1.  

(5 points) LifeCo’s ALM Report indicates a need to rebalance the assets supporting the non-traditional life segment.

(a) Describe the constraints on asset sales when rebalancing this portfolio.

(b) Evaluate LifeCo’s investment strategy for this segment, and recommend any necessary changes.
2. \textit{(9 points)} LifeCo is considering selling its closed block of Institutional Pension (GIC).

\begin{center}
| Liability book value (in $ millions) | $1,500 |
| Maturity (in years)                  | 2      |
| Annual liability crediting rate      | 6.60\% |
| 1 year risk-free rate                | 1\%    |
\end{center}

You are given the following information:

\begin{center}
\begin{tabular}{lcc}
Scenario 1 & Scenario 2 \\
1 year risk-free rate & 3\% & 2\% \\
1 year forward & 3\% & 2\% \\
Withdrawal rate at the end of year 1 & 4\% & 2\% \\
Withdrawal rate at the end of year 2 & 100\% & 100\% \\
Probability & 30\% & 70\% \\
\end{tabular}
\end{center}

There are no new deposits.

(a) \textit{(4 points)} Compare the option pricing method and the actuarial appraisal method for estimating the fair value of liabilities.

(b) \textit{(2 points)} Outline practical considerations in applying the option pricing method.

(c) \textit{(3 points)} Calculate the fair value of liabilities using the option pricing method with the risk-free interest rate for discounting.
3. (9 points) You are LifeCo’s Investment Actuary and a member of the ALM Committee. The Committee is currently reviewing the risk exposures of the Non-Traditional Life portfolio as contained in your December 31 ALM Report.

- You have recently switched LifeCo’s ALM reporting to effective duration from modified duration
- Your effective duration calculations use a base yield curve and a yield curve shocked by 1 basis point
- No explicit hedge has been set up for the minimum credited interest guarantees
- LifeCo’s head of Investments has proposed the sale of $50 million (in present value) of 15-year zero-coupon bonds in order to increase cash holdings to partially address the key rate duration mismatch

(a) Describe the limitations of LifeCo’s reported effective durations as an interest rate risk measure.

(b) Estimate the economic impact of a 100bp drop in interest rates based on your reported effective durations.

(c) Compare this estimate with your reported “margin squeeze” impact and briefly explain reasons for any differences to the ALM committee.

(d) Describe any interest rate ‘bets’ evident from the reported partial durations.

(e) Estimate the revised partial durations following this proposed transaction.

(f) Estimate the revised impact of margin squeeze following this proposed transaction.

(g) Explain how the potential margin squeeze should be incorporated into any assessment of interest rate bets.
4. (9 points) LifeCo wants to modify the design of its Equity Linked GIC product to reduce the impact of a large decline in the stock market just before maturity. LifeCo believes that these modifications should be done in such a way that the participation rate is as high as possible and the profit margin remains the same.

The Company is also concerned about the cost of purchasing call options for the Equity Linked GICs and is considering applying the dynamic hedging program it developed for its Variable Annuities.

(a) Compare the options embedded in LifeCo’s Equity Linked GIC and Variable Annuities.

(b) Recommend potential changes to the Equity Linked GIC product that would meet LifeCo’s goals for the redesigned product.

(c) Describe the process used to evaluate the cost and efficiency of dynamic hedging.

(d) Evaluate LifeCo’s Variable Annuity dynamic hedging program as an alternative to purchasing call options for its Equity Linked GIC product.

(e) Contrast the hedging of Equity Linked GICs as a stand-alone product to that of an investment option of a Variable Universal Life product.
5. *(8 points)* You are constructing a model that will be used for dynamically hedging the guaranteed minimum death benefits (GMDB) on a variable annuity portfolio. The following models have been proposed:

(i) Lognormal

(ii) Regime Switching

(iii) Time Series Model with GARCH Volatility

(iv) Empirical

(v) Wilkie

(vi) Stable Distribution

(a) *(3 points)* Evaluate each model for modeling equity returns.

(b) *(5 points)* Assess the suitability of each model for dynamic hedging and recommend a model to be used for dynamically hedging the GMDB.
6. (7 points) ABC Life has issued a 2-year GIC that has no policyholder options and pays interest at maturity.

