1. For a stationary AR(2) process, you are given:

\[ \rho_1 = 0.5 \]
\[ \rho_2 = -0.2 \]

Calculate \( \phi_2 \).

(A) \(-0.8\)
(B) \(-0.6\)
(C) \(-0.2\)
(D) \(0.6\)
(E) \(0.8\)
2. You are given the following claim data for automobile policies:

    200  255  295  320  360  420  440  490  500  520  1020

Calculate the smoothed empirical estimate of the 45th percentile.

(A) 358
(B) 371
(C) 384
(D) 390
(E) 396
3. You are given:

(i) The number of claims made by an individual insured in a year has a Poisson distribution with mean $\lambda$.

(ii) The prior distribution for $\lambda$ is gamma with parameters $\alpha = 1$ and $\theta = 1.2$.

Three claims are observed in Year 1, and no claims are observed in Year 2.

Using Bühlmann credibility, estimate the number of claims in Year 3.

(A) 1.35

(B) 1.36

(C) 1.40

(D) 1.41

(E) 1.43
4. In a study of claim payment times, you are given:

(i) The data were not truncated or censored.

(ii) At most one claim was paid at any one time.

(iii) The Nelson-Aalen estimate of the cumulative hazard function, $H(t)$, immediately following the second paid claim, was $23/132$.

Determine the Nelson-Aalen estimate of the cumulative hazard function, $H(t)$, immediately following the fourth paid claim.

(A) 0.35

(B) 0.37

(C) 0.39

(D) 0.41

(E) 0.43
5. You fit the following model to eight observations:

\[ Y = \alpha + \beta X + \varepsilon \]

You are given:

\[ \hat{\beta} = 2.065 \]

\[ \sum (X_i - \bar{X})^2 = 42 \]

\[ \sum (Y_i - \bar{Y})^2 = 182 \]

Determine \( R^2 \).

(A) 0.48
(B) 0.62
(C) 0.83
(D) 0.91
(E) 0.98
6. The number of claims follows a negative binomial distribution with parameters $\beta$ and $r$, where $\beta$ is unknown and $r$ is known. You wish to estimate $\beta$ based on $n$ observations, where $\bar{x}$ is the mean of these observations.

Determine the maximum likelihood estimate of $\beta$.

(A) $\frac{\bar{x}}{r^2}$

(B) $\frac{\bar{x}}{r}$

(C) $\bar{x}$

(D) $r\bar{x}$

(E) $r^2\bar{x}$
7. You are given the following information about a credibility model:

<table>
<thead>
<tr>
<th>First Observation</th>
<th>Unconditional Probability</th>
<th>Bayesian Estimate of Second Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/3</td>
<td>1.50</td>
</tr>
<tr>
<td>2</td>
<td>1/3</td>
<td>1.50</td>
</tr>
<tr>
<td>3</td>
<td>1/3</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Determine the Bühlmann credibility estimate of the second observation, given that the first observation is 1.

(A) 0.75  
(B) 1.00  
(C) 1.25  
(D) 1.50  
(E) 1.75
8. For a survival study, you are given:

(i) The Product-Limit estimator $\hat{S}(t_0)$ is used to construct confidence intervals for $S(t_0)$.

(ii) The 95% log-transformed confidence interval for $S(t_0)$ is $(0.695, 0.843)$.

Determine $\hat{S}(t_0)$.

(A) 0.758  
(B) 0.762  
(C) 0.765  
(D) 0.769  
(E) 0.779
9. You are given the following information about an AR(1) model with mean 0:

\[ \rho_2 = 0.215 \]

\[ \rho_3 = -0.100 \]

\[ y_T = -0.431 \]

Calculate the forecasted value of \( y_{T+1} \).

(A) – 0.2
(B) – 0.1
(C) 0.0
(D) 0.1
(E) 0.2
10. A random sample of three claims from a dental insurance plan is given below:

\[ 225 \quad 525 \quad 950 \]

Claims are assumed to follow a Pareto distribution with parameters \( \theta = 150 \) and \( \alpha \).

Determine the maximum likelihood estimate of \( \alpha \).

(A) Less than 0.6
(B) At least 0.6, but less than 0.7
(C) At least 0.7, but less than 0.8
(D) At least 0.8, but less than 0.9
(E) At least 0.9
11. An insurer has data on losses for four policyholders for 7 years. The loss from the \( i^{th} \) policyholder for year \( j \) is \( X_{ij} \).

You are given:

\[
\sum_{i=1}^{4} \sum_{j=1}^{7} (X_{ij} - \overline{X}_i)^2 = 33.60
\]

\[
\sum_{i=1}^{4} (\overline{X}_i - \overline{X})^2 = 3.30
\]

Using nonparametric empirical Bayes estimation, calculate the Bühlmann credibility factor for an individual policyholder.

(A) Less than 0.74

(B) At least 0.74, but less than 0.77

(C) At least 0.77, but less than 0.80

(D) At least 0.80, but less than 0.83

(E) At least 0.83
12. For the three variables $Y$, $X_2$ and $X_3$, you are given the following sample correlation coefficients:

\[ r_{YX_2} = 0.6 \]
\[ r_{YX_3} = 0.5 \]
\[ r_{X_2X_3} = 0.4 \]

Calculate $r_{YX_2,X_3}$, the partial correlation coefficient between $Y$ and $X_2$.

(A) 0.50  
(B) 0.55  
(C) 0.58  
(D) 0.64  
(E) 0.73
13. Losses come from an equally weighted mixture of an exponential distribution with mean $m_1$, and an exponential distribution with mean $m_2$.

Determine the least upper bound for the coefficient of variation of this distribution.

(A) 1
(B) $\sqrt{2}$
(C) $\sqrt{3}$
(D) 2
(E) $\sqrt{5}$
14. You are given the following information about a commercial auto liability book of business:

(i) Each insured’s claim count has a Poisson distribution with mean $\lambda$, where $\lambda$ has a gamma distribution with $\alpha = 1.5$ and $\theta = 0.2$.

(ii) Individual claim size amounts are independent and exponentially distributed with mean 5000.

(iii) The full credibility standard is for aggregate losses to be within 5% of the expected with probability 0.90.

Using classical credibility, determine the expected number of claims required for full credibility.

