losses and exposures. Assume the base class is Good Protection Class with a New Roof \((x_1,y_1)\). Assume initially that \(y_1 = 1.00\) and \(y_2 = 1.30\).

<table>
<thead>
<tr>
<th>Age of Roof</th>
<th>Total Losses</th>
<th>Protection Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Good ((x_1))</td>
</tr>
<tr>
<td>New ((y_1))</td>
<td>100,000</td>
<td>255,000</td>
</tr>
<tr>
<td>Old ((y_2))</td>
<td>360,000</td>
<td>650,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Age of Roof</th>
<th>Exposures</th>
<th>Protection Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Good ((x_1))</td>
</tr>
<tr>
<td>New ((y_1))</td>
<td>200</td>
<td>500</td>
</tr>
<tr>
<td>Old ((y_2))</td>
<td>600</td>
<td>1000</td>
</tr>
</tbody>
</table>

a. (2.75 points)

Calculate the first iteration of relativities for Protection Class and Age of Roof, using the Maximum Likelihood Multiplicative Model (Balance Principle).

b. (2.25 points)

Calculate the first iteration of relativities for Protection Class and Age of Roof, using the Least Squares Multiplicative Model.
25. (2 points)

a. (1 point)

Explain why one-way analysis of risk classification relativities can produce indicated relativities that are inaccurate and inconsistent with the data.

b. (1 point)

Describe an approach to calculating risk classification relativities that would reduce the error produced by a one-way analysis.
26. (5 points)

Using the following set of data for a group of large risks, determine whether the current or proposed experience rating plan is better. Assume the risks are all of the same premium size.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.80</td>
<td>0.62</td>
<td>0.90</td>
<td>0.77</td>
<td>0.81</td>
<td>0.91</td>
</tr>
<tr>
<td>B</td>
<td>0.85</td>
<td>0.70</td>
<td>0.96</td>
<td>0.92</td>
<td>0.95</td>
<td>0.92</td>
</tr>
<tr>
<td>C</td>
<td>0.87</td>
<td>0.85</td>
<td>0.97</td>
<td>0.86</td>
<td>0.79</td>
<td>0.97</td>
</tr>
<tr>
<td>D</td>
<td>0.92</td>
<td>0.82</td>
<td>0.95</td>
<td>0.80</td>
<td>0.84</td>
<td>0.90</td>
</tr>
<tr>
<td>E</td>
<td>0.94</td>
<td>0.94</td>
<td>0.98</td>
<td>0.81</td>
<td>0.87</td>
<td>0.90</td>
</tr>
<tr>
<td>F</td>
<td>0.99</td>
<td>0.95</td>
<td>0.97</td>
<td>0.88</td>
<td>0.91</td>
<td>0.96</td>
</tr>
<tr>
<td>G</td>
<td>1.00</td>
<td>0.90</td>
<td>1.03</td>
<td>1.03</td>
<td>0.97</td>
<td>1.07</td>
</tr>
<tr>
<td>H</td>
<td>1.07</td>
<td>0.99</td>
<td>1.02</td>
<td>1.05</td>
<td>1.02</td>
<td>1.08</td>
</tr>
<tr>
<td>I</td>
<td>1.09</td>
<td>1.04</td>
<td>1.04</td>
<td>0.99</td>
<td>0.97</td>
<td>1.03</td>
</tr>
<tr>
<td>J</td>
<td>1.12</td>
<td>1.01</td>
<td>1.05</td>
<td>1.08</td>
<td>0.95</td>
<td>1.03</td>
</tr>
</tbody>
</table>

CONTINUED ON NEXT PAGE
PAGE - 26 -
27. (3 points)

Company A operates in Alabama and all of its workers are classified in workers compensation class code 6216. Given the following data about Company A's workers compensation policy and using the NCCI experience rating manual, determine the standard premium for the policy effective July 1, 2003.

Manual Rate for Class 6216 = $5.00

<table>
<thead>
<tr>
<th>Annual Policy Effective</th>
<th>Payroll</th>
<th>Claim Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1, 1997</td>
<td>$1,500,000</td>
<td>Audited</td>
</tr>
<tr>
<td>July 1, 1998</td>
<td>$1,600,000</td>
<td>Audited</td>
</tr>
<tr>
<td>July 1, 1999</td>
<td>$1,750,000</td>
<td>Audited</td>
</tr>
<tr>
<td>July 1, 2000</td>
<td>$1,800,000</td>
<td>Audited</td>
</tr>
<tr>
<td>July 1, 2001</td>
<td>$1,850,000</td>
<td>Audited</td>
</tr>
<tr>
<td>July 1, 2002</td>
<td>$1,900,000</td>
<td>Audited</td>
</tr>
<tr>
<td>July 1, 2003</td>
<td>$2,000,000</td>
<td>Estimated</td>
</tr>
</tbody>
</table>

Loss History:

<table>
<thead>
<tr>
<th>Claim Number</th>
<th>Accident Date</th>
<th>Claim Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>September 25, 1997</td>
<td>150,000</td>
</tr>
<tr>
<td>2</td>
<td>October 30, 1998</td>
<td>3,000</td>
</tr>
<tr>
<td>3</td>
<td>July 5, 1999</td>
<td>7,500</td>
</tr>
<tr>
<td>4</td>
<td>June 30, 2000</td>
<td>150,000</td>
</tr>
<tr>
<td>5</td>
<td>March 1, 2001</td>
<td>2,500</td>
</tr>
<tr>
<td>6</td>
<td>April 4, 2001</td>
<td>26,000</td>
</tr>
<tr>
<td>7</td>
<td>June 25, 2002</td>
<td>5,500</td>
</tr>
<tr>
<td>8</td>
<td>June 28, 2002</td>
<td>2,400</td>
</tr>
<tr>
<td>9</td>
<td>August 24, 2002</td>
<td>3,300</td>
</tr>
</tbody>
</table>
28. (3 points)

a. (1.5 points)

In "Fundamentals of Individual Risk Rating," Gillam and Snader provide the following transformed equation for the basic experience rating formula:

\[ M = 1 + Z \times \frac{(A - E)}{E} \]

Give a verbal explanation of this formula.

b. (1.5 points)

State either algebraically or verbally the conditions credibility should satisfy for the credibility system to operate effectively in experience rating.
State how Lee interprets the Table M charge difference and show graphically that his statement is true. Clearly label the graph and all notation.
30. (5 points)

Given the following information, calculate the Table M and Table L charges at loss ratios from 0% to 120%, using increments of 10%.

Round all calculations to three decimal points.

<table>
<thead>
<tr>
<th>Unlimited Loss</th>
<th></th>
<th>Limited Loss</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>Number of Risks</td>
<td>Ratio</td>
<td>Number of Risks</td>
</tr>
<tr>
<td>0.10</td>
<td>1</td>
<td>0.10</td>
<td>1</td>
</tr>
<tr>
<td>0.20</td>
<td>3</td>
<td>0.20</td>
<td>3</td>
</tr>
<tr>
<td>0.30</td>
<td>4</td>
<td>0.30</td>
<td>4</td>
</tr>
<tr>
<td>0.40</td>
<td>2</td>
<td>0.40</td>
<td>5</td>
</tr>
<tr>
<td>0.50</td>
<td>3</td>
<td>0.50</td>
<td>4</td>
</tr>
<tr>
<td>0.60</td>
<td>4</td>
<td>0.60</td>
<td>6</td>
</tr>
<tr>
<td>0.70</td>
<td>4</td>
<td>0.70</td>
<td>4</td>
</tr>
<tr>
<td>0.80</td>
<td>4</td>
<td>0.80</td>
<td>5</td>
</tr>
<tr>
<td>0.90</td>
<td>4</td>
<td>0.90</td>
<td>2</td>
</tr>
<tr>
<td>1.00</td>
<td>3</td>
<td>1.00</td>
<td>5</td>
</tr>
<tr>
<td>1.10</td>
<td>6</td>
<td>1.10</td>
<td>1</td>
</tr>
<tr>
<td>1.20</td>
<td>2</td>
<td>1.20</td>
<td>0</td>
</tr>
</tbody>
</table>
31. (3 points)