<table>
<thead>
<tr>
<th>Single Premium</th>
<th>100 million</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payout in 2 years</td>
<td>110 million</td>
</tr>
<tr>
<td>Initial Assets (MV)</td>
<td>100 million of 3-year zero coupon corporate bonds</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Treasury zero-coupon rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term, Years</td>
</tr>
<tr>
<td>Year</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Corporate bond spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond Spread Curve</td>
</tr>
<tr>
<td>Term, Years</td>
</tr>
<tr>
<td>1 year</td>
</tr>
<tr>
<td>0.1%</td>
</tr>
</tbody>
</table>

Assume:
- the bond spread curve does not change from issue
- expenses are negligible
- GIC issues occur at the beginning of the year
- payouts occur at the end of the year

(a) Compute the Total Returns in year 1 for both assets and liabilities using the Total Return Approach.

(b) Disaggregate the Total Returns into their components, e.g. C-risks.

(c) Outline the advantages of the Total Return Approach over the book-value approach.
7. (9 points) The graph below shows the distribution of the daily returns of a particular equity portfolio in one year with a total of 254 trading days. For example, there were 65 days with daily return between 0.00% and 0.50%. The average daily return is 0.04% and the standard deviation of the daily return is 1.07%. The current value of the portfolio is $10 million.

(a) Explain the objective of a Value at Risk (VaR) calculation

(b) Calculate:

(i) The one day VaR at a 95% confidence level using the above histogram.

(ii) The 10 day VaR at a 95% confidence level assuming a normal distribution.

(c) Evaluate the advantages and disadvantages of these methods of calculation and the Monte Carlo Simulation approach.

(d) Describe how to test the accuracy of the alternative models.

(e) List the limitations of VaR as a measure of risk, and explain how the Conditional Tail Expectation approach and stress testing might complement VaR as a risk measure.

(f) Calculate the 95% Conditional Tail Expectation based upon the distribution in the graph above using interval midpoints as estimates of average values.
8. (4 points) Acme Motors offers the following investment options to the participants in its Defined Contribution plan:

1. A U.S. equity fund
2. An international equity fund
3. A fixed income fund

You are considering adding a Stable Value Fund option.

(a) Explain why a Stable Value Fund would be offered as an option.

(b) List the risks to Acme Motors and its participants associated with the Stable Value Fund option.

(c) List the risks to the issuer of the underlying GIC or BIC contracts that support Acme Motor’s DC plan’s stable value fund.

(d) Propose ways the issuer can manage these risks.
9.  

(3 points) You are given a 5-year, BB rated zero-coupon bond with par value of 100.

You are given the following information

The 1-year transition matrix is:

<table>
<thead>
<tr>
<th>Initial Rating</th>
<th>Rating at Year end (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AA</td>
</tr>
<tr>
<td>AA</td>
<td>95</td>
</tr>
<tr>
<td>A</td>
<td>2.5</td>
</tr>
<tr>
<td>BB</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
</tbody>
</table>

The 1-year forward zero-coupon curve is:

<table>
<thead>
<tr>
<th>Category</th>
<th>1-Year</th>
<th>2-Year</th>
<th>3-Year</th>
<th>4-Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>3.5</td>
<td>4.1</td>
<td>4.7</td>
<td>5.0</td>
</tr>
<tr>
<td>A</td>
<td>3.7</td>
<td>4.3</td>
<td>4.9</td>
<td>5.3</td>
</tr>
<tr>
<td>BB</td>
<td>4.0</td>
<td>4.7</td>
<td>5.3</td>
<td>5.7</td>
</tr>
<tr>
<td>B</td>
<td>5.6</td>
<td>6.0</td>
<td>6.8</td>
<td>7.4</td>
</tr>
<tr>
<td>C</td>
<td>10.0</td>
<td>12.0</td>
<td>11.0</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Using the CreditMetrics approach:

(a) Calculate the possible 1 year forward values of the bond.

(b) Calculate the credit VaR at the 99% confidence level.