(A) 2165

(B) 2381

(C) 3514

(D) 7216

(E) 7938
15. An insurance company uses a proportional hazards model to investigate whether to have different premium rates for two different classes of drivers.

You are given:

(i) The model has a single covariate: $Z = 1$ if the driver is in class 1, $Z = 0$ if the driver is in class 2.

(ii) The model is $h(t|Z) = h_0(t) \exp(\beta Z)$, where $h_0(t)$ is an arbitrary baseline hazard rate and $\beta$ is the parameter.

(iii) The estimated relative risk for drivers in class 1 compared to drivers in class 2 is 1.822.

(iv) The information matrix is $I(\beta) = 3.968$, where $\beta$ is the partial maximum likelihood estimate of $\beta$.

You use Wald’s test to test the hypothesis $\beta = 0$.

Determine the value of the test statistic.

(A) 0.7

(B) 0.9

(C) 1.4

(D) 2.2

(E) 5.7
16. Which of the following statements about stationary mixed autoregressive-moving average models is true?

(A) A necessary condition for stationarity is that each parameter $\phi_i$ must have an absolute value less than 1.

(B) The autocorrelation function approaches $\phi_i$ as the displacement increases.

(C) The difference between adjacent forecasted values approaches $\delta$ as the number of periods ahead increases.

(D) The forecasted values approach the mean as the number of periods ahead increases.

(E) These models are particularly well-suited to long forecasting horizons.
17. You are given:

(i) A sample of claim payments is:

\[ 29 \quad 64 \quad 90 \quad 135 \quad 182 \]

(ii) Claim sizes are assumed to follow an exponential distribution.

(iii) The mean of the exponential distribution is estimated using the method of moments.

Calculate the value of the Kolmogorov-Smirnov test statistic.

(A) 0.14

(B) 0.16

(C) 0.19

(D) 0.25

(E) 0.27
18. You are given:

(i) Annual claim frequency for an individual policyholder has mean $\lambda$ and variance $\sigma^2$.

(ii) The prior distribution for $\lambda$ is uniform on the interval $[0.5, 1.5]$.

(iii) The prior distribution for $\sigma^2$ is exponential with mean 1.25.

A policyholder is selected at random and observed to have no claims in Year 1.

Using Bühlmann credibility, estimate the number of claims in Year 2 for the selected policyholder.

(A) 0.56
(B) 0.65
(C) 0.71
(D) 0.83
(E) 0.94
19. You study the time between accidents and reports of claims. The study was terminated at time 3.

You are given:

<table>
<thead>
<tr>
<th>Time of Accident</th>
<th>Time between Accident and Claim Report</th>
<th>Number of Reported Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>11</td>
</tr>
</tbody>
</table>

Use the Product-Limit estimator to estimate the conditional probability that the time between accident and claim report is less than 2, given that it does not exceed 3.

(A) Less than 0.4
(B) At least 0.4, but less than 0.5
(C) At least 0.5, but less than 0.6
(D) At least 0.6, but less than 0.7
(E) At least 0.7
20. You study the impact of education and number of children on the wages of working women using the following model:

\[ Y = a + b_1 E + b_2 F + c_1 G + c_2 H + \varepsilon \]

where

\[ Y = \ln(wages) \]

\[ E = \begin{cases} 
1 & \text{if the woman has not completed high school} \\
0 & \text{if the woman has completed high school} \\
-1 & \text{if the woman has post-secondary education} 
\end{cases} \]

\[ F = \begin{cases} 
0 & \text{if the woman has not completed high school} \\
-1 & \text{if the woman has post-secondary education} \\
1 & \text{if the woman has completed high school} 
\end{cases} \]

\[ G = \begin{cases} 
0 & \text{if the woman has 1 or 2 children} \\
-1 & \text{if the woman has more than 2 children} \\
1 & \text{if the woman has no children} 
\end{cases} \]

\[ H = \begin{cases} 
0 & \text{if the woman has more than 2 children} \\
1 & \text{if the woman has 1 or 2 children} \\
-1 & \text{if the woman has no children} 
\end{cases} \]

Determine the expected difference between \( \ln(wages) \) of a working woman who has post-secondary education and more than 2 children and \( \ln(wages) \) of the average for all working women.

(A) \( a - b_1 - b_2 \)

(B) \( b_1 + b_2 \)

(C) \( -b_1 - b_2 \)

(D) \( a - b_1 - b_2 + c_2 \)

(E) \( -b_1 - b_2 - c_1 - c_2 \)
21. You are given:

(i) The prior distribution of the parameter $\Theta$ has probability density function:

$$\pi(\theta) = \frac{1}{\theta^2}, \quad 1 < \theta < \infty$$

(ii) Given $\Theta = \theta$, claim sizes follow a Pareto distribution with parameters $\alpha = 2$ and $\theta$.

A claim of 3 is observed.

Calculate the posterior probability that $\Theta$ exceeds 2.

(A) 0.33
(B) 0.42
(C) 0.50
(D) 0.58
(E) 0.64
22. You are given:

(i) 

<table>
<thead>
<tr>
<th>t</th>
<th>y_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>1.3</td>
</tr>
</tbody>
</table>

(ii) \( y_t = 0 \), for \( t < 0 \)

(iii) \( \alpha = 0.6 \)

Use double exponential smoothing to determine \( \bar{y}_2 \).

(A) 0.96
(B) 0.99
(C) 1.16
(D) 1.20
(E) 1.33
23. You are given:

(i) Losses follow an exponential distribution with mean $\theta$.

(ii) A random sample of 20 losses is distributed as follows:

<table>
<thead>
<tr>
<th>Loss Range</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 1000]</td>
<td>7</td>
</tr>
<tr>
<td>(1000, 2000]</td>
<td>6</td>
</tr>
<tr>
<td>(2000, $\infty$)</td>
<td>7</td>
</tr>
</tbody>
</table>

Calculate the maximum likelihood estimate of $\theta$.

(A) Less than 1950
(B) At least 1950, but less than 2100
(C) At least 2100, but less than 2250
(D) At least 2250, but less than 2400
(E) At least 2400
24. You are given:

(i) The amount of a claim, \( X \), is uniformly distributed on the interval \([0, \theta]\).

(ii) The prior density of \( \theta \) is \( \pi(\theta) = \frac{500}{\theta^2}, \quad \theta > 500 \).

