

COURSE 6
MORNING SESSION

SECTION A – WRITTEN ANSWER

****BEGINNING OF COURSE 6****
MORNING SESSION

- 1.** (5 points) You are given the following information:

Stock	Price at Beginning of Period	Price at End of Period	Number of Shares Outstanding
I	10	15	100
II	9	10	125
III	15	15	200

- (a) Describe and compare:
- (i) Price-weighted index
 - (ii) Market-value-weighted index
 - (iii) Equally-weighted index
- (b) Calculate the percentage change over the period for:
- (i) Price-weighted index
 - (ii) Market-value-weighted index
 - (iii) Equally-weighted index
- (c) Calculate the end of period price for Stock III that results in an equivalent percentage increase in the price-weighted index and the market-value-weighted index.

Show all work.

2. (6 points) An investor has the following securities available for investment:

	Expected Annual Return	Standard Deviation of Annual Return
Stock I	9%	13%
Stock II	12%	20%
T-bills	5%	0%

The covariance between the two risky assets is 0.93%.

Construct the optimal portfolio using the three available assets, assuming:

- (i) an investor's degree of risk aversion is 4
- (ii) an investor's degree of risk aversion is 2

Show all work.

3. (10 points) You are given the following information:

- all options have nine months to expiry
- all options have a strike price of 49
- current stock price is 50
- volatility is 30%
- risk-free rate is 5% per annum

Using the binomial option pricing model and a three-month step, calculate the cost of

- (i) a European put;
- (ii) an American put;
- (iii) a European call.

Show all work.

4. (5 points)

- (a) Describe the obligations of the trustee of a pension fund.
- (b) Describe the key considerations in selecting an appropriate pension funding method.

5. (6 points) You are given the following financial data for Company ABC:

- capital and surplus: 800,000
- total company assets:

Bond	Book Value	C1 Factor
AAA	2,000,000	0.3%
BB	5,000,000	4%
B	3,000,000	12%

- other required capital components:

Risk	Base	Required Capital Factor
C2	150,000,000	0.1%
C3	4,000,000	1%
C4	500,000	2%

- required capital components for whole life block:

Risk	Base	Required Capital Factor
C1	8,000,000	6%
C2	100,000,000	0.1%
C3	3,000,000	1%
C4	400,000	2%

- ceding allowance for a reinsurance treaty is 3% of assets transferred
 - C1 required capital factor for assets ceded to reinsurer is 0.5%
 - required capital formula: $C4 + \sqrt{C2^2 + (C1 + C3)^2}$
 - Risk Based Capital ratio = Available capital ÷ Required capital
- (a) Describe the weaknesses of using the required capital formula when comparing two companies that have healthy Risk Based Capital ratios.
- (b) Evaluate the impact on required capital if you entered into a 50% coinsurance agreement for the whole life business.
- (c) Evaluate the impact on required capital of upgrading all assets to a minimum rating of BB.
- (d) Recommend which capital management action the company should use.

Show all work.

6. (5 points) You are given the following information about three stocks in a multiple stock universe using the single-index model:

	Beta	Mean Excess Return	Standard Deviation
Stock I	1.25	12%	40%
Stock II	0.75	5%	25%
Stock III	1.75	12%	40%
Market	1.00	8%	20%

The risk-free rate is 6% per year.

- Explain why the single-index model is an effective tool for portfolio optimization.
- Determine the value of the items in the input list required for the development of a Markowitz efficient frontier.
- Calculate the expected return and standard deviation for a portfolio consisting of equal proportions of Stock I, Stock II, and Stock III.

Show all work.

7. (8 points) You are given the following information for company XYZ:

- it only sells deferred annuities with rates guaranteed to age 65
- its target market is young professionals under the age of 30
- policy surrenders are paid at the greater of book value and market value
- products are credited with new money interest rates, which are currently at historical lows
- currently the liabilities are supported by fixed income securities

XYZ is considering investing up to 50% of assets supporting the liabilities in equity investments.

XYZ has implemented an annual process to monitor duration mismatch between assets and liabilities.

Analyze XYZ's interest rate risk management practices and, if appropriate, recommend changes to current practices to help minimize this risk.

**COURSE 6
MORNING SESSION**

SECTION B – MULTIPLE CHOICE

1. You are given the following characteristics of a zero-coupon bond:

- term to maturity: 5 years
- yield: 8% on a semi-annual basis

Calculate the original-issue discount.

- (A) 31.9%
- (B) 32.4%
- (C) 46.3%
- (D) 53.7%
- (E) 67.6%

2. You are given the following information about a trading day on the New York Stock Exchange (NYSE):

- advances: 2,000
- declines: 1,500
- advancing volume: 300,000
- declining volume: 600,000

Calculate the trin statistic and indicate if the NYSE is considered bearish or bullish for that given day.

- (A) 0.38, bearish
- (B) 0.38, bullish
- (C) 2.00, bullish
- (D) 2.67, bearish
- (E) 2.67, bullish

3. You are given the following information as of January 1, 2002:

- stock price: 67
- call option price: 2
- call option strike price: 72
- call option expiry: January 1, 2003
- rate of return on a one-year T-bill: 4% annual effective

Stock Price on January 1, 2003	Probability
76	0.60
64	0.40

Calculate the risk premium for the call option.

- (A) 0.05
- (B) 0.16
- (C) 0.32
- (D) 0.40
- (E) 0.46

4. You are given the following information for a portfolio consisting of three stocks:

	Weight (W_i)	Beta (b_i)	Standard Deviation (s_i)
Stock I	20%	1.2	11%
Stock II	40%	0.9	14%
Stock III	40%	1.0	20%

The standard deviation of the deviation of the common factor from its expected value (s_F) is 15%.

Using a single factor arbitrage pricing model, calculate the nonsystematic risk standard deviation for this portfolio.

- (A) 10.0%
- (B) 15.0%
- (C) 15.8%
- (D) 16.2%
- (E) 18.0%

5. You are given the following information for a 15-year callable bond:

- annual coupon rate: 9% payable semi-annually
- price: 95.32
- effective duration: 3.17
- convexity measure (C): (67.31)

$$C = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2}$$

Calculate the price of the bond after a 50 basis point increase in interest rates.

- (A) 93.65
- (B) 93.97
- (C) 95.32
- (D) 96.67
- (E) 96.99

6. You are given the following information with respect to a single-period securities model:

$$S(0) = [10 \quad P \quad 10]$$
$$S(1) = \begin{bmatrix} 11 & 33 & 0 \\ 11 & 0 & 0 \\ 11 & 0 & 22 \end{bmatrix}$$

Determine the value of P which makes the model arbitrage-free.