(c) Calculate the capital charge using the value obtained in (b).
10.  (3 points) DM Life is considering hedging its credit risk with Credit Default Swaps (CDS).

(a) Explain FAS 133’s rules for qualifying for hedge accounting.

(b) Formulate a hedge strategy that would qualify for hedge accounting under FAS 133.
11. (3 points) An associate at your firm follows a select group of options. From time to time he feels he can develop information and views in terms of expected return and risk that are not reflected in current market prices.

(a) Explain why observed option prices might differ from those predicted by the Black-Scholes model.

(b) Explain whether a Black-Scholes option model would be useful in evaluating a specific option investment strategy.
12. (4 points) You are given the following statistics on a real estate portfolio.

<table>
<thead>
<tr>
<th>Property Type</th>
<th>Portfolio Weighting</th>
<th>Index Weighting</th>
<th>Portfolio Returns</th>
<th>Index Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warehouse</td>
<td>15%</td>
<td>15%</td>
<td>5.50%</td>
<td>6.00%</td>
</tr>
<tr>
<td>Apartment</td>
<td>15%</td>
<td>15%</td>
<td>3.00%</td>
<td>4.00%</td>
</tr>
<tr>
<td>Retail</td>
<td>50%</td>
<td>40%</td>
<td>7.88%</td>
<td>7.50%</td>
</tr>
<tr>
<td>Office</td>
<td>20%</td>
<td>30%</td>
<td>4.25%</td>
<td>4.00%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
<td>6.06%</td>
<td>5.70%</td>
</tr>
</tbody>
</table>

The real estate portfolio manager has made the following assertion:

“When it comes to real estate investing, I know exactly which properties are great performers!”

He also provided the following two recommendations:

- Sell some of the apartment holdings because the occupancy rates are high in all of the apartment buildings in the portfolio leaving little room left for any major price appreciation.
- Invest in a retail complex being developed on the outskirts of town because of the potential for price appreciation.

(a) Evaluate the validity of the real estate portfolio manager’s assertion.

(b) Critique each of the recommendations.
13. (5 points) You are the Mortgage Backed Security analyst at ABC Life Insurance Company. ABC’s chief economist, Jane Schmeau, is projecting that interest rates will rise sharply over the next 12 months and then continue to rise steadily into the foreseeable future. Ms. Schmeau makes the following statement:

“Because it is solely the current level of interest rates that determines the rate of mortgage refinancing, the rate of mortgage prepayments will drop sharply in my projected interest rate scenario”.

(a) (1 point) Assess Ms. Schmeau’s statement.

(b) (4 points) Describe the characteristics, advantages and disadvantages of each of the following securities based upon Ms. Schmeau’s economic projection. Select one of these bonds for purchase.

(i) A newly issued PAC with a 7 year average life, a collar of 100%-250% PSA, and a 6 year lockout period
(ii) An intermediate pay sequential PAC with a 7 year average life and 6 years of prepayment lockout
(iii) Z bond
(iv) Z-jump bond
14.  (7 points) Your company is considering adding a Market Value Adjustment (MVA) to its SPDA. The SPDA has the following characteristics:

- surrender charges decline over 5 years
- minimum guaranteed credited rate of 3.0%
- initial crediting rate is 4.0% fixed for 5 years and reset annually thereafter using a new-money rate

The MVA Factor being considered uses the following formula:

\[
\left[ \frac{1+ j}{1+i} \right]^{T-t} \text{ for } i > j
\]

where:

- \(i\) is the current market rate
- \(j\) is the fixed crediting rate
- \(T-t\) is the fixed rate period remaining

You asked a student to calculate the duration of the SPDA without the MVA. The student’s immediate response is:

“This product is sold as a 5 year CD in the bank channel, it will behave like a zero coupon bond with 5 year maturity and effective duration close to 5 years.”

(a) Assess the student’s response.

(b) Describe the benefit of the MVA feature from an ALM perspective.

(c) Compare the effective duration of SPDA with and without the MVA.

(d) Describe in what situation a return of premium feature applied before the surrender charge will be in-the-money with 4 more years of initial guarantee remaining.

(e) Assess the impact on effective duration of the return of premium feature.