A workers compensation insured is subject to the following retrospective rating plan with a per occurrence loss limitation of $50,000.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated Standard Premium</td>
<td>$100,000</td>
</tr>
<tr>
<td>Expected Losses</td>
<td>$61,000</td>
</tr>
<tr>
<td>Selected Minimum Premium Factor</td>
<td>0.5</td>
</tr>
<tr>
<td>Selected Maximum Premium Factor</td>
<td>1.4</td>
</tr>
<tr>
<td>Loss Conversion Factor</td>
<td>1.15</td>
</tr>
<tr>
<td>ELF for $50,000 Limit</td>
<td>0.36</td>
</tr>
<tr>
<td>Expenses from the expense ratio table</td>
<td>0.23</td>
</tr>
<tr>
<td>State Hazard Group Differential</td>
<td>0.9</td>
</tr>
<tr>
<td>State Loss Based Assessments</td>
<td>9.5%</td>
</tr>
<tr>
<td>State Premium Tax Rate</td>
<td>4.8%</td>
</tr>
</tbody>
</table>

a. (2.5 points)

Determine each of the following.

i. Expected Loss Group based on the 2001 Table of Expected Loss Ranges
   
ii. Table of Insurance Charges Entry Difference

b. (0.5 point)

What is the premium discount implied in this plan?
32. (2 points)

Explain why the Table of Insurance Charges in the NCCI Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance varies by expected loss group.
33. (3 points)

You are helping an underwriter price a retrospectively-rated workers compensation policy with a per occurrence limit on losses contributing to the aggregate limit. The policy specifies no minimum aggregate limit. Between last year and this year, the customer has doubled its work force without changing the nature of what it does. Assume that this is the only change between the expiring and renewing policy (i.e. assume no inflation, trends, or benefit changes; also assume no changes to selected limits or deductibles).

How would each of the following change, if at all, due to the increase in exposure? Explain your answers.

a. (1 point)

Premium discount, as a percentage of manual premium.

b. (1 point)

Insurance charge, as a percentage of expected losses.

c. (1 point)

Excess loss pure premium factor (excess losses as a percentage of expected ground-up losses).
34. (1 point)

Explain why the average loss and expense payout period is considerably longer for excess workers compensation policies than for large deductible workers compensation policies.
35. (3 points)

Given the following information, calculate the insurance charge (in dollars) for a large deductible workers compensation policy.

Manual Premium $10,000,000
Standard Premium $9,000,000
Expected Unlimited Loss Ratio 80%
State Hazard Group Relativity 0.90
Deductible $250,000
Excess Loss Pure Premium Factor 0.200
Excess Loss Factor 0.160
Aggregate Limit on Deductible $6,750,000

Table of Expected Loss Ranges

<table>
<thead>
<tr>
<th>Expected Loss Group</th>
<th>Range of Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>6,371,884</td>
</tr>
<tr>
<td></td>
<td>7,282,153</td>
</tr>
<tr>
<td>17</td>
<td>7,282,154</td>
</tr>
<tr>
<td></td>
<td>8,394,704</td>
</tr>
<tr>
<td>16</td>
<td>8,394,705</td>
</tr>
<tr>
<td></td>
<td>10,012,960</td>
</tr>
<tr>
<td>15</td>
<td>10,012,961</td>
</tr>
<tr>
<td></td>
<td>11,934,639</td>
</tr>
<tr>
<td>14</td>
<td>11,934,640</td>
</tr>
<tr>
<td></td>
<td>14,311,453</td>
</tr>
<tr>
<td>13</td>
<td>14,311,454</td>
</tr>
<tr>
<td></td>
<td>17,638,992</td>
</tr>
</tbody>
</table>

Table of Insurance Charges

<table>
<thead>
<tr>
<th>Entry Ratio</th>
<th>18</th>
<th>17</th>
<th>16</th>
<th>15</th>
<th>14</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.08</td>
<td>0.3000</td>
<td>0.2941</td>
<td>0.2884</td>
<td>0.2827</td>
<td>0.2771</td>
<td>0.2714</td>
</tr>
<tr>
<td>1.17</td>
<td>0.2793</td>
<td>0.2730</td>
<td>0.2667</td>
<td>0.2606</td>
<td>0.2545</td>
<td>0.2484</td>
</tr>
<tr>
<td>1.55</td>
<td>0.2073</td>
<td>0.2764</td>
<td>0.2661</td>
<td>0.2557</td>
<td>0.2453</td>
<td>0.2349</td>
</tr>
<tr>
<td>1.66</td>
<td>0.1900</td>
<td>0.2522</td>
<td>0.2417</td>
<td>0.2310</td>
<td>0.2203</td>
<td>0.2096</td>
</tr>
<tr>
<td>2.21</td>
<td>0.1299</td>
<td>0.1690</td>
<td>0.1583</td>
<td>0.1476</td>
<td>0.1369</td>
<td>0.1261</td>
</tr>
</tbody>
</table>
36. (2 points)

Provide two reasons why insurers may be subject to adverse selection by insureds that purchase high liability limits.
37. (2 points)

Using the information below, what is the pure premium for a $1,000,000 basic limits policy?

| Frequency                          | 0.10  
|------------------------------------|-------
| Severity of $2,000,000 policy      | $130,000  
| Increased Limit Factor of $5,000,000 limit | 1.75  
| Pure Premium of $3,000,000 excess of $2,000,000 policy | $5,000  

38. (4 points)

You have the following information for a lognormal loss distribution:

<table>
<thead>
<tr>
<th>Attachment Point (Times the Mean)</th>
<th>Excess of Loss Dist (% of Total Limits Loss) CV = 3.0</th>
<th>Excess of Loss Dist (% of Total Limits Loss) CV = 4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>70%</td>
<td>73%</td>
</tr>
<tr>
<td>1.0</td>
<td>55%</td>
<td>60%</td>
</tr>
<tr>
<td>1.5</td>
<td>46%</td>
<td>52%</td>
</tr>
<tr>
<td>2.0</td>
<td>40%</td>
<td>46%</td>
</tr>
<tr>
<td>2.5</td>
<td>35%</td>
<td>41%</td>
</tr>
<tr>
<td>3.0</td>
<td>31%</td>
<td>37%</td>
</tr>
<tr>
<td>4.0</td>
<td>25%</td>
<td>32%</td>
</tr>
<tr>
<td>5.0</td>
<td>21%</td>
<td>27%</td>
</tr>
<tr>
<td>10.0</td>
<td>11%</td>
<td>16%</td>
</tr>
<tr>
<td>15.0</td>
<td>7%</td>
<td>12%</td>
</tr>
<tr>
<td>20.0</td>
<td>5%</td>
<td>9%</td>
</tr>
</tbody>
</table>

The actual coefficient of variation underlying the observed loss distribution is 3.0. The unlimited expected value of a claim underlying this distribution is $25,000. The pure premium for the basic limits policy of $50,000 is $3,000.

a. (1 point)

What is the increased limit factor for a $75,000 limit?

b. (1 point)

What is the pure premium of a $125,000 limit policy?

c. (2 points)

What is the dollar amount of the error in the pure premium if the coefficient of variation for this distribution is incorrectly estimated to be 4.0 when calculating a $400,000 excess of $100,000 policy?
39. (2 points)

You are given the following for a single policy having a one-year term:

- Present value of net cash flows (premiums less loss, expense, and dividend payments) at policy inception is $2.
- Expected loss and loss expenses are $40 and are paid in one payment at exactly 18 months after policy inception.
- The risk-free discount rate is 8.00%.