- (A) 14
- (B) 15
- (C) 16
- (D) 17
- (E) 18

7-16. Each of questions 7 through 16 consists of two lists. In the list at the left are two items, lettered X and Y. In the list at the right are three items, numbered I, II, and III. ONE of the lettered items is related in some way to EXACTLY TWO of the numbered items. Indicate the related items using the following answer code:

	<u>Lettered Item</u>	<u>Is Related to Numbered Items</u>
(A)	X	I and II only
(B)	X	II and III only
(C)	Y	I and II only
(D)	Y	I and III only
(E)	The correct answer is not given by (A), (B), (C) or (D).	

- | | | |
|-----------|--------------------------|---|
| 7. | X. Asian call options | I. Payoffs depend on the average price of the underlying asset during the life of the option. |
| | Y. Lookback call options | II. Guarantees the purchase of the asset at the lowest price during the life of the option. |
| | | III. Can use averages for the exercise price. |
| 8. | X. Cliquet option | I. Guaranteed exchange-rate contracts. |
| | Y. Quanto option | II. A series of standard call options that pays the annual increase in the underlying assets. |
| | | III. The strike resets at the beginning of each year. |

- 9.** X. Interest rate corridor I. The purchase of a cap at one strike rate and the sale of a floor at a lower strike rate.
- Y. Interest rate collar II. The purchase of a cap at one strike rate and the sale of another cap at a higher strike rate.
- III. Sometimes described as swapping into a bond.
-
- 10.** X. Zero-coupon convertible bond I. Sacrifice yield
- Y. Putable convertible bond II. Greater credit risk
- III. Lower premium
-
- 11.** X. Modified duration I. Allows for changing cash flows as interest rates change.
- Y. Effective duration II. Does not allow for changing cash flows as interest rates change.
- III. Not an appropriate measure for callable bonds.

- 14.** X. Parallel shift interest rate generators
- I. Based on lognormal or Markov chain processes.
- Y. Non-parallel shift interest rate generators
- II. Used for New York 7 scenarios.
- III. Works well with classical immunization theory.
-
- 15.** X. Contingent immunization
- I. Reduces the risk associated with nonparallel shifts of a sloped yield curve.
- Y. Combination matching
- II. Liquidity needs are provided for in the initial cash flow matched period.
- III. A blend of active management with immunization.
-
- 16.** X. Risk Based Capital planning
- I. Reduce outstanding short-term debt.
- Y. Liquidity planning
- II. Portfolio diversification.
- III. Public perception management.

17-25. These questions consist of an assertion in the left-hand column and a reason in the right-hand column. Code your answer to each question by blackening space:

- (A) If both the assertion and the reason are true statements, and the reason is a correct explanation of the assertion.
- (B) If both the assertion and the reason are true statements, but the reason is NOT a correct explanation of the assertion.
- (C) If the assertion is a true statement, but the reason is a false statement.
- (D) If the assertion is a false statement, but the reason is a true statement.
- (E) If both the assertion and the reason are false statements.

17. ASSERTION

Commercial mortgage-backed securities have higher prepayment risks than residential mortgage-backed securities.

BECAUSE

REASON

Commercial mortgage-backed securities are often balloon loans.

18. ASSERTION

A portfolio consisting of negatively correlated assets offers better risk-return opportunities than individual component securities on their own.

BECAUSE

REASON

The standard deviation of a portfolio is less than the weighted average of the standard deviations of its component securities.

17-25. These questions consist of an assertion in the left-hand column and a reason in the right-hand column. Code your answer to each question by blackening space:

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- (C) If the assertion is a true statement, but the reason is a false statement.
- (D) If the assertion is a false statement, but the reason is a true statement.
- (E) If both the assertion and the reason are false statements.

23. ASSERTION

Option exposures can be managed by matching expected asset and liability cash flows.

BECAUSE

REASON

The asset and liability cash flow matching technique determines a dollar value for the interest rate exposures that exist.

24. ASSERTION

Option-based portfolio insurance strategies are time variant.

BECAUSE

REASON

With respect to option-based portfolio insurance strategies, the optimal mix of risky and riskless positions depends on the time left before the horizon is reached.

25. ASSERTION

The Multiple Asset Performance strategy is equivalent to purchasing an option that allows the buyer to choose the asset to call or buy at a guaranteed price.

BECAUSE

REASON

A Multiple Asset Performance option valuation-based approach does not require asset return forecasts.

26. You are given the following information with respect to a callable bond:

- par amount: 1,000
- term to maturity: 3 years
- annual coupon rate: 6% payable annually
- value of embedded call option: 20

Term	Annual Spot Interest Rates
1	7%
2	8%
3	9%

Calculate the value of the bond.

- (A) 906
- (B) 926
- (C) 930
- (D) 950
- (E) 1,000

27. You are given the following information with respect to a non-callable bond:

- par amount: 1,000
- term to maturity: 4 years
- annual coupon rate: 8% payable annually

Time	1-Year Annual Forward Interest Rates	
	Scenario X	Scenario Y
0	7%	7%
1	7%	6%
2	8%	7%
3	10%	5%

Each interest rate scenario has an equal probability of occurring.

Calculate the value of the bond.

- (A) 1,000.00
- (B) 1,018.40
- (C) 1,022.80
- (D) 1,030.39
- (E) 1,031.07

28. You are given the following information with respect to a multiplicative binomial branching model:

- the short rate one year from now will be either:

$$r_{t+1}^u = r_t \times (1 + \textit{gamma}), \text{ or}$$

$$r_{t+1}^d = r_t \div (1 + \textit{gamma}), \text{ with equal probability}$$

- volatility: 20%
- current short term interest rate: 6%
- notional amount of a 2-year interest rate collar: 100

Calculate the value of a 2-year interest rate collar with strike levels of 5% and 8%.

- (A) 0.141
- (B) 0.187
- (C) 0.328
- (D) 0.348
- (E) 0.368

29. You are given the following information:

- expected market return: 12%
- standard deviation of market return: 10%
- risk-free rate: 4%
- utility function of the investor: $U_M = E(R_M) - \frac{\sigma(R_M)^2}{5}$

Calculate the optimal percentage the investor would invest in the market.

- (A) 10%
- (B) 20%
- (C) 30%
- (D) 40%
- (E) 50%

30. You are given the following information:

- immunization target: 9.0%
- minimum return acceptable to the fund sponsor: 6.0%
- worst case return for an actively managed portfolio: 2.5%

Calculate the minimum proportion of the initial portfolio (X) that should be actively managed.