Two claims, \( x_1 = 400 \) and \( x_2 = 600 \), are observed. You calculate the posterior distribution as:

\[
f(\theta | x_1, x_2) = 3 \left( \frac{600^3}{\theta^4} \right), \quad \theta > 600
\]

Calculate the Bayesian premium, \( \mathbb{E}(X_3 | x_1, x_2) \).

(A) 450  
(B) 500  
(C) 550  
(D) 600  
(E) 650
25-26. *Use the following information for questions 25 and 26.*

The claim payments on a sample of ten policies are:

2 3 3 5\(^+\) 6 7\(^+\) 9 10\(^+\)

\(^+\) indicates that the loss exceeded the policy limit

25. Using the Product-Limit estimator, calculate the probability that the loss on a policy exceeds 8.

(A) 0.20
(B) 0.25
(C) 0.30
(D) 0.36
(E) 0.40
25-26. (Repeated for convenience) Use the following information for questions 25 and 26.

The claim payments on a sample of ten policies are:

\[ 2 \ 3 \ 3 \ 5 \ \ ^{+}5 \ 6 \ 7 \ \ ^{+}7 \ 9 \ 10 \ ^{+} \]

+ indicates that the loss exceeded the policy limit

26. You use the log-rank test to test the hypothesis that losses follow a Weibull distribution with survival function:

\[ S_0(x) = e^{-(x/\beta)^\gamma}, \ 0 < x < \infty \]

Determine the result of the test.

(A) Reject at the 0.005 significance level.
(B) Reject at the 0.010 significance level, but not at the 0.005 level.
(C) Reject at the 0.025 significance level, but not at the 0.010 level.
(D) Reject at the 0.050 significance level, but not at the 0.025 level.
(E) Do not reject at the 0.050 significance level.
27. For the multiple regression model $Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \epsilon$, you are given:

(i) $N = 3120$

(ii) $\text{TSS} = 15000$

(iii) $H_0: \beta_4 = \beta_5 = \beta_6 = 0$

(iv) $R^2_{UR} = 0.38$

(v) $\text{RSS}_R = 5565$

Determine the value of the $F$ statistic for testing $H_0$.

(A) Less than 10

(B) At least 10, but less than 12

(C) At least 12, but less than 14

(D) At least 14, but less than 16

(E) At least 16
28. You are given the following observed claim frequency data collected over a period of 365 days:

<table>
<thead>
<tr>
<th>Number of Claims per Day</th>
<th>Observed Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>122</td>
</tr>
<tr>
<td>2</td>
<td>101</td>
</tr>
<tr>
<td>3</td>
<td>92</td>
</tr>
<tr>
<td>4+</td>
<td>0</td>
</tr>
</tbody>
</table>

Fit a Poisson distribution to the above data, using the method of maximum likelihood.

Regroup the data, by number of claims per day, into four groups:

\[0 \quad 1 \quad 2 \quad 3+\]

Apply the chi-square goodness-of-fit test to evaluate the null hypothesis that the claims follow a Poisson distribution.

Determine the result of the chi-square test.

(A) Reject at the 0.005 significance level.
(B) Reject at the 0.010 significance level, but not at the 0.005 level.
(C) Reject at the 0.025 significance level, but not at the 0.010 level.
(D) Reject at the 0.050 significance level, but not at the 0.025 level.
(E) Do not reject at the 0.050 significance level.
29. You are given the following joint distribution:

<table>
<thead>
<tr>
<th>X</th>
<th>Θ</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

For a given value of Θ and a sample of size 10 for X:

\[ \sum_{i=1}^{10} x_i = 10 \]

Determine the Bühlmann credibility premium.

(A) 0.75
(B) 0.79
(C) 0.82
(D) 0.86
(E) 0.89
30. Which of the following is not an objection to the use of $R^2$ to compare the validity of regression results under alternative specifications of a multiple linear regression model?

(A) The $F$ statistic used to test the null hypothesis that none of the explanatory variables helps explain variation of $Y$ about its mean is a function of $R^2$ and degrees of freedom.

(B) Increasing the number of independent variables in the regression equation can never lower $R^2$ and is likely to raise it.

(C) When the model is constrained to have zero intercept, the ratio of regression sum of squares to total sum of squares need not lie within the range [0,1].

(D) Subtracting the value of one of the independent variables from both sides of the regression equation can change the value of $R^2$ while leaving the residuals unaffected.

(E) Because $R^2$ is interpreted assuming the model is correct, it provides no direct procedure for comparing alternative specifications.
31. You are given:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr[X = x]</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The method of moments is used to estimate the population mean, \( \mu \), and variance, \( \sigma^2 \), by \( \overline{X} \) and \( S_n^2 = \frac{\sum (X_i - \overline{X})^2}{n} \), respectively.

Calculate the bias of \( S_n^2 \), when \( n = 4 \).

(A) \(-0.72\)
(B) \(-0.49\)
(C) \(-0.24\)
(D) \(-0.08\)
(E) \(0.00\)
32. You are given four classes of insureds, each of whom may have zero or one claim, with the following probabilities:

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Claims</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>II</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>III</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>IV</td>
<td>0.1</td>
<td>0.9</td>
</tr>
</tbody>
</table>

A class is selected at random (with probability ¼), and four insureds are selected at random from the class. The total number of claims is two.

If five insureds are selected at random from the same class, estimate the total number of claims using Bühlmann-Straub credibility.