What is the risk-adjusted rate being used to discount the loss and loss expense reserves?
40. (3 points)

You are the commissioner of insurance for a state that uses a return-on-sales guideline to regulate personal auto liability rates. Due to recent large rate increases, several consumer protection organizations have accused you of allowing insurers to earn excessive returns and are advocating a switch to a return-on-equity guideline. What are three arguments you can use to support the current guideline?
A financial analyst has been asked to determine the optimum capital structure of an insurance company. He believes that the value of the firm is determined by the expected earnings stream and the rate at which that stream is capitalized by the market. To assist him in his determination, he expresses the return-on-equity for the insurance company as:

\[ T/S = I/A + R/S \times (I/A + U/R) \]

Where
- \( T \) - total return to the insurer
- \( I \) - investment income
- \( U \) - underwriting income
- \( P \) - premium income
- \( A \) - total assets
- \( R \) - reserves and other liabilities
- \( S \) - stockholders' equity

Note that all income items are on an after-tax basis and the equity in the unearned premium reserves is incorporated into the stockholders' equity.

a. (1.5 points)

The financial analyst notices that expressing the return-on-equity in this manner allows the reserves, \( R \), to be viewed as non-equity capital. Explain.

b. (1.5 points)

However, the analyst also realizes that the analysis of reserve as capital is not as straightforward as the analysis of debt capital. Describe two reasons why he would consider it more difficult.

c. (1 point)

Discuss the two items which determine the value of the firm in the formula and point out where they interact.
42. (5 points)

You are responsible for setting the appropriate premium levels for general liability insurance policies at your company and you have decided to use the Risk-adjusted Discounted Cash Flow Technique. You have also determined that the following assumptions are appropriate for the typical general liability policy written by your company:

- Premium is collected in full on the effective date of the policy.
- The following underwriting expenses are paid on the effective date of the policy: Commission in the amount of 10% of premium and fixed expenses in the amount of $20.
- You expect to pay $70 in loss and loss-adjustment expenses as follows: $40 at the end of year one and $30 at the end of year two.
- Taxes on investment income and underwriting profit are paid at the end of each year at a rate of 35%.
- The risk-free rate of return is 6% and the pre-tax return on the investment portfolio is also 6%.
- The discount rate that is appropriate for discounting loss and loss-adjustment expenses is 3%.
- For tax purposes, the company is required to discount losses at 7%.
- The initial amount of equity allocated to write a policy is $21 and this allocation is reset at the end of each year to be proportional to undiscounted loss and loss-adjustment expense reserves.

a. (4.5 points)

Using the Risk-adjusted Discounted Cash Flow Technique, what is the required premium on the typical policy?

b. (0.5 point)

What is the underwriting profit margin associated with this premium?
43. (2 points)

You believe the standard deviation of losses is an appropriate measure of risk. You are given the following loss distribution:

<table>
<thead>
<tr>
<th>Amount of Loss</th>
<th>Probability of Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 0</td>
<td>90%</td>
</tr>
<tr>
<td>$1,000</td>
<td>9%</td>
</tr>
<tr>
<td>$10,000</td>
<td>1%</td>
</tr>
</tbody>
</table>

You have selected 10% of standard deviation as the appropriate risk load. Your basic limit is $2,000. There are no expenses. What is the increased limit factor with risk load for a $5,000 policy limit?
44. (3 points)

a. (2 points)

Identify two advantages and two disadvantages of the Calendar Year Return on Equity method of calculating underwriting profit provisions.

b. (1 point)

Name a method that would correct at least one of the disadvantages listed in part a. above and identify what disadvantage it corrects.
45. (2 points)

An actuary for XYZ Insurance Co. was in the process of preparing a rate filing but instead of using traditional ratemaking methods decided to use a discounted cash flow method. Assume $40 of surplus is assigned to support writing the policy. Also assume that expenses are incurred and paid when a policy is written and taxes are paid at the end of the year.

- $r_f = 6.0\%$, the risk-free rate
- $r_a = 3.5\%$, the risk-adjusted rate
- tax rate: 40\%
- expenses per policy: $25
- expected loss paid at the end of 1 year: $70

Using the above information, determine the total undiscounted pre-tax income expected to arise out of writing the policy.
46. (2 points)

Stone, in "A Theory of Capacity and the Insurance of Catastrophe Risks," describes three precepts as the foundation for rational insurance company behavior. Identify and describe two of the three precepts, and provide an example when describing each.
47. (3 points)

An insurance company believes that risk load should be equal to 0.1% of the variance of the pure premium, and that the frequency of a loss follows a Poisson distribution. For a given insured, the insurance company derived the following risk loads for various layers of coverage:

<table>
<thead>
<tr>
<th>Retention</th>
<th>Policy Limit</th>
<th>Expected Losses</th>
<th>Risk Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0</td>
<td>$100</td>
<td>$15</td>
<td>$1</td>
</tr>
<tr>
<td>$100</td>
<td>$400</td>
<td>$10</td>
<td>$5</td>
</tr>
<tr>
<td>$500</td>
<td>$500</td>
<td>$5</td>
<td>$3</td>
</tr>
</tbody>
</table>

a. (2 points)

Calculate the risk load for a ground-up policy with a limit of $1000.

b. (1 point)

Assuming that $500 is the basic limit, calculate the increased limit factor with risk load for a $1000 policy limit.
48. (2 points)

You are given the following information:

Risk-free interest rate is 3%.
Return on market portfolio is 10%.
Capital requirement is 25% of discounted reserve.

You are considering the following deal, which has zero net present value.

You will assume a loss reserve of $100 (undiscounted), which is expected to be paid 1 year from now. You will also receive $99 in cash for the assumption of risk.

a. (1 point)

What is your expected Return on Equity on that deal?

b. (1 point)

Your company has beta of 1.5. You also believe that CAPM accurately estimates the required return. Would you take the above-mentioned deal? Explain. (Ignore taxes.)
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
</tr>
<tr>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>C</td>
</tr>
<tr>
<td>6</td>
<td>D</td>
</tr>
<tr>
<td>7</td>
<td>D</td>
</tr>
<tr>
<td>8</td>
<td>E</td>
</tr>
<tr>
<td>9</td>
<td>B</td>
</tr>
<tr>
<td>10</td>
<td>C</td>
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<td>B</td>
</tr>
<tr>
<td>12</td>
<td>A</td>
</tr>
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<td>13</td>
<td>E</td>
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<td>Invalid</td>
</tr>
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<td>15</td>
<td>E</td>
</tr>
<tr>
<td>16</td>
<td>A</td>
</tr>
<tr>
<td>17</td>
<td>D</td>
</tr>
<tr>
<td>18</td>
<td>C</td>
</tr>
<tr>
<td>19</td>
<td>A, D</td>
</tr>
<tr>
<td>20</td>
<td>E</td>
</tr>
</tbody>
</table>
Please note that the following sample essay solutions are examples of candidate answers that were given full credit for the Fall 2003 exam. In cases where we accepted significantly different answers for full credit, we have included multiple examples.

The sample essay solutions below do not include all candidate answers for which we did award or might have awarded full credit.

**Question 21—Example 1**

**Method 1**  
Chi-Square Test

**Null Hypothesis - H₀:** risk parameters do not shift over time

- Group data into appropriate intervals
- Calculate the overall expected value
- Then calculate for each interval, \(X^2 = \frac{(A - E)^2}{E}\); Where A = actual observation and E = expected observation
- Sum up \(X^2\) for all intervals
- If the total \(X^2\) is > \(X^2\) value from the \(X^2\) table (based on degrees of freedom), then reject the null hypothesis that parameters do not shift over time. Therefore, accept the alternative hypothesis that risk parameters shift over time
- If total \(X^2\) < \(X^2\) value based on degrees of freedom, then accept null hypothesis

**Method 2**

Correlation test

- Group data by pairs based on time lag
- Calculate correlation for each pair
- Calculate the average correlation by time lag
- If the correlation decreases as time lag increases, then risk parameters shift over time

**Question 21—Example 2**

1. **Chi-Square Test**

Break up the experience period into 5 year intervals, then calculate

\[(Actual \ Losses - Expected \ Losses)^2/(Expected \ Losses)\]

for each interval, then sum them. Compare the sum to the \(X^2\) statistic for (# intervals - 1). If the sum exceeds the statistic, risk parameters do shift over time.
2. Correlation Test

Compute the correlation between loss ratios for a risk for every possible pair of years during the historical period. Then average the correlations for all pairs with a given difference in years.

If the average correlation decreases as the difference in years increases, then risk parameters do shift over time.

Question 22—Example 1

a) and b)

<table>
<thead>
<tr>
<th>Class</th>
<th>EP</th>
<th># Claims</th>
<th>freq.</th>
<th>rel. freq.</th>
<th>Cred = 1 - rel. freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 3 or more</td>
<td>375.0</td>
<td>200</td>
<td>.533</td>
<td>.888</td>
<td>.112</td>
</tr>
<tr>
<td>A+X+Y = 1 or more</td>
<td>412.5</td>
<td>232</td>
<td>.562</td>
<td>.937</td>
<td>.063</td>
</tr>
<tr>
<td>and Total</td>
<td>450.0</td>
<td>270</td>
<td>.600</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

a) = .112
b) = .063
c) 1) individual accident propensities change over time
   2) population of class changes over time as risks enter & leave

Question 22—Example 2

a) Average Frequency:

\[
\frac{(200 + 12+ 20 + 38)}{(375,000 + 15,000 + 22,500 + 37,500)} = 0.0006
\]

Class A Frequency = \( \frac{200}{375,000} = 0.000533 \)

\[0.8889 = Z * 0 + (1 - Z) * 1 = 1 - Z\]

\[Z_3 = 1 - 0.8889 = 0.1111\]

b) Class A + X + Y Frequency:

\[
\frac{(200 + 12 + 20)}{(375,000 + 15,000 + 22,500)} = 0.00056
\]

\[Z_4 = 1 - 0.9374 = 0.0626\]
c)  
   1. the probability of loss for an individual insured changes from time to time during the year and from one year to another.  
   2. Marked skewness of individual insureds' probability of loss within a class.