(f) Assess the impact of minimum guaranteed rate on the effective duration of the SPDA with MVA.
15. *(8 points)* BMC Olympiad Inc. is looking for a potential acquirer within the next 12 months. You have been asked to assess the company’s near-term credit risk exposure before proceeding with an appraisal analysis of BMC Olympiad’s operating businesses.

You are given the following information:

- Company’s credit rating: BBB
- Risk-free rate: 5% per annum compounded continuously
- Market value of company’s assets today: $21 billion
- Market value of company’s equity today: $5 billion
- Company’s debt due to be repaid including interest one year from now: $17 billion
- Volatility of equity returns: 80%
- Volatility of asset returns: 20%
- A company with similar credit risk has five-year corporate zero-coupon bonds trading at 350 basis points above risk-free rate
- Assumed recovery rate in the event of a default: 40% (as percent of bond’s no-default value)

(a) Estimate the risk-neutral probability that the company will default on its debt using Merton’s model.

(b) Determine the expected loss on the debt and the expected recovery in the event of a default.

(c) Compare the default probability produced by Merton’s model versus the annualized risk-neutral default probability inherent in the company’s current corporate bond pricing. Explain possible reasons for discrepancy between these two estimates of default probabilities.
16. (8 points) BH Life is a small insurance company that has sold a product with a minimum return guarantee. The investment guarantee is equivalent to a European put option on the S&P 500 index with a notional amount equal to 5,000 times the index.

You are given the following information:

- Risk free rate is 2.5%
- Current index value is 1200
- Strike price is 1100
- Time to maturity is 1 year
- Black Scholes value of the put option with implied volatility of 22% is $242,900
- No dividends
- Trading costs equal 0.10%
- Proceeds are invested at the risk-free rate
- Risk-free rate and volatility do not change during the year

The firm wishes to delta hedge the risk on the put with quarterly rebalancing by using shares of a fund that track the S&P 500.

(a) (2 points) Calculate the expected profit at the time the product is sold assuming that expected volatility is 22% and the put option was priced using a volatility of 25% per year.

(b) (1 point) Calculate the initial hedge position the firm should hold for delta neutrality if the volatility assumed in their hedge is 22%.

(c) (3 points) Calculate the firm’s net gain or loss on the transaction, assuming that over the course of the year the S&P 500 takes the following path given below, and that the volatility assumed in their hedge is 22%. N(d1) factors are provided for European options with the same maturity date and strike price and a volatility of 22%.

<table>
<thead>
<tr>
<th>Time</th>
<th>Index Level</th>
<th>N(d1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point of sale</td>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>End of Quarter 1</td>
<td>1250</td>
<td>0.8051</td>
</tr>
<tr>
<td>End of Quarter 2</td>
<td>1150</td>
<td>0.6700</td>
</tr>
<tr>
<td>End of Quarter 3</td>
<td>1050</td>
<td>0.3783</td>
</tr>
<tr>
<td>End of Quarter 4</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

(d) (2 points) Describe other risk management strategies the firm can use.
17. (4 points) You are considering investing in government issued fixed income securities in the three countries listed below.

You are given the following information:

<table>
<thead>
<tr>
<th>Country</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continent</td>
<td>Europe</td>
<td>Asia</td>
<td>Asia</td>
</tr>
<tr>
<td>Population</td>
<td>3M</td>
<td>400M</td>
<td>50M</td>
</tr>
<tr>
<td>Oil Exports</td>
<td>1B</td>
<td>150B</td>
<td>5B</td>
</tr>
<tr>
<td>Agriculture Exports</td>
<td>1B</td>
<td>150B</td>
<td>15B</td>
</tr>
<tr>
<td>Agriculture Domestic</td>
<td>3B</td>
<td>450B</td>
<td>40B</td>
</tr>
<tr>
<td>Manufacturing Exports</td>
<td>8B</td>
<td>150B</td>
<td>300B</td>
</tr>
<tr>
<td>Manufacturing Domestic</td>
<td>13B</td>
<td>600B</td>
<td>240B</td>
</tr>
<tr>
<td>Total GDP</td>
<td>26B</td>
<td>1500B</td>
<td>600B</td>
</tr>
<tr>
<td>Government type</td>
<td>Stable Democracy</td>
<td>Stable Democracy</td>
<td>Emerging Democracy</td>
</tr>
<tr>
<td>Inflation Range</td>
<td>2%-20%</td>
<td>0%-30%</td>
<td>8%-16%</td>
</tr>
<tr>
<td>Infant deaths per 1000</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Life expectancy</td>
<td>75</td>
<td>70</td>
<td>65</td>
</tr>
</tbody>
</table>