(A) 2.0
(B) 2.2
(C) 2.4
(D) 2.6
(E) 2.8
33. The following results were obtained from a survival study, using the Product-Limit estimator:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\hat{S}(t)$</th>
<th>$\sqrt{\hat{V}[\hat{S}(t)]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>0.957</td>
<td>0.0149</td>
</tr>
<tr>
<td>25</td>
<td>0.888</td>
<td>0.0236</td>
</tr>
<tr>
<td>32</td>
<td>0.814</td>
<td>0.0298</td>
</tr>
<tr>
<td>36</td>
<td>0.777</td>
<td>0.0321</td>
</tr>
<tr>
<td>39</td>
<td>0.729</td>
<td>0.0348</td>
</tr>
<tr>
<td>42</td>
<td>0.680</td>
<td>0.0370</td>
</tr>
<tr>
<td>44</td>
<td>0.659</td>
<td>0.0378</td>
</tr>
<tr>
<td>47</td>
<td>0.558</td>
<td>0.0418</td>
</tr>
<tr>
<td>50</td>
<td>0.360</td>
<td>0.0470</td>
</tr>
<tr>
<td>54</td>
<td>0.293</td>
<td>0.0456</td>
</tr>
<tr>
<td>56</td>
<td>0.244</td>
<td>0.0440</td>
</tr>
<tr>
<td>57</td>
<td>0.187</td>
<td>0.0420</td>
</tr>
<tr>
<td>59</td>
<td>0.156</td>
<td>0.0404</td>
</tr>
<tr>
<td>62</td>
<td>0.052</td>
<td>0.0444</td>
</tr>
</tbody>
</table>

Determine the lower limit of the 95% linear confidence interval for $x_{0.75}$, the 75th percentile of the survival distribution.

(A) 32
(B) 36
(C) 50
(D) 54
(E) 56
You fit an AR(2) model to a series of 100 observations.

You are given:

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\hat{r}_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.01</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>-0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>-0.03</td>
</tr>
<tr>
<td>6</td>
<td>-0.13</td>
</tr>
<tr>
<td>7</td>
<td>-0.23</td>
</tr>
<tr>
<td>8</td>
<td>-0.05</td>
</tr>
<tr>
<td>9</td>
<td>-0.01</td>
</tr>
<tr>
<td>10</td>
<td>0.05</td>
</tr>
<tr>
<td>11</td>
<td>-0.04</td>
</tr>
<tr>
<td>12</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Calculate the Box-Pierce $Q$ statistic based on the first twelve residual autocorrelations.

(A) 9.0  
(B) 9.3  
(C) 9.6  
(D) 9.9  
(E) 10.2
35. With the bootstrapping technique, the underlying distribution function is estimated by which of the following?

(A) The empirical distribution function
(B) A normal distribution function
(C) A parametric distribution function selected by the modeler
(D) Any of (A), (B) or (C)
(E) None of (A), (B) or (C)
36. You are given:

<table>
<thead>
<tr>
<th>Number of Claims</th>
<th>Probability</th>
<th>Claim Size</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{1}{5} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( \frac{3}{5} )</td>
<td>25</td>
<td>( \frac{1}{5} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{5} )</td>
<td>50</td>
<td>( \frac{2}{5} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>( \frac{2}{5} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>( \frac{1}{5} )</td>
</tr>
</tbody>
</table>

Claim sizes are independent.

Determine the variance of the aggregate loss.

(A) 4,050
(B) 8,100
(C) 10,500
(D) 12,510
(E) 15,612
You are given:

(i) Losses follow an exponential distribution with mean $\theta$.
(ii) A random sample of losses is distributed as follows:

<table>
<thead>
<tr>
<th>Loss Range</th>
<th>Number of Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 – 100]</td>
<td>32</td>
</tr>
<tr>
<td>(100 – 200]</td>
<td>21</td>
</tr>
<tr>
<td>(200 – 400]</td>
<td>27</td>
</tr>
<tr>
<td>(400 – 750]</td>
<td>16</td>
</tr>
<tr>
<td>(750 – 1000]</td>
<td>2</td>
</tr>
<tr>
<td>(1000 – 1500]</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>

Estimate $\theta$ by matching at the 80th percentile.

(A) 249  
(B) 253  
(C) 257  
(D) 260  
(E) 263
38. You fit a two-variable linear regression model to 20 pairs of observations.

You are given:

(i) The sample mean of the independent variable is 100.

(ii) The sum of squared deviations from the mean of the independent variable is 2266.

(iii) The ordinary least-squares estimate of the intercept parameter is 68.73.

(iv) The error sum of squares (ESS) is 5348.

Determine the lower limit of the symmetric 95% confidence interval for the intercept parameter.

(A) $-273$

(B) $-132$

(C) $-70$

(D) $-8$

(E) $-3$
39. You are given:

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Insureds</th>
<th>Claim Count Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3000</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>0</td>
</tr>
</tbody>
</table>

A randomly selected insured has one claim in Year 1.

Determine the expected number of claims in Year 2 for that insured.

(A) 1.00
(B) 1.25
(C) 1.33
(D) 1.67
(E) 1.75
You are given the following information about a group of policies:

<table>
<thead>
<tr>
<th>Claim Payment</th>
<th>Policy Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>

Determine the likelihood function.

(A) \( f(50) f(50) f(100) f(100) f(500) f(1000) \)
(B) \( f(50) f(50) f(100) f(100) f(500) f(1000) / [1-F(1000)] \)
(C) \( f(5) f(15) f(60) f(100) f(500) f(500) \)
(D) \( f(5) f(15) f(60) f(100) f(500) f(500) / [1-F(1000)] \)
(E) \( f(5) f(15) f(60) [1-F(100)] [1-F(500)] f(500) \)

**END OF EXAMINATION**
<table>
<thead>
<tr>
<th>Test Item</th>
<th>Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
</tr>
<tr>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
</tr>
<tr>
<td>8</td>
<td>E</td>
</tr>
<tr>
<td>9</td>
<td>E</td>
</tr>
<tr>
<td>10</td>
<td>B</td>
</tr>
<tr>
<td>11</td>
<td>D</td>
</tr>
<tr>
<td>12</td>
<td>A</td>
</tr>
<tr>
<td>13</td>
<td>C</td>
</tr>
<tr>
<td>14</td>
<td>B</td>
</tr>
<tr>
<td>15</td>
<td>C</td>
</tr>
<tr>
<td>16</td>
<td>D</td>
</tr>
<tr>
<td>17</td>
<td>E</td>
</tr>
<tr>
<td>18</td>
<td>E</td>
</tr>
<tr>
<td>19</td>
<td>B</td>
</tr>
<tr>
<td>20</td>
<td>E</td>
</tr>
<tr>
<td>21</td>
<td>E</td>
</tr>
<tr>
<td>22</td>
<td>B</td>
</tr>
<tr>
<td>23</td>
<td>B</td>
</tr>
<tr>
<td>24</td>
<td>A</td>
</tr>
<tr>
<td>25</td>
<td>D</td>
</tr>
<tr>
<td>26</td>
<td>D</td>
</tr>
<tr>
<td>27</td>
<td>D</td>
</tr>
<tr>
<td>28</td>
<td>C</td>
</tr>
<tr>
<td>29</td>
<td>D</td>
</tr>
<tr>
<td>30</td>
<td>A</td>
</tr>
<tr>
<td>31</td>
<td>C</td>
</tr>
<tr>
<td>32</td>
<td>C</td>
</tr>
<tr>
<td>33</td>
<td>D</td>
</tr>
<tr>
<td>34</td>
<td>A</td>
</tr>
<tr>
<td>35</td>
<td>A</td>
</tr>
<tr>
<td>36</td>
<td>B</td>
</tr>
<tr>
<td>37</td>
<td>A</td>
</tr>
<tr>
<td>38</td>
<td>D</td>
</tr>
<tr>
<td>39</td>
<td>B</td>
</tr>
<tr>
<td>40</td>
<td>E</td>
</tr>
</tbody>
</table>
Question # 1
Answer: B