**Question 23—Example 1**

a. Both the High & Low risks will demand full coverage, Q*. Companies will offer the high risks \( ?_H \) (actuarial fair rate) & they will offer the low risks \( ?_L \) (actuarial fair rate). Since both risks seek to optimize their utility they will select the intersection of their demand curve and the rate offered.

b. The financial position is worse. The high risks will still demand full coverage since \( ? \) is below their demand curve. The low risks will demand, \( Q_L \), less than full coverage since their demand curve intersects \( ? \) at D (or \( Q_L \)). The financial position is worse because the benefit from addition premium from low risks is less than the premium loss from the high risks.

**Question 23—Example 2**

a. \( Q^* \) will be demanded, since both risks pay their own actuarially fair rates, and it lies on their demand curves at points A & B, for high and low, respectively.

b. Low risks will reduce their coverage to \( Q_L \) while high risks will continue to demand \( Q^* \), but at a much lower rate \( ? \), not \( ?_H \)

The companies financial position will deteriorate given the premium increases from the low risks, from \( ?_L \) to \( ? \), will not be enough to overcome the losses and premium reductions from the high risk insureds.

**Question 24—Example 1**

a.

Freq = Loss / Exposure

<table>
<thead>
<tr>
<th></th>
<th>Good</th>
<th>Avg</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>500</td>
<td>510</td>
<td>600</td>
</tr>
<tr>
<td>Old</td>
<td>600</td>
<td>650</td>
<td>1,000</td>
</tr>
</tbody>
</table>

Rel Freq = Freq / Base Class Freq

<table>
<thead>
<tr>
<th></th>
<th>Good</th>
<th>Avg</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>1.00</td>
<td>1.02</td>
<td>1.2</td>
</tr>
<tr>
<td>Old</td>
<td>1.2</td>
<td>1.3</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Method of Max Likelihood:

\[ x_i = \frac{\sum n_{i,j} r_{i,j}}{\sum n_{i,j} y_j} \]

\[ y_j = \frac{\sum n_{i,j} r_{i,j}}{\sum n_{i,j} x_i} \]

\[ x_1 = \frac{200 \times 1 + 600 \times 1.2}{200 \times 1 + 600 \times 1.3} = \frac{920}{980} = 0.939 \]

\[ x_2 = \frac{500 \times 1.02 + 1000 \times 1.3}{500 \times 1 + 1000 \times 1.3} = \frac{1810}{1800} = 1.006 \]

\[ x_3 = \frac{100 \times 1.2 + 200 \times 2.0}{100 \times 1.0 + 200 \times 1.3} = \frac{520}{360} = 1.444 \]

And

\[ y_1 = \frac{200 \times 1 + 500 \times 1.02 + 100 \times 1.2}{600 \times 0.939 + 500 \times 1.006 + 100 \times 1.444} = \frac{830}{835.2} = 0.994 \]

\[ y_2 = \frac{600 \times 1.2 + 1000 \times 1.3 + 200 \times 2.0}{600 \times 0.939 + 1000 \times 1.006 + 200 \times 1.444} = \frac{2420}{1858.2} = 1.302 \]

b. Least Squares:

\[ x_i = \frac{\sum n_{i,j} r_{i,j} y_j}{\sum n_{i,j} y_j^2} \]

\[ y_j = \frac{\sum n_{i,j} r_{i,j} x_i}{\sum n_{i,j} x_i^2} \]

\[ x_1 = \frac{200 \times 1 \times 1 + 600 \times 1.2 \times 1.3}{200 \times 1^2 + 600 \times 1.3^2} = \frac{1136}{1214} = 0.936 \]

\[ x_2 = \frac{500 \times 1.02 \times 1 + 1000 \times 1.3 \times 1.3}{500 \times 1^2 + 1000 \times 1.3^2} = \frac{2200}{2190} = 1.005 \]

\[ x_3 = \frac{100 \times 1.2 \times 1 + 200 \times 2 \times 1.3}{100 \times 1^2 + 200 \times 1.3^2} = \frac{640}{438} = 1.461 \]

And
\[
y_1 = \frac{200 \times 1 \times 0.936 + 500 \times 1.02 \times 1.005 + 100 \times 1.2 \times 1.461}{200 \times 0.936^2 + 500 \times 1.005^2 + 100 \times 1.461^2} = \frac{875.07}{893.68} = 0.979
\]
\[
y_2 = \frac{600 \times 1.2 \times 0.936 + 1000 \times 1.3 \times 1.005 + 200 \times 2 \times 1.461}{600 \times 0.936^2 + 1000 \times 1.005^2 + 200 \times 1.461^2} = \frac{2564.82}{1962.59} = 1.307
\]

Question 24—Example 2

a. Balance Principle:
\[
\sum_i n_{i,j} (r_{i,j} - x_i y_j) = 0
\]
\[
\sum_j n_{i,j} (r_{i,j} - x_i y_j) = 0
\]

Solutions found iteratively using:
\[
x_i = \frac{\sum_j n_{i,j} r_{i,j}}{\sum_j n_{i,j} y_j}
\]
\[
y_j = \frac{\sum_i n_{i,j} r_{i,j}}{\sum_i n_{i,j} x_i}
\]

Where,
- \(n_{i,j}\) : are the exposures
- \(r_{i,j}\) : are the loss cost relativities

<table>
<thead>
<tr>
<th>Loss/Exposure</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_1)</td>
<td>500</td>
<td>510</td>
<td>600</td>
</tr>
<tr>
<td>(y_2)</td>
<td>600</td>
<td>650</td>
<td>1,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loss Cost Relativities (r_{i,j})</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_1)</td>
<td>1</td>
<td>1.02</td>
<td>1.2</td>
</tr>
<tr>
<td>(y_2)</td>
<td>1.2</td>
<td>1.3</td>
<td>2</td>
</tr>
</tbody>
</table>

Then:
Question 25—Example 1

a. One-way analysis assumes that the variables used in classification are independent from each other. This is usually false in reality.

b. The methods explained by Holler in “Something Old, Something New” considers all the variables used in classification at the same time, using iteration procedure. These methods include Least Squares Model, Maximum Likelihood Model, Chi-
Square Model, Balance Principle Model. For each of these model, one can assume a multiplicative, an additive or a combination of them as a relationship between variables. For example, the iteration formula for Least Square Multiplicative Model is

\[
 x_i = \frac{\sum n_{i,j}r_{i,j}y_j}{\sum n_{i,j}y_j^2}
\]

and

\[
 y_j = \frac{\sum n_{i,j}r_{i,j}x_i}{\sum n_{i,j}x_i^2}
\]

With an initial value of \( x_i \) or \( y_j \), we can do iterations on the value of \( x_i \) and \( y_j \) until they converge to a certain value. These values are the proper relativities.

**Question 25—Example 2**

a. It ignores the fact that relativity factors may not be independent. Class factors may be correlated and actual relativities could be even higher than products of one way relativities

b. Use least squares to minimize the squared error in relativities

Set

\[
 x_i = \frac{\sum n_{i,j}r_{i,j}y_j}{\sum n_{i,j}y_j^2}
\]

and

\[
 y_j = \frac{\sum n_{i,j}r_{i,j}x_i}{\sum n_{i,j}x_i^2}
\]

Use \( x_i \)'s to calculate \( y_j \)'s & vice versa

Repeat iteratively until values converge

**Question 26—Example 1**

Two methods to test the plan performance

1. The ability to identify differences among risks – The manual loss ratios disperse wider, the better.

2. How the plan corrects those differences. This method is to look at standard loss ratios of each risk group. If the standard loss ratios are closer to unity, the plan works better

From the manual loss ratios and standard loss ratios for two plans, divide risks into 5 groups based on the mods.
Current plan:

<table>
<thead>
<tr>
<th></th>
<th>Manual LR</th>
<th>Standard LR</th>
</tr>
</thead>
<tbody>
<tr>
<td>A,B</td>
<td>0.66</td>
<td>0.93</td>
</tr>
<tr>
<td>C,D</td>
<td>0.835</td>
<td>0.96</td>
</tr>
<tr>
<td>E,F</td>
<td>0.945</td>
<td>0.975</td>
</tr>
<tr>
<td>G,H</td>
<td>0.945</td>
<td>1.025</td>
</tr>
<tr>
<td>I,J</td>
<td>1.025</td>
<td>1.045</td>
</tr>
</tbody>
</table>

Proposed plan:

<table>
<thead>
<tr>
<th></th>
<th>Manual</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>A,D</td>
<td>0.825</td>
<td>0.905</td>
</tr>
<tr>
<td>C,E</td>
<td>0.83</td>
<td>0.935</td>
</tr>
<tr>
<td>F,B</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>I,G</td>
<td>0.97</td>
<td>1.05</td>
</tr>
<tr>
<td>H,J</td>
<td>0.985</td>
<td>1.055</td>
</tr>
</tbody>
</table>

Solution:
1. Current plan is better for identifying differences, since manual loss ratios are more dispersed.
2. Current plan is better for correct differences, since standard loss ratios are closer to unity.