- (A) $X < 40\%$
- (B) $40\% \leq X < 45\%$
- (C) $45\% \leq X < 50\%$
- (D) $50\% \leq X < 55\%$
- (E) $55\% \leq X < 60\%$

****END OF COURSE 6**
MORNING SESSION**

COURSE 6
AFTERNOON SESSION

****BEGINNING OF COURSE 6****
AFTERNOON SESSION
Beginning with question 8

8. (4 points) Describe the risks associated with investing in fixed income securities.

9. (5 points) You are given the following information:

	Duration	Projected Cash Flows				
		Year 1	Year 2	Year 3	Year 4	Year 5
Liabilities	4.2	210	69	445	180	1980
Assets	4.3	194	254	41	200	2200

Universe of available assets for investment:

- 90-day T-bills
- 2-year bonds with annual coupons of 5%
- 3-year bonds with annual coupons of 6%
- 5-year bonds with annual coupons of 10%

Determine the necessary asset transactions to cash flow match the projected liability cash flows.

Show all work.

10. (8 points) The Chief Financial Officer (CFO) of a large corporation is considering offering an innovative “collared floater” with the following features:

- value at issue: par
- par amount: 10 million
- term to maturity: 5 years
- coupon: semi-annual payment and reset, 6-month LIBOR + 0.50%
- minimum coupon: 7.5%
- maximum coupon: 12.5%

The CFO intends to use derivative instruments to convert this collared floater into synthetic fixed-rate funding. The following quotes for five-year, semi-annual settlement interest rate swaps, caps and floors on 6-month LIBOR are obtained from a market maker in derivative products:

	Bid	Ask
Swaps for LIBOR	8.65%	8.75%
Interest Rate Cap at 12.0%	0.65%	0.75%
Interest Rate Cap at 12.5%	0.50%	0.60%
Interest Rate Cap at 13.0%	0.35%	0.45%
Interest Rate Floor at 7.0%	0.80%	0.90%
Interest Rate Floor at 7.5%	0.95%	1.05%
Interest Rate Floor at 8.0%	1.10%	1.20%

- (a) Determine the specific combination of transactions which result in a synthetic fixed rate of funding.
- (b) Explain why this combination works.
- (c) Calculate the effective (all-in) interest cost for this synthetic fixed-rate funding.
- (d) Identify the situations when credit risk is a concern to the corporation in this transaction.

Show all work.

11. *(6 points)*

- (a) Define a floating-rate security and describe its features.
- (b) Describe the yield spread measures used to evaluate floating-rate securities.
- (c) Describe the factors affecting the price of floating-rate securities.

12. (8 points) ABC Financial is considering two opportunities for capital investment for the upcoming fiscal year:

- a deposit-taking business earning a guaranteed return of 10% per year
- a life insurance business earning one of two possible returns: 20%, or -20% per year.

ABC is also considering a one-year reinsurance agreement that would eliminate any loss on its life insurance business. The single premium would be paid at the beginning of the fiscal year.

- (a) Compare the features of the reinsurance contract and an option contract.
- (b) Calculate the reinsurance premium for the year by applying risk-neutral valuation to solve for the replicating trading strategy for the reinsurance contract.
- (c) Assume that the life insurance business now has a third possible outcome of catastrophic loss, where all of the capital investment in the life insurance business is lost.

You are given the following:

- The unit Arrow-Debreu (e_1) price is its upper bound price less $\frac{1}{12}$
- $S(1)^{-1} = \begin{bmatrix} 0 & 0 & \frac{10}{11} \\ 0 & \frac{5}{4} & -\frac{5}{4} \\ 1 & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$

Calculate the new reinsurance premium using risk-neutral valuation.

- (d) Explain how your reinsurance premium calculations would be affected if the deposit taking business earned a guaranteed return of 20% per year.

Show all work.

13. (4 points) You are an investment actuary managing the pension assets of a small Canadian company.

- (a) Describe the key considerations in setting the Statement of Investment Policies.
- (b) Describe possible investment vehicles.

14. (5 points) You are given the following securities:

60-day T-bill: face amount: 1,000

150-day T-bill: face amount: 1,000

Stock of ABC Corporation:

current price: 25
 dividend rate: 6%, payable continuously
 The amount of dividend payment is constant, regardless
 of changes in stock price.

European call option on the ABC stock:

current price: 1
 strike price: 30
 time to exercise date: 60 days
 d_1 0.7

European put option on the ABC stock:

current price: 6
 strike price: 30
 time to exercise date: 60 days

Futures contract:

underlying security: 90-day T-bill
 time to delivery date: 60 days
 face amount: 1,000
 current price: 984

Cumulative normal distribution:

Z	- 1.4	- 0.7	0	0.7	1.4
$N(Z)$	0.0808	0.242	0.5	0.758	0.9192

Calculate the current market price of the 150-day T-bill.

Show all work.

****END OF COURSE 6****

Course 6
May 2002

Multiple-Choice Answer Key

1	B
2	D
3	C
4	A
5	A
6	A
7	E
8	B
9	D
10	B
11	B
12	B
13	A
14	B
15	C
16	D
17	D
18	A
19	D
20	A
21	B
22	B
23	E
24	A
25	B

26	A
27	E
28	C
29	B
30	C
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May 2002 Course 6 Solutions

Question #1

- a) price-weighted index: can be replicated by buying one of every stock in index
- equal to the sum of the prices of all stocks in the index, divided by the number of stocks.
 - the divisor is adjusted for stock splits and stock dividends >10% so that the index is unaffected by the event.
 - gives more weight to more expensive stocks

market-value weighted index: can be replicated by buying amounts of each stock in proportion to the outstanding market value of each stock.

- stocks with large market value get more weight
- no adjustments need to be made for splits/dividends

equally-weighted index: can be replicated by buying equal dollar amounts of each stock.

- need to constantly re-balance portfolio as stock prices change to maintain equal weight
- can just take average of individual stock returns.

b)

	t_0	t_1	% change
Price-weighted index	$(10+9+15)/3=11.3333$	$(15+10+15)/3 = 13.3333$	$\left(\frac{13.3333}{11.3333} - 1\right) \times 100\% = +17.647\%$
Market value weighted	$(10 \times 100 + 9 \times 125 + 15 \times 200) = 5125$	$(15 \times 100 + 10 \times 125 + 15 \times 200) = 5750$	$\left(\frac{5750}{5125} - 1\right) \times 100\% = +12.195\%$
Equally-weighted			$\left[\left(\frac{15}{10} + \frac{10}{9} + \frac{15}{15}\right) \frac{1}{3} - 1\right] \times 100\% = +20.370\%$

c) $PWI = MVWI$

$$\frac{(15+10+X)}{3(11.3333)} = \frac{(15 \times 100 + 10 \times 125 + X \times 200)}{5125}$$

$$5125(25+X) = 3(11.3333)(2750+200X)$$

$$128,125 + 5125X = 93500 + 6800X$$

$$34,625 = 1675X$$

$$X = 20.67$$

∴ if the end of period price for stock III was \$20.67, the percentage increase for the price-weighted index and the market value-weighted index would be equal.