$$\rho_1 = \frac{\phi_1}{1 - \phi_2} = 0.5$$

$$\rho_2 = \phi_2 + \frac{\phi_1^2}{1 - \phi_2} = -0.2$$

Solving simultaneously gives:

$$\phi_1 = 0.8$$
$$\phi_2 = -0.6$$

Question # 2
Answer: C

$$g = [12(.45)] = [5.4] = 5; \quad h = 5.4 - 5 = 0.4.$$  

$$\hat{\pi}_{45} = .6x_{(5)} + .4x_{(6)} = .6(360) + .4(420) = 384.$$  

Question # 3
Answer: D

\(N\) is distributed \(\text{Poisson}(\lambda)\)

\(\mu = E(\lambda) = \alpha \theta = 1(1.2) = 1.2.\)

\(\nu = E(\lambda) = 1.2; \quad a = Var(\lambda) = \alpha \theta^2 = 1(1.2)^2 = 1.44.\)

\(k = \frac{1.2}{1.44} = \frac{5}{6}; \quad Z = \frac{2}{\frac{2 + 5}{6}} = \frac{12}{17}.\)

Thus, the estimate for Year 3 is

$$\frac{12}{17}(1.5) + \frac{5}{17}(1.2) = 1.41.$$  

Note that a Bayesian approach produces the same answer.
**Question # 4**
Answer: C

At the time of the second failure,

\[ \hat{H}(t) = \frac{1}{n} + \frac{1}{n-1} = \frac{23}{132} \Rightarrow n = 12. \]

At the time of the fourth failure,

\[ \hat{H}(t) = \frac{1}{12} + \frac{1}{11} + \frac{1}{10} + \frac{1}{9} = .3854. \]

**Question # 5**
Answer: E

\[ R^2 = \hat{\beta}^2 \sum \frac{x_i^2}{y_i^2} = 2.065^2 \cdot \frac{42}{182} = .9841. \]

**Question # 6**
Answer: B

The likelihood is:

\[ L = \prod_{j=1}^{n} \frac{r(r+1)\cdots(r+x_j-1)\beta^{x_j}}{x_j!(1+\beta)^{r+x_j}} \propto \prod_{j=1}^{n} \beta^{x_j}(1+\beta)^{-r-x_j}. \]

The loglikelihood is:

\[ l = \sum_{j=1}^{n} \left[ x_j \ln \beta - (r + x_j) \ln(1 + \beta) \right] \]

\[ l' = \sum_{j=1}^{n} \left[ \frac{x_j}{\beta} - \frac{r + x_j}{1 + \beta} \right] = 0 \]

\[ 0 = \sum_{j=1}^{n} \left[ x_j(1 + \beta) - (r + x_j)\beta \right] = \sum_{j=1}^{n} x_j - nr \beta \]

\[ 0 = nx - nr\hat{\beta}; \quad \hat{\beta} = \frac{n}{r}. \]
Question # 7
Answer: C

The Bühlmann credibility estimate is $Zx + (1 - Z)\mu$ where $x$ is the first observation. The Bühlmann estimate is the least squares approximation to the Bayesian estimate. Therefore, $Z$ and $\mu$ must be selected to minimize

$$\frac{1}{3}[Z + (1 - Z)\mu - 1.5]^2 + \frac{1}{3}[2Z + (1 - Z)\mu - 1.5]^2 + \frac{1}{3}[3Z + (1 - Z)\mu - 3]^2.$$  

Setting partial derivatives equal to zero will give the values. However, it should be clear that $\mu$ is the average of the Bayesian estimates, that is,

$$\mu = \frac{1}{3}(1.5 + 1.5 + 3) = 2.$$  

The derivative with respect to $Z$ is (deleting the coefficients of $1/3$):

$$2(-Z + .5)(-1) + 2(.5)(0) + 2(Z - 1)(1) = 0$$

$Z = .75.$

The answer is

$.75(1) + .25(2) = 1.25.$

Question # 8
Answer: E

The confidence interval is $\hat{\theta}(t_0)^{1/\theta}, \hat{\theta}(t_0)^{\theta}$. Taking logarithms of both endpoints gives the two equations

$$\ln .695 = -.36384 = \frac{1}{\theta}\ln \hat{S}(t_0)$$

$$\ln .843 = -.17079 = \theta \ln \hat{S}(t_0).$$

Multiplying the two equations gives

$$0.06214 = [\ln \hat{S}(t_0)]^2$$

$$\ln \hat{S}(t_0) = -.24928$$

$$\hat{S}(t_0) = .77936.$$  

The negative square root is required in order to make the answer fall in the interval $(0,1).$
Question # 9
Answer: E

Because $\rho_k = \phi^k$ there are a number of ways to get the value of $\phi$.