Question 26—Example 2

Quintiles Test: Current Plan

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Risks</th>
<th>Avg Man L/R</th>
<th>Avg Std L/R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A,B</td>
<td>(0.62 + 0.7)/2 = 0.66</td>
<td>0.93</td>
</tr>
<tr>
<td>2</td>
<td>C,D</td>
<td>0.835</td>
<td>0.96</td>
</tr>
<tr>
<td>3</td>
<td>E,F</td>
<td>0.945</td>
<td>0.975</td>
</tr>
<tr>
<td>4</td>
<td>G,H</td>
<td>0.945</td>
<td>1.025</td>
</tr>
<tr>
<td>5</td>
<td>I,J</td>
<td>1.025</td>
<td>1.045</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.882</td>
<td>0.987</td>
</tr>
</tbody>
</table>

V (Man L/R)  
0.015976  
= (0.7939 − 0.882^2) = 0.015976

V (St L/R)
0.97596 – 0.981^2 = 0.001791

Ratio V(SD L/R) to V(M L/R) = 0.001791 / 0.015976 = 0.112

Proposed Plan

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Risks</th>
<th>Avg Man L/R</th>
<th>Avg St L/R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A,D</td>
<td>0.825</td>
<td>0.905</td>
</tr>
<tr>
<td>2</td>
<td>E,C</td>
<td>0.83</td>
<td>0.935</td>
</tr>
<tr>
<td>3</td>
<td>F,B</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>I,G</td>
<td>0.97</td>
<td>1.05</td>
</tr>
<tr>
<td>5</td>
<td>H,J</td>
<td>0.985</td>
<td>1.055</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.908</td>
<td>0.977</td>
</tr>
</tbody>
</table>

V (Man L/R) = 0.82911 – 0.908^2 = 0.004646

V (SD L/R) = 0.95848 – 0.977^2 = 0.003951

Ratio V(SD L/R) to V (M L/R) = 0.003951 / 0.004646 = 0.8504

Since the current plan has lower ratio of V(SD L/R) to V (M L/R), better plan by quintiles test.

Question 26—Example 3

Gillam presents Meyers Efficiency Test.

Calculate (Variance(Modified loss ratios) / Variance(Unmodified loss ratios)). The smaller the result, the better the Plan is.

Current Plan:

\[ E(\text{Mod. LR}) = \frac{9.87}{10} = 0.987. \]
\[ E(\text{Mod LR}^2) = \frac{9.7617}{10} = 0.97617. \]
\[ \text{Var (Mod (Standard) LR)} = 0.002. \]

\[ \text{Var (Unmodified (Manual))} = 0.0168. \]
\[ \text{Statistic} = \frac{0.002}{0.0168} = 0.12. \]

Proposed:

\[ \text{Var (Mod LR)} = 0.004481. \]
\[ \text{Var (Unmod LR)} = 0.005336. \]
\[ \text{Statistic} = \frac{0.004481}{0.005336} = 0.84. \]
The statistic for the current plan (0.12) is less than the statistic for the proposed (0.84). So the current plan is better.

**Question 27—Example 1**

We need to use 3 policy years of data with a 1 year lag between the end of the experience period and the effective date of the rates. Thus our experience period is July 1, 1999 to June 30, 2002.

This means that claims 3 to 8 are included.

**Actual Primary Losses**

\[ A_p = 0.30(5,000) + 5,000 + .30(2,500) + 5,000 + 5,000 + .30(2,400) = 17,970 \]

Since the State Accident Limit is 111,500,

\[ A_e = 0.3(7500-5000) + (111,500 – 5,000) + 0.3(0) + (26,000 – 5,000) + (5,500 – 5,000) + 0.3(0) = 128,750 \]

**Expected Losses**

\[ E = (\text{Payroll} / 100) \times (\text{ELR}) = (1,750,000 + 1,800,000, + 1,850,000) / (100) \times 5.06 \text{ – according to class code} \]

\[ = 273,240 \]

**Primary Expected Losses**

\[ E_p = E \times D-\text{Ratio} = 273,240 \times .22 = 60,112.80 \]

**Excess Expected Losses**

\[ E_e = E - E_p = 273,240 - 60,112.80 = 213,127.20 \]

Since \( E = 273,240 \), \( W = .32 \), \( B = 37.825 \)

**Mod**

\[ \text{Mod} = (A_p + W \times A_e + (1-W) \times E_e + B) / (E + B) \]

\[ = (17,970 + .32(128,750) + (1 - .32)(213,127.20) + 37.825) / (273,240 + 37,825) = .77772 \]

**Standard Premium**

\[ = (\text{Manual rate}) \times \text{(Payroll} / 100) \times \text{Modification) } \]

\[ = (5.00)(2,000,000/100)(.77772) \]

\[ = 77,772 \]

**Question 27—Example 2**

Manual Premium = Payroll x class rate = 2,000,000 x 5 / 100 = 100,000

Looking up in the manual for Alabama, Class 6216:

\[ \text{ELR} = 5.06 \text{ and D-Ratio} = 0.22 \]

The Expected loss: 99-01 is the Experience Period: \((1,750,000 + 1,800,000 + 1,850,000) \times 5.06 / 100 = 273,240 \)
Looking up in the table of weighting values: \( W = 0.32, B = 37,825 \)

We derive

Expected Primary Losses = 0.22 x 273,240 = 60,113
Expected Excess Losses = 273,240 – 60,113 = 213,127

Only losses from the experience period (policies inception 07/01/99, 07/01/00 & 07/01/01) should be included. Claims 3 to 8 only

Per Accident limitation: 111,500 (see manual)
Assume the primary loss cap is 5,000.

Non-Medical Primary Losses = 5,000 + 5,000 + 5,000 = 15,000
Medical Excess Losses = 106,500 + 21,000 + 500 = 128,000
   The per accident limit applies

Medical Primary = 30% x [5,000 + 2,500 2,400] = 2,970
Medical Excess = 30% x [2,500] = 750

Therefore,
Actual Primary Losses = 17,970
Actual Excess Losses = 128,750

\[
\text{Mod} = \frac{17,970 + 0.32 \times 128,750 + (1 - 0.32) 213,127 + 37,825}{273,240 + 37,825}
\]

\[
\text{Mod} = .7777
\]

And the standard premium is 100,000 x .7777 = 77,770

**Question 28—Example 1**

a) It says the mod is a standard rate plus a credit or debit depending on whether \( A > E \), stabilized by a value of \( Z/E \).

b) \( 0 \leq Z \leq 1 \)
\( \frac{dZ}{dE} \geq 0 \)
\( \frac{d}{dE} (Z/E) < 0 \)

**Question 28—Example 2**

1. \( M = 1 + Z/E (A - E) \)
a. Unity represents the manual rate  
b. \((A - E)\) is the charge/credit for experience  
c. \(Z/E\) moderates the effect of any single large claim

2. a. \(0 \leq Z \leq 1\)  
b. \(\frac{d}{dE} (Z) \geq 0\) (must not decrease)  
c. \(\frac{d}{dE} (Z/E) < 0\)

**Question 29—Example 1**

Lee states that the Table M charge difference is equal to the difference between the expected retro premium and the minimum premium.