$$E[r_c] = .05 + .7464(1.03824 - .05) = 9.017\%$$

$$s_c = .7464(.13427) = 10.02\%$$

ii) for investor with $A = 2$

$$y^* = \frac{10.3824 - 5}{.01(2)(13.427)^2} = 149.28\%$$

Assuming that this investor can borrow at the risk free rate, the optimal portfolio is 149.28% in P and borrowing 49.28% @ $r_f = .05$

$$E[r_c] = .05 + 1.4928(1.03824 - .05) = 13.035\%$$

$$s_c = 1.4928(.13427)$$

$$= .2004$$

invest	1.4928(.5392) = 80.49%	in stock I
	1.4928(.4608) = 68.79%	in stock II
borrow	49.28 @ $r_f = 5\%$	

If investor cannot borrow at risk-free rate, then he will invest such that his utility function is maximized with the following weights.

$$W_I = \frac{E[r_I] - E[r_{II}] + .01A(s_{II}^2 - Cov(r_I, r_{II}))}{.01A(s_I^2 + s_{II}^2 - 2Cov(r_I, r_{II}))}$$

$$W_{II} = 1 - W_I$$

$$= \frac{9 - 12 + .01(2)(20^2 - .93)}{.01(2)(20^2 + 13^2 - .93(2))}$$

$$= \frac{4.9814}{11.3428} = .4392$$

$$W_{II} = 56.08\%$$

Invest 43.92% in stock I
56.08% in stock II

$$E(r_p) = 10.6824 \qquad U = 10.274$$

$$s_p^2 = .0204 \Rightarrow s_p = 14.29\%$$

Question #3

(i) For the continuous binomial option pricing model

$$u = e^{s\sqrt{h}} \text{ where } h = \text{time in years} \\ = 3/12 = 1/4$$

$$u = e^{0.3\sqrt{0.25}} = 1.16183$$

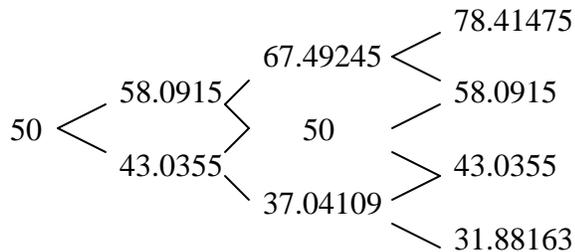
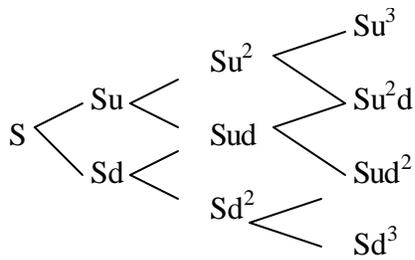
$$d = u^{-1} = 0.86071$$

p = probability of an up-move

$$= \frac{e^{rh} - d}{u - d} \\ = \frac{e^{0.05(0.25)} - 0.86071}{1.16183 - 0.86071} \\ = 0.50435$$

$$q = 1 - p = 1 - 0.50435 = 0.49565$$

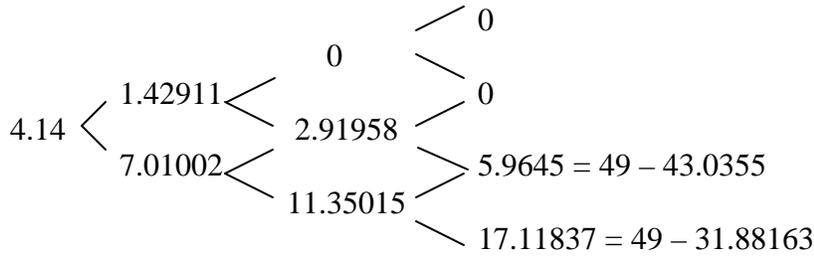
Calculate the stock price tree



European put payoffs at time 3 and values- only exercised at maturity

$$\text{Payoff} = \text{Max} (\text{Strike} - \text{stock}, 0) \\ = \text{Max} (49 - \text{stock}, 0)$$

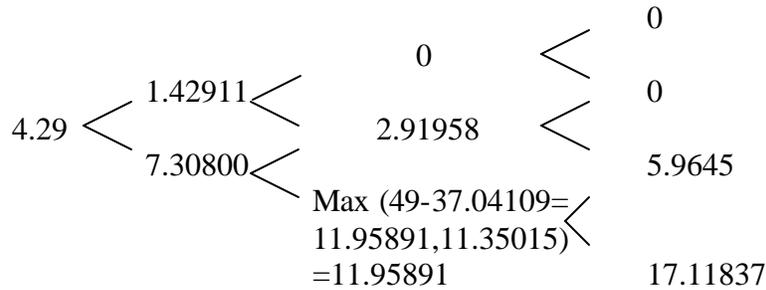
European put



Value of any node earlier than time 3

$$V = [V_H P + V_L (1 - p)]e^{-rh} \quad \text{Value of the European put} = 4.14$$

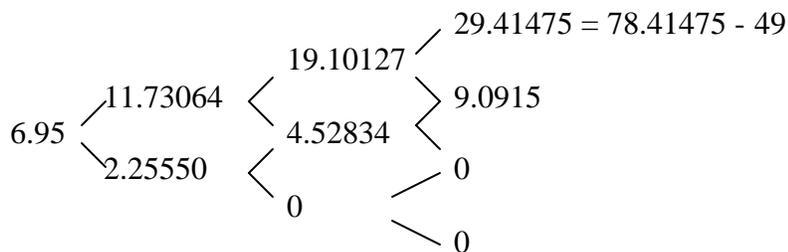
- (ii) American put- can be optimal to exercise at any time if intrinsic value is greater than value, then exercise (substitute intrinsic value for value)



Value of the American put = 4.29

- (iii) European call

$$\begin{aligned} \text{Payoff} &= \text{Max} (\text{stock} - \text{strike}, 0) && \text{only exercised at maturity} \\ &= \text{Max} (\text{stock} - 49, 0) \end{aligned}$$



Value of the European call = 6.95

Value of any node earlier than time 3

$$V = [V_H P + V_L (1 - p)]e^{-rh}$$

Question #4

Loyalty

- The trustee should act to the best interest of the participants of the fund, avoid any contradicting interest.
- Should not manage fund to the interest of the plan sponsors.