$\phi = .215^{1/2} = -.46368; \quad \phi = (.1)^{1/3} = -.46416; \quad \phi = \frac{-1}{.215} = -.46512.$

Also, because the mean is zero, $\delta$ must be zero. Then (using the first choice for $\phi$),

$\hat{y}_{r,i} = -.46368(-.431) + 0 = .1998.$

Question # 10
Answer: B

The likelihood is:

$L = \frac{\alpha^{150^\alpha}}{(150 + 225)^{\alpha+1}} \frac{\alpha^{150^\alpha}}{(150 + 525)^{\alpha+1}} \frac{\alpha^{150^\alpha}}{(150 + 950)^{\alpha+1}}$

$= \frac{\alpha^{150^\alpha}}{(375\cdot675\cdot1100)^{\alpha+1}}.$

The loglikelihood is:

$l = 3\ln \alpha + 3\alpha \ln 150 - (\alpha + 1) \ln(375\cdot675\cdot1100)$

$l' = \frac{3}{\alpha} + 3 \ln 150 - \ln(375\cdot675\cdot1100) = \frac{3}{\alpha} - 4.4128$

$\hat{\alpha} = 3 / 4.4128 = .6798.$

Question # 11
Answer: D

For this problem, $r = 4$ and $n = 7$. Then,

$\hat{\nu} = \frac{33.60}{4(7-1)} = 1.4 \quad \text{and} \quad \hat{a} = \frac{3.3}{4-1} - \frac{1.4}{7} = .9.$

Then,

$k = \frac{1.4}{.9} = \frac{14}{9}; \quad Z = \frac{7}{7 + (14/9)} = \frac{63}{77} = .82.$
Question # 12  
Answer: A

\[ r_{x_2; x_3} = \frac{r_{x_2x_3} - r_{x_2}r_{x_3}}{\sqrt{1-r^2_{x_2}} \sqrt{1-r^2_{x_3}}} = \frac{.6 - .5 \cdot .4}{\sqrt{1-.5^2} \sqrt{1-.4^2}} = .504 \]

Question # 13  
Answer: C

For a mixture variable, raw moments are weighted averages of the individual moments. Thus,

\[ E(X) = .5m_1 + .5m_2 \text{ and } E(X^2) = .5(2m_1^2) + .5(2m_2^2). \]

The square of the coefficient of variation is

\[ \frac{Var(X)}{E(X)^2} = \frac{E(X^2) - E(X)^2}{E(X)^2} = \frac{m_1^2 + m_2^2}{.25(m_1 + m_2)^2} - 1. \]

Divide numerator and denominator by \( m_2^2 \) and let \( r = m_1 / m_2 \). The square of the coefficient of variation becomes

\[ \frac{r^2 + 1}{.25(r + 1)^2} - 1. \]

Setting the derivative equal to zero yields \( r = 1 \), however, this value minimizes the function (at a value of 1). There are no other critical points. Looking at the endpoints (\( r = 0 \) and \( r = \text{infinity} \)) the limiting value is 3, which is the maximum. Therefore, the least upper bound for the coefficient of variation is the square root of 3.

Question # 14  
Answer: B

\( X \) is the random sum \( Y_1 + Y_2 + \ldots + Y_N \).
\( N \) has a negative binomial distribution with \( r = \alpha = 1.5 \) and \( \beta = \theta = 0.2 \).

\[ E(N) = r \beta = 0.3 \]
\[ Var(N) = r \beta (1 + \beta) = 0.36 \]
\[ E(Y) = 5000 \]
\[ Var(Y) = 25,000,000 \]
\[
E(X) = 0.3 \times 5000 = 1500
\]
\[
Var(X) = 0.3 \times 25,000,000 + 0.36 \times 25,000,000 = 16,500,000
\]

Number of exposures (insureds) required for full credibility
\[
n_{FULL} = (1.645 / 0.05)^2 \times 16,500,000 / (1500)^2 = 7937.67.
\]

Number of expected claims required for full credibility
\[
E(N) \times n_{FULL} = 0.3 \times 7937.67 = 2381.
\]

**Question # 15**
**Answer: C**

The estimated relative risk is
\[
\frac{h(t \mid Z = 1)}{h(t \mid Z = 0)} = \frac{h_0(t)e^b}{h_0(t)} = e^b = 1.822 \Rightarrow b = .6.
\]

For the single covariate case, the Wald test for testing \(H_0: \beta = 0\) reduces to:
\[
(b - 0)^2 I(b) = (.6)^2 (3.968) = 1.43.
\]

**Question # 16**
**Answer: D**

See pages 535-7, the bottom of page 567 and the top of page 568. The only correct statement – and the correct answer – is (D).

**Question # 17**
**Answer: E**

| \(x\) | \(F_n(x)\) | \(F_n(x^-)\) | \(F_0(x)\) | \(|F_n(x) - F_0(x)|\) | \(|F_n(x^-) - F_0(x)|\) |
|---|---|---|---|---|---|
| 29 | 0.2 | 0 | 0.252 | 0.052 | 0.0252 |
| 64 | 0.4 | 0.2 | 0.473 | 0.073 | 0.273 |
| 90 | 0.6 | 0.4 | 0.593 | 0.007 | 0.193 |
| 135 | 0.8 | 0.6 | 0.741 | 0.059 | 0.141 |
| 182 | 1.00 | 0.8 | 0.838 | 0.162 | 0.038 |

where:
\[
\hat{\theta} = \bar{x} = 100 \quad \text{and} \quad F_0(x) = 1 - e^{-x/100}.
\]

The maximum value from the last two columns is 0.273.
Question # 18
Answer: E

\[ \mu = E(\lambda) = 1; \quad \nu = E(\sigma^2) = 1.25; \quad a = Var(\lambda) = 1/12. \]

\[ k = \nu / a = 15; \quad Z = \frac{1}{1+15} = \frac{1}{16}. \]

Thus, the estimate for Year 2 is

\[ \frac{1}{16}(0) + \frac{15}{16}(1) = .9375. \]

Question # 19
Answer: B

Time must be reversed, so let \( T \) be the time between accident and claim report and let \( R = 3 - T \). The desired probability is

\[ \Pr(T < 2 \mid T \leq 3) = \Pr(3 - R < 2 \mid 3 - R \leq 3) = \Pr(R > 1 \mid R \geq 0) = \Pr(R > 1). \]

The product-limit calculation is:

<table>
<thead>
<tr>
<th>( R )</th>
<th>( Y )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>55</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td>43</td>
<td>43</td>
</tr>
</tbody>
</table>

The estimate of surviving past (reversed) time 1 is \((31/40)(32/55) = .4509\).