(1) Charge at maximum = D  
Charge at minimum = D + C  
Difference = C

(2) Retro Premium = A + B + C  
Minimum Premium = A + B  
Difference = C

**Question 29—Example 2**

Charge difference = \(X_H - X_G = (e + E - H/T) / cE\)  
= Guaranteed Cost Premium − Minimum Premium  
without taxes divided by the converted expected losses.
$X_H - X_G = \text{Areas 2} + 3 - \text{Area 3} = \text{Area 2}.

GCP – H/T = 1 + 2 + 3.

= (4 + 1 + 2) – (4 + 1)

= 2.

Question 30—Example 1

<table>
<thead>
<tr>
<th>Unlimited # of Risks</th>
<th>Risks Above</th>
<th>Double Sum Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>0.10</td>
<td>1</td>
<td>39</td>
</tr>
<tr>
<td>0.20</td>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>0.30</td>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>0.40</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>0.50</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>0.60</td>
<td>4</td>
<td>23</td>
</tr>
<tr>
<td>0.70</td>
<td>4</td>
<td>19</td>
</tr>
<tr>
<td>0.80</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>0.90</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>1.00</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>1.10</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>1.20</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

ELR = 0.705
### Question 30—Example 2

Expected Unlim LR = \( \frac{\text{Sum (#Risks} \times \text{Unlim LR})}{\text{Sum (#Risks)}} \) = 0.705

Expected Lim LR = 0.605

LER = \( (1 - \frac{0.605}{0.705}) \) = 0.142

Table

<table>
<thead>
<tr>
<th>Entry Ratio</th>
<th>#Risks</th>
<th>%Above</th>
<th>Layer Width</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.000</td>
<td>40</td>
<td>1.000</td>
<td>0.142</td>
</tr>
<tr>
<td>0.142</td>
<td>1</td>
<td>39</td>
<td>0.975</td>
<td>0.142</td>
</tr>
<tr>
<td>0.284</td>
<td>3</td>
<td>36</td>
<td>0.900</td>
<td>0.142</td>
</tr>
<tr>
<td>0.426</td>
<td>4</td>
<td>32</td>
<td>0.800</td>
<td>0.142</td>
</tr>
<tr>
<td>0.567</td>
<td>5</td>
<td>30</td>
<td>0.750</td>
<td>0.142</td>
</tr>
<tr>
<td>0.709</td>
<td>6</td>
<td>27</td>
<td>0.675</td>
<td>0.142</td>
</tr>
<tr>
<td>0.851</td>
<td>7</td>
<td>23</td>
<td>0.575</td>
<td>0.142</td>
</tr>
<tr>
<td>0.993</td>
<td>8</td>
<td>19</td>
<td>0.475</td>
<td>0.142</td>
</tr>
<tr>
<td>1.135</td>
<td>9</td>
<td>15</td>
<td>0.375</td>
<td>0.142</td>
</tr>
<tr>
<td>1.277</td>
<td>10</td>
<td>11</td>
<td>0.275</td>
<td>0.142</td>
</tr>
<tr>
<td>1.418</td>
<td>11</td>
<td>8</td>
<td>0.200</td>
<td>0.142</td>
</tr>
<tr>
<td>1.560</td>
<td>12</td>
<td>2</td>
<td>0.050</td>
<td>0.142</td>
</tr>
<tr>
<td>1.702</td>
<td>13</td>
<td>0</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table
\[
\begin{array}{ccccccc}
L & 0.0 & 0.000 & 0 & 40 & 1.000 & 0.142 & 1.000 \\
& 0.1 & 0.142 & 1 & 39 & 0.975 & 0.142 & 0.858 \\
& 0.2 & 0.284 & 3 & 36 & 0.900 & 0.142 & 0.720 \\
& 0.3 & 0.426 & 4 & 32 & 0.800 & 0.142 & 0.592 \\
& 0.4 & 0.567 & 5 & 27 & 0.675 & 0.142 & 0.479 \\
& 0.5 & 0.709 & 4 & 23 & 0.575 & 0.142 & 0.383 \\
& 0.6 & 0.851 & 6 & 17 & 0.425 & 0.142 & 0.301 \\
& 0.7 & 0.993 & 4 & 13 & 0.325 & 0.142 & 0.241 \\
& 0.8 & 1.135 & 5 & 8 & 0.200 & 0.142 & 0.195 \\
& 0.9 & 1.277 & 2 & 6 & 0.150 & 0.142 & 0.167 \\
& 1.0 & 1.418 & 5 & 1 & 0.025 & 0.142 & 0.145 \\
& 1.1 & 1.560 & 1 & 0 & 0 & 0.142 & 0.142 \\
& 1.2 & 1.702 & 0 & 0 & 0 & 0.142 & 0.142 \\
\end{array}
\]

Question 31—Example 1

a) (i) ELG based on 61,000 x .9 x 1+.8 LER

\[
\text{LER} = \text{ELF} = \frac{.36}{1-\text{LER}} = .590 \\
\text{ELR} = \frac{61/100}{1-.590} = \frac{61Kx.9x1+.8(0.590)}{1-.590} \\
= 54900 x 3.590 \\
= 197,104 \\
\text{ELG} = 44
\]

(ii) \( r_G - r_H = \frac{G-H}{cE’T} = \frac{1.4-.5}{1.15 x (0.61-.36) \times 1.126} = 2.780 \)

\[
T = \frac{.2 + (1+\mu)E}{.2 + (1.095)E} \times \frac{1}{1 - t} \\
= \frac{.2 + (1.095)E}{.2 + (1.095)E} \times \frac{1}{1 - .048} \\
= 1.126
\]

(iii) \( X_H - X_G = \frac{e+H}{cE’T} = \frac{.23 + .61 - .5/1.126}{1.15 x (0.61-.36)} = 1.377 \)
b) \( (e+E)T = 1 - D \)
\[
(0.23 + 0.61) \times 1.126 = 1 - D
\]
\[
0.946 = 1 - D
\]
\[
D = 0.054 = 5.4\%
\]

Question 31—Example 2

a) \( LER = 0.36 / 0.61 = 0.59 \Rightarrow AF = (1 + 0.8LER) / (1 - LER) = 3.59 \)

\( \hat{E} = \text{Adj. } E = 61,000 \times 0.9 \times 3.59 = 197,091 \Rightarrow ELG = 44 \)

\( T = \frac{[0.2 + 0.61(1.0 + 0.095)]}{[(0.2 + 0.61)(1.0 - 0.048)]} \)

\[
r_G - r_H = \frac{G - H}{c\hat{E}T} = \frac{1.4 - 0.5}{1.15(0.61 - 0.36)1.126} = 2.78
\]

\[
x_H - x_G = \frac{(e + E) - H / T}{c\hat{E}} = \frac{(0.23 + 0.61) - 0.5 / 1.126}{0.25 \cdot 1.150} = 1.377
\]

b) \( 1 - D = (e + E)T = 0.946 \Rightarrow D = 5.4\% \)

Question 32—Example 1

Simply, because there is more variability in the loss ratio distribution of smaller insureds than there is in the loss ratio distribution of larger insureds. The charges reflect the different loss ratio distributions.

If there were just one set of charges, it would overcharge the large insureds and undercharge the small insureds.

Question 32—Example 2

The skewness of the loss distribution decreases as the size of the risk increases. The different columns of Table M account for this.

Question 33—Example 1

A. Premium discount should increase. For a larger risk, the expenses are a smaller % of manual premium.

B. Insurance charge should decrease. The risk now has higher expected losses, and larger expected loss is less skewly distributed.
C. This should not change. The excess loss pure premium factor is determined by individual loss distribution, which does not change when the exposure base is merely doubled (without any other change.)

Question 33—Example 2

A. The absolute amount of the premium discount should increase because the expenses should not increase in the same proportion as the increase in business. There will be a larger discount factor, therefore decreasing standard premium.
B. The insurance charge will increase significantly because there will be twice as many losses able to contribute to the maximum.
C. Because the per occurrence limit is not changing the excess loss pure premium factor should not change.

Question 34—Example 1

Large deductible policies include a much larger provision for expense than do excess policies, since claims servicing is not included in the cost of an excess policy. Therefore most cost associated with an excess policy is loss, which may pay out over 10+ years. The average payout period of a large deductible policy is tempered by the fact that many expenses are paid closer to the time of writing, thus decreasing the average payout period.

Question 34—Example 2

LDD policies cover claims handling & loss reimbursement. The expense payments are much quicker than loss payments therefore lowering the avg duration of payout. Excess policies only handle loss reimbursement which on long tailed lines such as WC, has longer payout.