Care and Diversify

- The trustee should manage the fund with care, taking into account economic outlook and participants interest.
- Should diversify the fund accordingly to reduce the risk of the pensioners unless prove diversification is not prudent.

Impartiality

- The trustee should not perform anything to the interest of a particular group in the pension fund.
- Should treat every participant the same and do anything to the interest to all participants equally.

Delegate

- Should delegate fund management or investment according to any suitable parties, to help manage the fund.
- But should bear in mind the responsibility and liability even delegate to the other parties.

Follow Statutory Constraints

- Should follow all regulation constraints.
- Any accounting issues, valuation method
- To comply to any tax rules and investment guidelines

Make the Property Productive

- Should manage the fund such that the investments are productive, or should maintain comfortable surplus for the pensioners.
- Should set-up objective and try to carry it out.

Re Co-Trustee

- Make sure all co-trustees have the same objective and same management mechanisms.
- All co-trustees follow the guidelines and manage or invest the fund in the same manner.

Act According to the Trust Agreement

- Should make sure is acting according to the terms written in the trust agreement.
- Should not carry out anything that is in contradiction to the trust agreement.
- Should manage trust in a prudent manner otherwise not stated in the trust agreement.

Purpose of funding

- finding purpose affect which method to use
- comply with purpose will produce better results

Plan sponsor's financial objective

- financial objectives should be taken into account, stable contribution or flexible contribution?
- Stabilize business profit or retain comfortable surplus?

Plan sponsor's business or industry

- each industry has different needs, so need to consider that and use the appropriate method

Type of benefits

- is the benefit (with employer contribution) DC or DB will affect the funding method used.
- % of salary or level for all?

Method of accrual of benefit

- Does past services take into account or is any benefits related
- All these affect funding method used

Regulatory constraints

- Should comply with any statutory rules
- Accounting or reporting or tax rules should be followed.

Funding method that require lowest cost may be more flexible to plan sponsors

Should avoid method that generates negative NC or negative UAL

Should avoid fluctuations to funded liability ratio.

May want to tap into lump sum benefit since those are usually discounted in a higher rate.

Question #5

- a.)
- Solvency test only
 - Based on formula that ignores many subtle but important issues
 - Cannot and does not reflect all strengths and weaknesses of company, such as:
 - Strong underwriting
 - Good contracts
 - Customer loyalty
 - Does not differentiate within a given class of investment vehicles.
 - Companies can manipulate the RBC formula

b.)
$$C1 = 0.003(2,000,000) + 0.04(5,000,000) + 0.12(3,000,000)$$
$$= 566,000$$

$$C2 = 150,000,000 \times 0.001 = 150,000$$

$$C3 = 4,000,000 \times 0.01 = 40,000$$

$$C4 = 500,000 \times 0.02 = 10,000$$

$$RBC = 10,000 + \sqrt{150,000^2 + (566,000 + 40,000)^2}$$
$$= 634,288$$

Before reinsurance, RBC ratio = Available Capital / RBC

$$= 800,000 / 634,288$$

$$= 126\%$$

Changes to RBC after reinsurance:

$$\Delta C1 = -8,000,000(6.0\% - 0.5\%)(50\%) = -220,000$$

$$C1 = 566,000 - 220,000 = 346,000$$

$$\Delta (C2) = -100,000,000(0.1\%)(50\%) = -50,000$$

$$C2 = 150,000 - 50,000 = 100,000$$

$$\Delta (C3) = -3,000,000(1.0\%)(50\%) = -15,000$$

$$C3 = 40,000 - 15,000 = 25,000$$

$$\Delta (C4) = 0$$

$$C4 = 10,000 - 0 = 10,000$$

$$\begin{aligned} \text{RBC} &= 10,000 + \text{SQRT}(100,000^2 + (346,000 + 25,000)^2) \\ &= 394,291 \end{aligned}$$

$$\text{Delta(Available capital)} = 4,000,000(50\%)(3\%) = 120,000$$

$$\text{Available capital} = 800,000 + 120,000 = 920,000$$

$$\begin{aligned} \text{After reinsurance, RBC Ratio} &= 920,000 / 394,241 \\ &= \mathbf{233\%} \end{aligned}$$

- c.) Upgrade all assets to minimum BB

$$\begin{aligned} \text{C1} &= 2,000,000(0.3\%) + 8,000,000(4.0\%) \\ &= 326,000 \end{aligned}$$

$$\begin{aligned} \text{RBC} &= 10,000 + \text{SQRT}(150,000^2 + (326,000 + 40,000)^2) \\ &= 405,545 \end{aligned}$$

$$\text{RBC Ratio} = \text{Available Capital} / \text{RBC}$$

$$= 800,000 / 405,545$$

$$= 197\%$$

- d.) I recommend the reinsurance option because it raises the RBC ratio more than upgrading assets does.

Question #6

- Markowitz model requires $(n^2-n)/2$ estimates of covariance and $2n$ mean /variances assumptions.
- Allows one analyst to study one sector and another to study another each relative to index.

- a.) single-index model:

$$R_i = \beta_i + \mathbf{b}_m + R_m + e_i$$

R_i refers to excess return of security i

R_m refers to excess return of market

$$\mathbf{b}_i \text{ will be the same with } \beta \text{ of CAPM } \left(\mathbf{b} = \frac{\text{cov}(r_i - r_m)}{\mathbf{s}_m^2} \right)$$

e_i will be the firm-specific factor

Advantages:

1. Using single index model only requires estimation of different e_i and $\text{cov}(r_i, r_M)$ for each security i , then estimate R_m , s_m^2 for the market portfolio. It will reduce calculation greater than CAPM.
2. Single-index model assumes specific security i 's excess return only related to excess market return and firm-specific factor e_i .

b). The values of Markowitz efficient frontier.