(a) Describe the risks of the potential investments and propose how to mitigate them.

(b) Rank the countries by their political stability and justify your ranking.
18. (6 points) You are a Risk Manager for a US-based trading company with international operations. The company has the following risk exposures:

- a contract to deliver 1,000 ounces of gold semi-annually for one year at a maximum price of $400 USD per ounce
- beginning of period 1 year LIBOR (in USD) on a $10,000,000 bank deposit payable at the end of the year
- 7,000,000 Euro receivable from financing a customer purchase due in 1 year

You are given the following information:

- Available hedging instruments include currency forwards, USD swaps, and gold options.
- Current exchange rate = .75 Euro / $1 USD
- Risk free rates USD Euro
  6 month 2.5% 4.0%
  1 year 3.0% 5.0%

<table>
<thead>
<tr>
<th>Swap rates (USD)</th>
<th>Floating rate</th>
<th>Fixed Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6 month LIBOR</td>
<td>2.75%</td>
</tr>
<tr>
<td></td>
<td>1 year LIBOR</td>
<td>3.25%</td>
</tr>
</tbody>
</table>

- Option price per 1 ounce gold contract with strike of USD 400:

<table>
<thead>
<tr>
<th></th>
<th>Buy Call</th>
<th>Sell Call</th>
<th>Buy Put</th>
<th>Sell Put</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 month</td>
<td>4.0</td>
<td>3.6</td>
<td>6.0</td>
<td>5.2</td>
</tr>
<tr>
<td>1 year</td>
<td>6.0</td>
<td>5.6</td>
<td>8.0</td>
<td>7.2</td>
</tr>
</tbody>
</table>

(a) Describe the advantages of managing risk of strategic exposures in general as defined in Chew.

(b) Propose a methodology to completely hedge the company against its strategic risks.

(c) Calculate the market value of the risks in USD.
19. (6 points) BB No-Show Inc, a US company, is negotiating to buy a Japanese company. The deal is expected to be closed in 3 months, with cash payment in Japanese Yen.

The chief financial officer of BB No-Show Inc has decided to buy an at-the-money call option on Japanese Yen to hedge against a sudden increase in Yen relative to the US dollar.

You are given the following information:

- Option type: at-the-money European call option.
- Maturity date of the option: 3 months (65 trading days)
- The option-holder has the right to buy 220 billion of Japanese Yen.
- US Treasury bond rates are 1% compounded continuously
- Japanese government bond rates are 0.05% compounded continuously
- Current exchange rate: 1 USD = 110 Japanese Yen
- Japanese Yen / USD exchange rate has daily volatility of 0.62%

An asset price $S$ follows the stochastic process

$$dS = \mu S dt + \sigma S dz$$

(a) Apply Ito’s lemma to derive the process followed by $G = S \exp \left( r(T - t) \right)$ where $r$ is the risk-free rate and $(T - t)$ is the time to maturity.

(b) Interpret the derived stochastic process if $G$ is a stock paying dividends at a continuous rate.

(c) Define the stochastic process of $G$ in the risk neutral world, assuming $G$ is the spot foreign exchange rate.