Question 35—Example 1

ELR based on SP = (.8)(9,000,000) = 7,200,000
Expected excess = (7,200,000)(.20) = 1,440,000
Expected limited = 7,200,000 - 1,440,000 = 5,760,000
Entry ratio = Agg/Exp. Limited = 6750/5760 = 1.17
ELG = (7,200,000)[(1+(.8)(.16/.8))]/(1-.16/.8)(.9)
   =(7,200,000)(1.45)(.9) = 9,396,000
ELG = 16
(.2667)(Exp. Limited) = (.2667)(5,760,000) = 1,536,192

Question 35—Example 2

Expected Losses = ELR*SP = .80 * 9M = 7,200,000
Expected XS Losses = ELF * SP = .16 * 9M = 1,440,000
Expected Primary = 7.2M – 1.44M = 5,760,000

Entry Ratio = Agg Limit / Expected Primary Losses = 6,750,000 / 5,760,000 = 1.17

Now find Expected Loss Group

E*SP*((1+.8(F/E))/(1-(F/E))*SH Group Diff
= .8*9M*((1+.8(.16/.8)/(1-.16/.8))*9 = 9,396,000
Therefore ELG = 16

Insur charge for ELG 16, entry ratio 1.17 = .2667

Insur charge (in $) = .2667*Deductible losses = .2667*5,760,000 = 1,536,192

Question 36—Example 1

1. Policy limits might influence claim settlements. In other words, juries may be more likely to award higher damages to a plaintiff when an insurance policy will cover this amount.

2. Insureds that are worse risks may be more likely to purchase higher policy limits, since they perceive these limits might be needed.

Question 36—Example 2

1) Jury awards may be influenced by policy limits. Higher policy limits may influence jury to award higher awards than the jury would have with smaller policy limits.

2) Poor risks, aware of their bad risk status, would be more willing to purchase higher limits knowing they’re more likely to need the high limits than an average risk.

Question 37—Example 1

Frequency = 0.10
Severity = 130,000
Pure Premium = 0.10 x 130,000 = 13,000 (PP for 2M policy)

Pure Premium for $5M Limit
= PP for $2M Limit + PP for $3M XS of $2M
= 13,000 + 5,000
= 18,000
So the Pure Premium for a $1M Basic Limit = 18,000 / 1.75 = 10,285.71

Question 37—Example 2

Answer:
PP of 3M * 2M = 5,000

Assume the pure premium for a 1M basic limits policy is X.

Increased limits factor for 2,000,000 = (130,000 * 0.1) / X = 13,000 / X

Therefore, 5,000 = X * [1.75 – (13,000/X)]
X = 10,285.7

Question 37—Example 3

E[n] = 0.01
E[x; 2,000,000] = 130000

ILF5MM = E[x; 5000000] / E[x; 1000000] = 1.75

PP = E[n] * [E[x; 5000000]] – [E[x; 2000000]]
5000 = 0.1 * [E[x; 5000000] – 130000]
E[x; 5000000] = 180000

E[x; 1000000] = E[x; 5000000] / ILF5MM
=180000 / 1.75 = 102857

PP1MM = E[x; 1000000] * E[n] = 10,286

Question 38

A) Entry ratio for 50,000 = 50K/25K = 2
   Entry ratio for 75,000 = 75K/25K = 3
   
   I(75) = (1-.31)/(1-.4) = \mathbf{1.15}

B) Entry ratio 125K = 125/25 = 5
   I(125) = (1-.21)/(1-.4) = 1.317
Pure Premium = (3,000)(1.317) = 3,951

C) Entry ratio at 100K = 4
    at 500K = 20

CV3:
    I(100K) = (1-.25)/(1-.4) = 1.25
    I(500K) = (1-.05)/(1-.4) = 1.583
    PP 400xs100 = (1.583-1.25)(3,000) = 999

CV4:
    I(100K) = (1-.32)/(1-.46) = 1.259
    I(500K) = (1-.09)/(1-.46) = 1.685
    PP 400xs100 = (1.685-1.259)(3,000) = 1,278

PP $ error = 1,278 – 999 = 279

Question 39—Example 1

\[ Z = e(R - i) = \frac{(1 + i) \cdot C}{V_m} \]

Given:
    C = 2
    \( i_r = .08 \)

\[ V_m = \frac{1}{2} L(1+f)(1+i-Z)^{\frac{1}{1-i}} = \frac{1}{2} (40)(1+1)(1+.08-Z)^{\frac{1}{1-.5}} = 40(1.08-Z)^{-1} \]

Substituting into the formula for \( Z \) yields:

\[ Z = \frac{(1+i) \cdot C}{V_m} = \frac{(1+.08) \cdot 2}{40(1.08-Z)^{-1}} = \frac{2.16 \cdot (1.08-Z)}{40} \]

Solving for \( Z \) yields \( Z = .0617 \)

Then, \( i_a = i_r - Z = .08 - .0617 = 1.83\% \)

Question 39—Example 2

From Bustic,
Sample Essay Solutions
CAS Exam 9
Fall 2003

\[ Z = \frac{A(B - L(1 + i - Z)^{-t})}{(1 + i - Z)^{-5-t}} \]

\[ A = \frac{1 + i}{.5(1 + f)L} \]

Given \( i=.08, f=1, \) and \( L=40, \) then \( A=.027 \)

\[ B = P(1+i)^{-u} - D(i+i)^{-d}, \] then

\[ B - L(1 + i - Z)^{-t} = 2, \] which is given in the problem.

Therefore,

\[ Z = \frac{(.027)(2)}{(1.08 - Z)^{-1}} = .054(1.08 - Z) \]

Solving for \( Z \) yields \( Z=.0617 \)

Then, \( i_i = i - Z = .08 - .0617 = 1.83\% \)

**Question 40—Example 1**

1. The ROE approach focuses on return on equity and forgoes rate equity. Return on sales reflects rate equity and evaluates insurers independent of their surplus. ROE penalizes insurers that have high surplus (make them charge low rates even though they provide more protection)

2. Allocating surplus by LOB and jurisdiction is not reasonable. All surplus of a cy stands behind each LOB & State. ROE treats a monoline insurer with 1MM surplus the same way it treats a 100MM surplus cy that allocates 1MM to a given LOB & state.

3. It represents markup and makes it easier to evaluate all insurers on the same basis.

**Question 40—Example 2**

1. Using equity requires allocation of equity to line and state. This is incorrect. All equity in a company theoretically can support writings in one line & state.

2. Two companies can be writing exactly same type of business with same experience but because one has a higher or lower leverage ratio, the rate indication would be different under a return on equity guideline. By requiring the allocation of surplus to line and jurisdiction, the ROE basis ignores the value inherent in unallocated surplus.
3. Return on sales has intuitive appeal. It’s like a mark-up to consumers. They understand if the profit on goods is x% of the price. Stating the profit in terms of equity means nothing to them.

Question 41

A) Reserve leverage – you can think of the insurer as borrowing from the policyholder with underwriting losses being the “interest” paid.

B1) The interest on the debt capital is fixed while the interest on reserve capital is not – it’s the underwriting gain or loss.

B2) Also, when you acquire more debt, the interest rate you pay tends to increase. It’s possible that, when it comes to reserve capital, the interest rate decreases the more you borrow. This happens because the total losses become more predictable.

C) Value of firm determined by returns and rate at which returns are discounted. As leverage (1+R/S) increases, the volatility of returns increases so the discount rate must also increase, which lower value of firm.