Question # 20
Answer: E

The solution depends on identifying the parameter \( a \) with the overall average, which requires the convention stated in Pindyck and Rubinfeld under the table on page 140. The convention also applies to subgroups. With this convention, the solution is the following conditional expectation:

\[ E(Y \mid E = -1, F = -1, G = -1, H = -1) - a = -b_1 - b_2 - c_1 - c_2. \]
Question # 21  
Answer: E  
The posterior density, given an observation of 3 is:

\[
\pi(\theta | 3) = \frac{f(3 | \theta)\pi(\theta)}{\int_{\theta}^{\infty} f(3 | \theta)\pi(\theta) d\theta} = \frac{2\theta^2}{(3 + \theta)^3} \frac{1}{\int_{\theta}^{\infty} 2(3 + \theta)^{-3} d\theta} = \frac{2(3 + \theta)^{-3}}{-(3 + \theta)^{-2}} \bigg|_{\theta}^{\infty} = 32(3 + \theta)^{-3}, \ \theta > 1.
\]

Then,

\[
\Pr(\Theta > 2) = \int_{2}^{\infty} 32(3 + \theta)^{-3} d\theta = -16(3 + \theta)^{-2} \bigg|_{2}^{\infty} = \frac{16}{25} = .64.
\]

Question # 22  
Answer: B  
Because all previous values are 0, previous single and double smoothed values are also zero. Then,

\[
\begin{align*}
\tilde{y}_0 &= .6(1) + .4(0) = .6 \\
\tilde{y}_1 &= .6(1.2) + .4(.6) = .96 \\
\tilde{y}_2 &= .6(1.3) + .4(.96) = 1.164 \\
\tilde{y}_0' &= .6(.6) + .4(0) = .36 \\
\tilde{y}_1' &= .6(.96) + .4(.36) = .72 \\
\tilde{y}_2' &= .6(1.164) + .4(.72) = .9864.
\end{align*}
\]

Question # 23  
Answer: B  
\[
L = F(1000)^7 [F(2000) - F(1000)]^6 [1 - F(2000)]^7 \\
= (1 - e^{-1000/\theta})^7 (e^{-1000/\theta} - e^{-2000/\theta})^6 (e^{-2000/\theta})^7 \\
= (1 - p)^7 (p - p^2)^6 (p^2)^7 \\
= p^{20} (1 - p)^{13}
\]

where \( p = e^{-1000/\theta} \). The maximum occurs at \( p = 20/33 \) and so \( \hat{\theta} = -1000 / \ln(20/33) = 1996.90 \).
Question # 24
Answer: A

\[ E(X \mid \theta) = \theta / 2. \]

\[ E(X_3 \mid 400,600) = \int_{400}^{600} E(X \mid \theta) f(\theta \mid 400,600) d\theta = \int_{400}^{600} \frac{\theta}{2} \frac{600^3}{\theta^4} d\theta = \frac{3(600^3)}{2} \frac{\theta^{-2}}{600} \bigg|_{400}^{600} \]

\[ = \frac{3(600^3)(600^{-2})}{4} = 450. \]

Question # 25
Answer: D

The data may be organized as follows:

<table>
<thead>
<tr>
<th>( t )</th>
<th>( Y )</th>
<th>( d )</th>
<th>( \hat{S}(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>1</td>
<td>( (9/10) = .9 )</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>2</td>
<td>( .9(7/9) = .7 )</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>1</td>
<td>( .7(6/7) = .6 )</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>1</td>
<td>( .6(4/5) = .48 )</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>1</td>
<td>( .48(3/4) = .36 )</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>1</td>
<td>( .36(1/2) = .18 )</td>
</tr>
</tbody>
</table>

Because the product-limit estimate is constant between observations, the value of \( \hat{S}(8) \) is found from \( \hat{S}(7) = .36 \).

Question # 26
Answer: D

As of time 10 there were 7 observed payments, so \( O = 7 \). For the Weibull distribution, the cumulative hazard function is \( H(x) = -\ln S(x) = x^2 / 25 \). Then

\[ E(Z) = Var(Z) = \sum_{i=1}^{10} \frac{x_i^2}{25} = \frac{1}{25} (4 + 9 + 9 + 25 + 25 + 36 + 49 + 49 + 81 + 100) = 15.48. \]

The chi-squared test statistic (with one degree of freedom) is \( (7 - 15.48)^2 / 15.48 = 4.645 \). From the tables, this leads to rejection at the 5% level, but not at the 2.5% level.
Question # 27
Answer: D

\[ ESS_R = 15,000 - 5,565 = 9,435 \]

\[ R_{UR}^2 = .38 = 1 - \frac{ESS_{UR}}{TSS} \Rightarrow ESS_{UR} = .62(15,000) = 9,300 \]

\[ F = \frac{(9,435 - 9,300)/3}{9,300/3,114} = 15.07. \]

Question # 28
Answer: C

The maximum likelihood estimate for the Poisson distribution is the sample mean:

\[ \hat{\lambda} = \bar{x} = \frac{50(0) + 122(1) + 101(2) + 92(3)}{365} = 1.6438. \]

The table for the chi-square test is:

<table>
<thead>
<tr>
<th>Number of days</th>
<th>Probability</th>
<th>Expected*</th>
<th>Chi-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( e^{-1.6438} ) = .19324</td>
<td>70.53</td>
<td>5.98</td>
</tr>
<tr>
<td>1</td>
<td>( 1.6438e^{-1.6438} ) = .31765</td>
<td>115.94</td>
<td>0.32</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1.6438^2 e^{-1.6438}}{2} ) = .26108</td>
<td>95.30</td>
<td>0.34</td>
</tr>
<tr>
<td>3+</td>
<td>.22803**</td>
<td>83.23</td>
<td>0.92</td>
</tr>
</tbody>
</table>

*365x(Probability) **obtained by subtracting the other probabilities from 1

The sum of the last column is the test statistic of 7.56. Using 2 degrees of freedom (4 rows less 1 estimated parameter less 1) the model is rejected at the 2.5% significance level but not at the 1% significance level.
Question # 29
Answer: D

\[ \mu(0) = \frac{.4(0) + .1(1) + .1(2)}{.6} = .5; \quad \mu(1) = \frac{.1(0) + .2(1) + .1(2)}{.4} = 1 \]

\[ \mu = .6(.5) + .4(1) = .7 \]

\[ a = .6(5^2) + .4(1^2) - .7^2 = .06 \]

\[ v(0) = \frac{.4(0) + .1(1) + .1(4)}{.6} - .5^2 = \frac{7}{12}; \quad v(1) = \frac{.1(0) + .2(1) + .1(4)}{.4} - 1^2 = .5 \]

\[ v = .6(7/12) + .4(.5) = 11/20 \]

\[ k = v/a = 55/6; \quad Z = \frac{10}{10 + 55/6} = \frac{60}{115} \]

Bühlmann credibility premium = \[ \frac{60}{115} + \frac{10}{115} + \frac{55}{115}(.7) = .8565 . \]

Question # 30
Answer: A

All the statements about \( R^2 \) are true, but only (A) is not raised as an objection to \( R^2 \).