	$Z(r)$	s_r
Stock I	$6\% + 12\% = 18\%$	40%
Stock II	$6\% + 5\% = 11\%$	25%
Stock III	$6\% + 12\% = 18\%$	4%
Market	$6\% + 8\% = 14\%$	25%

$$\text{cov}(r_I, r_{II}) = \mathbf{b}_1 \mathbf{b}_2 \cdot \mathbf{s}_m^2 = 1.25 \times 0.75 \times 20^2 = 375$$

$$\text{cov}(r_I, r_{III}) = \mathbf{b}_1 \mathbf{b}_3 \cdot \mathbf{s}_m^2 = 1.25 \times 1.75 \times 20^2 = 875$$

$$\text{cov}(r_{II}, r_{III}) = \mathbf{b}_2 \mathbf{b}_3 \cdot \mathbf{s}_m^2 = 0.75 \times 1.75 \times 20^2 = 525$$

\therefore covariance – variance matrix is:

$$\begin{vmatrix} 1,600 & 375 & 875 \\ 375 & 625 & 525 \\ 875 & 525 & 1,600 \end{vmatrix}$$

c).

$$Z_p = \frac{1}{3} \times 18\% + \frac{1}{3} \times 11\% + \frac{1}{3} \times 18\% = 15.67\%$$

$$\mathbf{s}_p^2 = \left(\frac{1}{3}\right)^2 \times 1,600 + \left(\frac{1}{3}\right)^2 \times 625 + \left(\frac{1}{3}\right)^2 \times 1,600 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 375 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 525 + 2 \times \frac{1}{3} \times \frac{1}{3} \times 875$$

$$= 819$$

$$\Rightarrow \mathbf{s}_p = 28.62\%$$

Question #7

According to the CIA guidance note the risk management practice issues are:

1. Rebalancing Practices
2. Liquidity Management Practices
3. Asset Options
4. Embedded liability options
5. Use of derivatives
6. Equity Investments
7. Long Duration Liabilities
8. Tax/Legislative Implications

I will now elaborate on each of these.

Rebalancing Practices

Must rebalance due to time, drift, and non-parallel interest rate shifts

Low risk factors for rebalancing

- Position reported frequently and process to rebalance in place
- Computer optimizing software used to rebalance
- Assets are liquid or can use derivatives
- History of quickly eliminating unwanted exposures

Liquidity Management Practices Low Risk Factors

1. MIS in place to understand different cash outflow scenarios of inforce
2. Don't rely on writing new business for liquidity
3. Sufficient liquid assets to meet adverse cash outflow demands
4. Disciplined investing in illiquid assets
5. Derivative expertise if use derivatives

Asset Options Low Risk Factors

1. No options can be exercised against the company
2. Potential options are accurately modeled

Embedded Liability Options Low Risk Factors

1. No discretionary withdrawals are allowed
2. Discretionary withdrawals are market value adjusted if they are permitted
3. Embedded liability options are accurately modeled
4. Policyholders can't annuitize at book value when they want to
5. Policyholders can't selectively use rate quotes
6. Ensure that surrender charges are reasonably high to prevent early surrenders and lessen exposure to company.

Use of Derivatives Low Risk Factors

Derivatives are good if you don't have liquid assets but they are risky because they are complex and highly leveraged.

1. Limits on purchasing derivatives without senior management approval
2. Significant expertise in company
3. Derivatives are accurately modeled in ALM process

Equity Investments

Risky because

- a. assets don't have guarantees; liabilities do
- b. expected return and volatility are difficult to measure
- c. they are often uncorrelated with interest rate volatility

Low risk factors for equity investments

- a. no equities are used
- b. if equities are used, they are segmented and used to back very long duration liabilities
- c. use very little junk bonds

Long Duration Liabilities

Often it is hard or impossible to find assets that have maturities greater than 30 years

Options to manage risk

- a. discount liability cash flows past 30 years to duration 30 and manage as a time 30 cash flow
- b. manage in total rate of return segment
- c. use derivatives to leverage asset sensitivity to match liability sensitivity

Low risk factors for long duration liabilities

- a. No liabilities issued that have maturities greater than 30 years
- b. Procedures in place to handle unique risk

Tax/Legislative Liabilities

Do not plan to manage interest rate using techniques that are bad for taxes or not permitted by regulators.

Analysis and Recommendations

Target market is professionals under 30, which means they will have long duration liabilities (ie, 73 yrs). I recommend that they reconsider this strategy because it will be tough to find assets to match the liabilities.

Policy surrenders should be not permitted or market value adjusted. If they pay the Max [book value, market value], they may put themselves at more risk.

The fact that liabilities are supported by fixed income securities is good because they are less risky. If 50% is moved to equities considerable interest rate risk may arise. I would not recommend such a big shift to equities.

Right now, they are measuring interest rate risk exposure annually. Interest rate risk exposure should be monitored on a day to day basis so that actions can be taken to correct bad exposures quickly. If the position is only reported once a year, you do not have a good handle on your exposure to interest rate risk. As such, I recommend they implement a process that measures exposure on a day to day basis.

Recommend liabilities being backed by assets should only represent statutory margins/required surplus.

Limitations of duration mismatch:

- do not measure exposure to non-parallel yield curve shift
- do not always capture the effects of options

A wide range of scenarios covering changes in level, terms structure, sector spread and default rates should be chosen when doing scenario testing.

Question #8

I. Market or Interest Rate risk

- biggest risk faced by an investor
- magnitude of price response to changes to interest rates
- magnitude depends on: maturity, coupons, embedded options

Duration is a summary measure of price sensitivity to yields

II. Reinvestment Risk

- risk that interest rates will decline and reinvestment of coupons will be at lower than current rates.
- risk larger for longer holding periods
- opposite risk to market risk with respect to yields
- when reinvestment and market risk offset each other over a time horizon it is referred to as immunization.

III. Call or Timing Risk

- magnitude depends upon the parameters of the call and current market conditions

IV. Credit or Default Risk

- risk that promised payments are not made.
- rating agencies include: Moody's and Standard & Poor's
- institutional lenders also do credit ratings
- investors will be concerned with any changes in credit ratings because this will impact yield spreads over treasuries.

V. Yield Curve or Maturity Risk

- risk that the shape of the yield curve changes in a manner different than anticipated
- hedges will no longer hold

VI. Inflation or Purchasing Power Risk

- bonds with fixed cash flows will be at risk due to unanticipated inflation
- the value of cash flows in real terms is a function of the rate of inflation

VII. Marketability or Liquidity Risk

- represents the ease at which an issue can be sold at or near its true value
- a common measure is the bid-ask spread
- not a major concern of the intent to hold to maturity
- spreads will widen during recession and narrow during expansion

VIII. Exchange Rate or Currency Risk

- risk when non-dollar denominational issues are held
- 2 risks: currency fluctuation & changing foreign interest rates

IX. Volatility Risk

- value of interest rate options increase when interest rate volatility increases

X. Legal or Political Risk

- expropriation
- change in tax status from tax free to taxable
- decision by a regulatory body that an instrument is unsuitable
- tax free vs. tax spread changing because of different tax rates

XI. Event Risk

- the ability of a borrower to repair may be jeopardized due to:
 - natural or industrial accident
 - corporate restructuring or takeover
- may be firm specific, or related to event risk of other firms or may be systematic (CAPM)

XII. Sector Risk

- yield spreads across different sectors respond differently to different events:
 - premium vs. discount
 - corporate vs. mortgage-booked, etc.

XIII. Basis Risk

- all risks other than market risk

Question #9

In order to cash flow match, each liability cash flow must be matched with sufficient assets. We first need to match the longest liabilities and work back.