Question 42—Example 1

(a)

\[
PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII)
\]

\[
P = 0.1P + 20 + \frac{40}{1.03} + \frac{30}{1.03^2} + \left(\frac{P - 0.1P - 20}{1.06}\right) \times (35\%) - \left(\frac{40 + 30/1.07}{1.03}\right) \times (35\%) - \left(\frac{30 - 30/1.07}{1.03^2}\right) \times (35\%)
\]

\[
+ \left(\frac{P - 0.1P - 20 + 21}{1.06}\right) \times (6\%) (35\%) + \left(\frac{0.9P - 20 - 40 + 9}{1.06^2}\right) \times (6\%) (35\%)
\]

\[
0.9P = 87.1128 + 0.2972P - 6.6038 - 23.1195 - 6.6475 + 0.0178P + 0.01981 + 0.0168P - 0.9532
\]

\[
0.5682P = 55.8078
\]

\[
P = $98.22
\]
(b)

Total losses and expenses = 70 + 0.1P + 20 = 99.82

Combined ratio = 99.82 / 98.22 = 101.63%

Underwriting profit margin = 100% - 101.63% = -1.63%

Question 42—Example 2

(a)

\[ P = 0.1P + 20 + \frac{40}{1.03} + \frac{30}{1.03^2} + \frac{(P - 0.1P - 20)(0.35)}{1.06} - \frac{(40 + 30/1.07)(0.35) - (30 - 30/1.07)(0.35)}{1.03} + \frac{(P - 0.1P - 20 + 21)(0.06)(0.35)}{1.07} + \frac{(0.9P - 20 - 40)(0.06)(0.35)}{1.06^2} + \frac{(40)(0.06)(0.35)}{1.03^2} \]

\[ P = 0.1P + 20 + 67.1128 + 0.2972P - 6.6038 - 23.1195 - 0.6475 + 0.0178P + 0.0198 + 0.0168P - 0.3738 + 0.1682 - 0.7918 \]

\[ 0.5682P = 55.7644 \]

\[ P = $98.14 \]

(b)

\[ 98.14 - (98.14)(0.1) - 20 - 40 - 30 = -1.674 \]

\[ -1.674 / 98.14 = -1.71\% \]

Question 43—Example 1

2000 Limit

\[ \begin{array}{c|c}
    \text{Prob} & \\
    0 & 90\% \Rightarrow E[L] = 110
\end{array} \]
5000 Limit

<table>
<thead>
<tr>
<th>Limit</th>
<th>Prob</th>
<th>( \sigma(L) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>90%</td>
<td>343.4</td>
</tr>
<tr>
<td>2000</td>
<td>1%</td>
<td></td>
</tr>
</tbody>
</table>

Loaded premium = \( 110 + 0.1 \times 343.4 = 144.3 \)

\[ \text{ILF} = \frac{196.6}{143.3} = 1.36 \]

**Question 43—Example 2**

Basic \( E[L] = 0.09 \times (1,000) + 0.01 \times (2,000) = 90 + 20 = 110 \)
\( Sd = \sqrt{0.9 \times 110^2 + 0.09 \times 890^2 + 0.01 \times 1890^2} = 343.37 \)

\[ E[L_2] = 0.09 \times (1,000) + 0.01 \times (5,000) = 140 \]
\( Sd = \sqrt{0.9 \times 140^2 + 0.09 \times 860^2 + 0.01 \times 1460^2} = 566.038 \)

\[ 140 + 0.1 \times (566.038) = 196.9 = 1.362 \]
\[ 110 + 0.1 \times (343.37) = 144.34 \]

**Question 44—Example 1**

a)
Advantages:

1. Use data from the Annual Statement & Insurance Expense Exhibit, which is readily available & easily verifiable.

2. Calculates a rate of return similar in some aspects to a GAAP return on equity, which is a measure of profitability in other industries.

Disadvantage:

1. Since it is a calendar year method, it is subject to distortions from rapid growth/decline in reserves or sudden changes in reserve adequacy.
2. Requires selection of a target rate of return.

b) Present Value Offset Method.
This method does not use calendar year data & thus corrects disadvantage #1 above.

Question 44—Example 2

a) Adv:
1. ROE calculated is comparable to GAAP ROE
2. Figures are readily available in the annual statement and IEE

Dis:
1. There is a need to select a target ROE, which requires assumptions
2. If Stat figures used instead of GAAP, then ROE is not comp to GAAP ROE

b) Under the Calendar Year Investment Income Offset Method no need to select a target ROE.

Question 44—Example 3

Advantages:
1. Using figures in IEE and Annual Statement, so data is easily verifiable.
2. Comparable to GAAP ROE which is widely used in other industries

Disadvantages:
1. Need to select target rate of return
2. Need to use leverage ratios by line, which may not be meaningful

b) Present Value Offset Method – no need to use leverage ratios—avoids #2 above.

Question 44—Example 4
a) Advantages:

1. Uses data readily available and verifiable from financial statements

2. Does not use traditional U/W profit provision, but links the UPP directly to the rate of return required by investors

Disadvantages:

1. Since this is a CY method, it is subject to distortions from loss volume growth, as well as reserve adequacy changes.

2. Target rate of return (TRR) has to be selected

b) DCF (discounted cash flows): approach calculates the fair premium directly and thereby does not require the selection of TRR (disadvantage #2).

Question 45

PV P = PVE + PVL + PVTUI + PVTII
PV P = 25 + 70/1.035 + ((P-25)/1.06 - 70/1.035 )×0.40 + (P-25+40)/1.06×0.40
P = 56.485 + 0.4P → P=94.14

- pretax UW Inc = 94.14 – 25 – 70 = -0.86
- pretax Inv Inc = (94.14 – 25 + 40) × 0.06 = 6.55
- total Inc = 5.69

Question 46—Example 1

1. Stability constraint- the probability must be less than P that the combined ratio exceeds X% of the target combined ratio in a given period.

Probability < .01 that the actual combined ratio exceeds a 96% combined ratio by more than 4%.

2. Survival Constraint- the probability must be less than P that losses +expenses will exceed new premium income + existing capital in a given period.

Probability < .01 that losses + expenses will exceed new premium of $100,000 and existing capital of $1,000,000
Question 46—Example 2

1. survival constraint
   
   must keep $\text{pr}[(\text{loss} + \text{expenses}) > (\text{premium} + \text{surplus})] < P_2$

   Mgmt identifies some probability to limit risk of insolvency.

   For example $P_2 = 10\%$ would mean that there must be less than 10% chance of loss + expense exceeding premium + surplus for the given risk portfolio.

2. Stability constraint
   
   Must keep $\text{pr}[\text{CR} > (\text{target CR} + X)] < P_1$
   
   Mgmt identifies $X: P_1$

   For example w/ $X = 5$ points : $P_1 = 1\%$

   The risk portfolio must have less than 1% probability of producing a combined ratio of 5 points higher than the company’s target combined ratio (e.g. target CR 90%)

   So $\text{pr}\text{(CR} > 95\%) < 1 \%; \ 95\% = 5 \text{ points} + \text{TCR}$

Question 47—Example 1

A) Reduction in variance in limit: $2*\text{retention}*\mathbb{E}\{h(x;r,j)\}$

   Reduction in variance for limit @100 = $2(100)(10) = 2000$
   
   Reduction in variance for limit @ 500 = $2(500)(5) = 5000$

   The extra risk load from layering is $(2000 + 5000)(.001) = 7$

   Total risk load = $9 + 7 = 16$

B) Premium for 500 limit = expected loss + given risk load + extra risk load from layering
   
   = $25 + 1 + 5 + (2000)(.001) = 33$

   Premium for 1000 limit = $30 + 1 + 5 + 3 + (7000)(.001) = 46$

   $\text{ILF} = 46/33 = 1.394$

Question 47—Example 2

Expected losses are additive, but risk loads are not

Reduction in variance = $2(\text{retention})(\text{expected value PP at top layer})$  

$\text{RL} = .001 \text{var}$
Var(500) – Var(100) – Var(4xs1) = 200(10)
.001[ ... ]

RL_{500} – RL_{100} – RL_{4xs1} = 2
RL_{500} – 1 – 5 = 2
Thus, RL_{500} = 8

Var(1000) – Var(500) – Var(5xs5) = 1000(5)
RL_{1000} – RL_{500} – RL_{500xs500} = 5
RL_{1000} – 8 – 5 = 5
A) RL_{1000} = 16

B) ILF_{1000} with RL = ((15+10+5) + 16)/((15+10) + 8) = 46/33 = 1.394

**Question 48—Example 1**

a.  e = .25  i_A = i – e ( R – i)
99 = 100v
v = .99  i_A = .0101
0.0101 = 0.03 – 0.25 (R - 0.03)
R = 0.1096 = exp ROE

b.  Req return  = R_F + B (R_m – R_F)
= 0.03 + 1.5(.1 - .03)
= 0.135

I would not take the deal because the expected ROE is below the required rate of return.

**Question 48—Example 2**

a.  99 = 100 / (1 + I_A)  I_A = 1%
I_A = .01 = i - e(R - i)
   .01 = .03 - .25 (R-.03)
R = .11

b.  B = 1.5
R_F + B (E_R_m - R_F)
= .03 + 1.5(.1-.03)
= .135

I would not take the above deal since the implicit ROE is less than that required by CAPM.