Question # 31
Answer: C

\[ \mu = .5(0) + .3(1) + .1(2) + .1(3) = .8 \]

\[ \sigma^2 = .5(0) + .3(1) + .1(4) + .1(9) - .64 = .96 \]

\[ E(S_n^2) = \frac{n-1}{n} \sigma^2 = \frac{3}{4}(.96) = .72 \]

\[ bias = .72 - .96 = -.24. \]
Question # 32
Answer: C

The four classes have means .1, .2, .5, and .9 respectively and variances .09, .16, .25, and .09 respectively.

Then,
\[ \mu = .25(.1 + .2 + .5 + .9) = .425 \]
\[ \nu = .25(.09 + .16 + .25 + .09) = .1475 \]
\[ a = .25(.01 + .04 + .25 + .81) = .425^2 = .096875 \]
\[ k = .1475 / .096875 = 1.52258 \]

\[ Z = \frac{4}{4 + 1.52258} = .7243 \]

The estimate is \([.7243(2/4) + .2757(425)] \cdot .5 = 2.40\).

Question # 33
Answer: D

The lower limit is determined as the smallest value such that
\[ \hat{S}(t) \leq .25 + 1.96\sqrt{\hat{V}[\hat{S}(t)]}. \]

At \( t = 50 \) the two sides are .360 and .25 + 1.96(.0470) = .342 and the inequality does not hold.

At \( t = 54 \) the two sides are .293 and .25 + 1.96(.0456) = .339 and the inequality does hold.

The lower limit is 54.

Question # 34
Answer: A

\[ Q = T \sum_{k=1}^{K} \hat{r}_k^2 = 100 \left( (-.01)^2 + .01^2 + \cdots + .10^2 \right) = 8.96 \]

Question # 35
Answer: A

The distribution used for simulation is given by the observed values.
Question # 36
Answer: B

First obtain the distribution of aggregate losses:

<table>
<thead>
<tr>
<th>Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/5</td>
</tr>
<tr>
<td>25</td>
<td>(3/5)(1/3) = 1/5</td>
</tr>
<tr>
<td>100</td>
<td>(1/5)(2/3)(2/3) = 4/45</td>
</tr>
<tr>
<td>150</td>
<td>(3/5)(2/3) = 2/5</td>
</tr>
<tr>
<td>250</td>
<td>(1/5)(2)(2/3)(1/3) = 4/45</td>
</tr>
<tr>
<td>400</td>
<td>(1/5)(1/3)(1/3) = 1/45</td>
</tr>
</tbody>
</table>

\[ \mu = (1/5)(0) + (1/5)(25) + (4/45)(100) + (2/5)(150) + (4/45)(250) + (1/45)(400) = 105 \]
\[ \sigma^2 = (1/5)(0^2) + (1/5)(25^2) + (4/45)(100^2) + (2/5)(150^2) \]
\[ + (4/45)(250^2) + (1/45)(400^2) - 105^2 = 8,100. \]

Question # 37
Answer: A

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 100</td>
<td>0.320</td>
</tr>
<tr>
<td>100 – 200</td>
<td>0.530</td>
</tr>
<tr>
<td>200 – 400</td>
<td>0.800</td>
</tr>
<tr>
<td>400 – 750</td>
<td>0.960</td>
</tr>
<tr>
<td>750 – 1000</td>
<td>0.980</td>
</tr>
<tr>
<td>1000 – 1500</td>
<td>1.000</td>
</tr>
</tbody>
</table>

At 400, \( F(x) = 0.8 = 1 - e^{\frac{-400}{\theta}} \); solving gives \( \theta = 248.53 \).

Question # 38
Answer: D

The sum of the squared values of the independent variable is \( 2266 + 20(100)^2 = 202,266 \). The value of \( s^2 \) is \( \frac{5348}{18} = 297.111 \). Then,

\[ s^2_a = s^2 \frac{\sum X_i^2}{N \sum x_i^2} = 297.111 \frac{202,266}{20(2266)} = 1326. \]

The 97.5\(^{th}\) percentile of a \( t \)-distribution with 18 degrees of freedom is 2.101, so the lower limit of the symmetric 95\% confidence interval for \( \alpha \) is

\[ 68.73 - 2.101 \sqrt{1326} = -7.78. \]
Question # 39
Answer: B

\[
\begin{align*}
\Pr(class1 \mid 1) &= \frac{(1/2)(1/3)}{(1/2)(1/3) + (1/3)(1/6) + (1/6)(0)} = \frac{3}{4} \\
\Pr(class2 \mid 1) &= \frac{(1/3)(1/6)}{(1/2)(1/3) + (1/3)(1/6) + (1/6)(0)} = \frac{1}{4} \\
\Pr(class3 \mid 1) &= \frac{(1/6)(0)}{(1/2)(1/3) + (1/3)(1/6) + (1/6)(0)} = 0 \\
\end{align*}
\]

because the prior probabilities for the three classes are 1/2, 1/3, and 1/6 respectively.

The class means are

\[
\begin{align*}
\mu(1) &= (1/3)(0) + (1/3)(1) + (1/3)(2) = 1 \\
\mu(2) &= (1/6)(1) + (2/3)(2) + (1/6)(3) = 2.
\end{align*}
\]

The expectation is

\[
E(X_2 \mid 1) = (3/4)(1) + (1/4)(2) = 1.25.
\]

Question # 40
Answer: E

The first, second, third, and sixth payments were observed at their actual value and each contributes \(f(x)\) to the likelihood function. The fourth and fifth payments were paid at the policy limit and each contributes \(1 - F(x)\) to the likelihood function. This is answer (E).