5th year: Assets > liabilities, 5 year assets must be sold

$$\text{sale of 5 year bond} = \frac{A_5 - L_5}{1 + \text{couponrate}} = \frac{2200 - 1980}{1 + 10\%} = 200 \quad \therefore \text{sell \$200 of par amount of 5 year bond}$$

Asset before sale	194	254	41	200	2200
Assets sold	<u>-20</u>	<u>-20</u>	<u>-20</u>	<u>-20</u>	<u>-20</u>
Adjusted Assets	174	234	21	180	1980

4th year: Assets = liabilities, the cash flows are matched

3rd year: Liabilities > Assets, 3 year assets must be purchased

$$\text{purchase of 3 year bond} = \frac{L_3 - A_3}{1 + \text{couponrate}} = \frac{445 - 21}{1 + 6\%} = 400 \quad \therefore \text{purchase \$400 of amount of 3 year bond}$$

Assets before purchase	174	234	21	180	1980
Asset purchased	<u>+24</u>	<u>+24</u>	<u>+424</u>	<u>+0</u>	<u>+0</u>
Adjusted assets	198	258	445	180	1980

2nd year: Assets > Liabilities, 2 year assets must be sold

$$\text{sale of 2 year bond} = \frac{A_2 - L_2}{1 + \text{couponrate}} = \frac{258 - 69}{1 + 5\%} = 180 \quad \therefore \text{sell } \$180 \text{ of par amount of } 2 \text{ year bond}$$

Assets before sale	198	258	445	180	1980
Assets sold	<u>-9</u>	<u>-189</u>	<u>0</u>	<u>0</u>	<u>0</u>
Adjusted assets	189	69	445	180	1980

1st year: Liabilities > Assets, 1 year asset must be purchased but there are no 1 year assets. Can purchase \$21 (210-189) of 90-day T-bills or mix T-bills and 2 year bond for better match.

Question #10

For this “collared floater”, the corporations payments will be:

If $L \geq 12\%$, 12.5%

If $7\% \leq L \leq 12\%$, $L + .5\%$

If $L \leq 7\%$, 7.5%

We want to pay instead a fixed rate of X%. Thus, we need:

If $L \geq 12\%$, $12.5\% - X\%$

If $7\% \leq L \leq 12\%$, $L + .5\% - X\%$

If $L \leq 7\%$, $7.5\% - X\%$

Let's try the middle one first:

This is a swap. We want to receive $L + .5\%$ and pay X%. This will be an off-market swap. Since we want to receive $L + .5\%$, we need to pay for the extra .5% for 5 years.

I'll assume we'll pay for it by paying an extra .5% on the swap as well, so that we'll pay $8.75\% + .5\% = 9.25\%$. This then must be our X.

So now, if for any L, we receive $L + .5\% - 9.25\%$.

So if $L \geq 12\%$, we want to get $17.5\% - 9.25\%$, so we don't need the extra $L - 12\%$. So we'll sell a cap at 12%. Similarly, we need to buy a floor at 7%. Then our total cash flow should equal what we desired:

If $L \geq 12\%$:

receive	$L + .5\% - 9.25\%$	on swap
pay	$\frac{L - 12\%}{.5\% - 9.25\% + 12\%} = 12.5\% - 9.25\%$	on cap

receive 3.25%, which is 12.5% - 9.25%, as hoped.

If $7\% \leq L \leq 12\%$, receive $L + .5\% - 9.25\%$ on swap, as hoped
neither cap nor floor pays

If $L \leq 7\%$,

receive	$L + .5\% - 9.25\%$	on swap
receive	$\frac{7\% - L}{5\% - 9.25\% + 7\%} = 7.5\% - 9.25\%$	on floor

= -1.75%, as hoped

What will this cost up-front?

- The swap is free, since both fixed and variable rates were adjusted.
- The cap will bring us income of $.65\%(10) = .065$ million.
- The floor will cost us $.9\%(10) = .09$ million, for a total upfront cost of $.09 - .065 = .025$ million.

Then, we'll pay a fixed-rate of 9.25% each year, on 10 million, is .925 million a year. Actually, since this is semi-annual, we'll pay $\frac{1}{2}(.925) = .4625$ million every 6 months.

Solve for i in
$$.025 + \frac{.4625}{\left(1 + \frac{i}{2}\right)} + \frac{.4625}{\left(1 + \frac{i}{2}\right)^2} + \dots + \frac{.4625}{\left(1 + \frac{i}{2}\right)^9} + \frac{10.4625}{\left(1 + \frac{i}{2}\right)^{10}} = 10$$

to calculate interest cost.

Credit risk is an issue for the swap we entered into and the floor that we bought. Thus, if Libor rises above 8.75%, we have to worry about the swap paying. If Libor drops below 7%, we have to worry about the floor paying.

Question #11

(a) Define a floating-rate security and describe its features

- a security whose coupon rate resets at predetermined dates
- coupon rate is based on a defined reference rate
- formula: coupon rate = reference rate +/- quoted margin
- quoted margin is the adjustment the issuer agrees to make to the reference rate
- Common reference rates include: Libor, T-Bills, or Prime rate
- Others include COFI
- There may be limits on the coupon rate including a cap or a floor, or both, which is a collar

(b) Describe the yield spread measures used to evaluate floating-rate securities

There are 4 margins commonly used:

1. Spread for Life also known as Simple Margin
 - Accounts for the amortization of discount/premium
2. Adjusted Simple Margin also called Effective Margin
 - An adjustment to spread for life
 - Adjustment accounts for the one-time cost of carry when a floater is purchased with borrowed funds
3. Adjusted Total Margin
 - Adds an additional adjustment to adjusted simple margin
 - This adjustment is the interest earned on the difference between the floater's par value and the carry-adjusted price
4. Discount Margin
 - Method that employs discounted cash flows
 - Procedure for calculating:
 - Project the floater's cash flows assuming the reference rate does not change
 - Select a margin
 - Discount the cash flows from the first step using the current reference rate and the selected margin
 - Compare the above present value to the security's price. If equal, the discount margin is the selected margin; if not equal, select a different margin and repeat until the present value equals the price

(c) Describe the factors affecting the price of floating-rate securities

There are 3 main factors:

1. The time until the next coupon reset date
 - The longer the time until the next reset, the more a floater behaves like a fixed-rate security

2. Change in the margin required by the market
 - After issue, the margin demanded by the market may increase/decrease and the price of the floater will decrease/increase respectively
 - The required margin will depend on:
 - Margin available in competitive markets
 - Credit quality of the issuer
 - Presence of embedded options
 - Liquidity of the issue

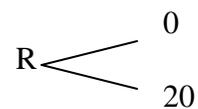
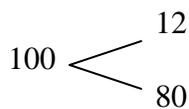
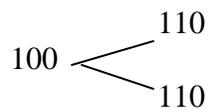
3. If a cap or floor exists, whether or not it is reached
 - If there is a cap, once the coupon rate rises above the cap, then the rate is limited and the floater will trade at a discount
 - The floater will trade more and more like a fixed-rate security, the further above the cap the market rate goes.

Question #12

- (a) -The reinsurance contract is similar to a put option on the life insurance business.
 -The reinsurance would eliminate the loss on the portfolio just like a protective put strategy does to a portfolio of stocks.
 -The reinsurance premium is payable at the beginning of the year just like the premium for a one-year option.

- (b) Answering the value of the life insurance business is 100, then

bank account



where R is the reinsurance premium

$$\text{Probability of up-move} = q = \frac{1.10 - 0.80}{1.20 - 0.80} = \frac{0.30}{.40} = 0.75$$

$$\text{then } R = \frac{1}{1.10} \{0.75(0) + 0.25(20.00)\} = 4.55$$

Therefore the reinsurance premium is 4.55 or 4.55% of the reinsured business

Replicating portfolio: $D - 100\Delta = R$

$$\begin{aligned} 1.10D - 120\Delta &= 0 \\ 1.10D - 80\Delta &= 20 \end{aligned}$$

$$1.10D = 120\Delta$$

$$D = \frac{120\Delta}{1.10} \Rightarrow 1.10\left(\frac{120\Delta}{1.10}\right) - 80\Delta = 20$$

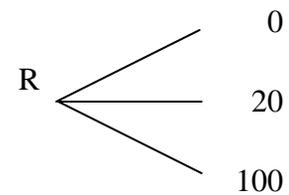
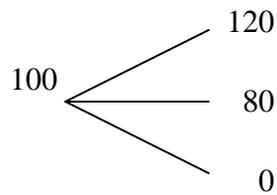
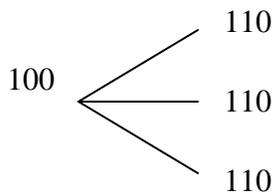
$$\Rightarrow 120\Delta - 80\Delta = 20$$

$$\Rightarrow 40\Delta = 20$$

$$\Rightarrow \Delta = \frac{1}{2} \quad \therefore D = \frac{120\left(\frac{1}{2}\right)}{1.10} = 54.55$$

Therefore the replicating portfolio is 54.55 in the deposit-taking business and a short position of 50 in the life insurance business

(c)



$$100 = 110y_1 + 110y_2 + 110y_3 \quad \Rightarrow \quad y_1 + y_2 + y_3 = \frac{1}{1.1}$$

$$100 = 120y_1 + 80y_2$$

$$\text{let } y_1 = s$$

$$\text{then } s + y_2 + y_3 = \frac{1}{1.1} \quad \Rightarrow \quad y_2 = \frac{1}{1.1} - s - y_3$$

$$\text{so } 100 = 120s + 80\left(\frac{1}{1.1} - s - y_3\right) = 120s + 72.73 - 80s - 80y_3$$

$$100 = 120s + 72.73 - 80s - 80y_3$$

$$80y_3 = 40s - 27.27$$

$$y_3 = 0.5s - 0.340875$$

$$y_2 = \frac{1}{1.1} - s - 0.5s + 0.340875 = 1.25 - 1.5s$$

We need to find y_1, y_2, y_3 in order to find $R = 0y_1 + 20y_2 + 100y_3$

$$y = [s \ 1.25 - 1.5s \ 0.5s - 0.340875]$$

$s > 0$	$1.25 - 1.5s > 0$	$1.25 - 1.5s < 1$	$0.5s - 0.340875 > 0$
$s < 1$	$1.25 > 1.5s$	$1.25 < 1 + 1.5s$	$0.5s > 0.340875$
	$s < \frac{1.25}{1.5} = \frac{5}{6}$	$0.25 < 1.5s$	$s > 0.68175$
		$s > \frac{0.25}{1.5} = \frac{1}{6}$	

$$\therefore 0.68175 < s < \frac{5}{6} \quad \text{but} \quad e_1 = y_1 = s_{\max} - \frac{1}{12} = \frac{5}{0} - \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$$

$$\Rightarrow y = [0.75 \ 0.125 \ 0.034125]$$

$$R = 20(0.125) + 100(0.034125) = 2.5 + 3.4125 = 5.9125$$

The new reinsurance premium is 5.9125 or 5.9125% of the reinsurance business

(d) If the guaranteed rate is 20% then $q = \frac{1.20 - 0.80}{1.20 - 0.80} = 1$

That means the risk-neutral probability of a down-move is zero. But the real probability is greater than 0. Therefore the 2 probability measures are not equivalent and the model would not be arbitrage free.

Using the guaranteed rate of 20%, the reinsurance premium would be 0 but there would be a non-zero probability of a positive pay-off from the reinsurance contract.

Question #13

- (a) Must define the following:
- target objective of the pension
 - liability characteristics
 - risk tolerance (relate to surplus requirement, risk of business, etc.)
 - liquidity
 - asset mix
 - availability of asset
 - regulation constraints and taxation
 - asset liability matching
 - procedure and authority
 - diversification of investment
 - how to review (appraise) of investment performance (can compare with benchmark, peer or liability cost, etc)
- (b) 3 possible vehicles
1. Segregate fund (fund by insurance company or trust)
 - pension fund own fixed income security
 - must be great size for diversity
 2. Unit of pool fund (by insurance company or combination of many small pension fund trust)
 - advantage: diversity, exposure to many asset, low cost, expertise available
 3. Fixed-income fund or insurance product
 - group annuity (lock return when purchase)
 - single premium group annuity
- internally managed fund

Question #14

$$S = 25, q = 6\%, X = 30, t = \frac{60}{360} = \frac{1}{6}, c = 1, p = 6$$

$$c = Se^{-qt} N(d_1) - Xe^{-rt} N(d_2)$$

$$p = Xe^{-rt} N(-d_2) - Se^{-qt} N(-d_1)$$

$$c + Xe^{-rt} = p + Se^{-qt}$$

$$1 + 30e^{-rt} = 6 + 25e^{-.06/6}$$

$$30e^{-rt} = 5 + 25e^{-.01}$$

$$e^{-rt} = .991708$$

Current market price of 150-day T-bill of 1,000 face amount

$$\begin{aligned} &= e^{-rt} \cdot (\text{price of futures contract on 90-day T-bill delivery in 60 days}) \\ &= .991708 \times 984 \\ &= 975.84 \end{aligned}$$