(d) Calculate the value of the call option using the applicable Black-Scholes formula for this call option on Japanese Yen.
20. (3 points) You are given the following information at time \( t = 0 \):

- Total assets supporting participating life = $74,081,822
- Total participating life liabilities = $66,673,640
- Maturity of policy = 10 years
- Guaranteed interest rate of the policy = 3\% \text{ continuously compounded}
- 10-year European call option values for an asset with current price $74,081,822:

<table>
<thead>
<tr>
<th>Strike price</th>
<th>Call Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$85,000,000</td>
<td>$8,520,220</td>
</tr>
<tr>
<td>$90,000,000</td>
<td>$7,625,000</td>
</tr>
<tr>
<td>$95,000,000</td>
<td>$4,502,535</td>
</tr>
<tr>
<td>$100,000,000</td>
<td>$1,204,330</td>
</tr>
<tr>
<td>$105,000,000</td>
<td>$820,300</td>
</tr>
</tbody>
</table>

Calculate the equilibrium participation level for policyholders.

**END OF EXAMINATION**

AFTERNOON SESSION
Solution 1

(a) Accounting
   Effects on different accounting basis

Tax
   Tax gain/loss carry forward position
   Netted within calendar year
   Netted between affiliates

EV/EVA
   Need to reproduce cash flows

ALM

Credited Rate
   Effect on credited rate

Policyholder Equity
   Effect on segment’s credit quality, maturity structure, concentration

(b) Description of UL product characteristics:
   Product has embedded options
   Liability cash flows are interest sensitive

Asset Mix

   Duration of 12 is CRAZY long. The company is exposed to
   HUGE disintermediation risk if interest rates rise.
   A/L dollar-duration mismatch is way outside Investment Policy
   Constraints
   Sell much of the government and public corporate portfolios,
   reinvest much shorter, to bring duration down toward 40

   Need to consider product liquidity needs
   Given privates’ low liquidity a higher quality would be appropriate
   As the privs would be very difficult to sell, reinvest any excess cash
   flow on maturity in short-dur, non-callable publics

Derivatives

Policy is in place to permit use
Solution 1 (continued)

Use to quickly correct duration mismatch

Protect with a floor or pay-float swap
Solution 2

(a) **Option pricing method:**
- Also known as direct method or multi-scenario method
- Generate stochastic economic scenarios
- Project liability CF along each scenario path
- Calculate pathwise liability PV by discounting liability CF along each path
- May reflect risk by adjusting either the discount rate or the projected liability CF
- If adjusting discount rate, may add a spread that reflects
  ⇒ Issuer’s credit
  ⇒ Quality of issuer’s bond portfolio
  ⇒ Risk contingency margin of the liability
  ⇒ Liquidity of the liability
- Assign probability to each scenario path
- Calculate fair value of liability as weighted sum of pathwise liability present values
- Not often used for transfer pricing for a block of insurance liabilities

**Actuarial Appraisal Methods:**
- Also known as indirect method
- Generate stochastic economic scenarios
- Project free CF under each scenario
- Free CF may reflect:
  ⇒ Investment earnings
  ⇒ income tax
  ⇒ RBC
- Calculate pathwise PV by discounting free CF at risk adjusted firm’s cost of capital
- Calculate DDE as average of pathwise PVs
- FV of liability = MV of assets – DDE – DTL
- Method of choice for transfer pricing for a block of insurance liabilities
- Reconcilable with Option Pricing Method under certain assumptions
Solution 2 (continued)

(b) 
- Volume of computation could be prohibitive
- System may not be available to do this routinely
- Expertise may be lacking
- Market data for calibration may not exist for certain insurance options

(c)

Scenario 1:
\[ L_{V0} \text{ (liability value at year 0)} = 1,500 \]
\[ L_{V1} \text{ before withdrawal (WD)} = 1,500 \times (1 + 6.6\%) = 1,599 \]
\[ CF_1 = 1,599 \times 4\% = 64 \]
\[ L_{V1} \text{ after WD} = 1,599 - 64 = 1,535 \]
\[ L_{V2} \text{ before WD} = 1,535 \times (1 + 6.6\%) = 1,636 \]
\[ CF_2 = 1,636 \]
\[ PVCF_{sce1} = 64 \div 1.01 + 1,636 \div \left[ \frac{1}{(1.01)(1.03)} \right] = 1,636 \]

Scenario 2:
\[ L_{V0} = 1,500 \]
\[ L_{V1} \text{ before WD} = 1,599 - 32 = 1,567 \]
\[ CF_1 = 1,599 \times 2\% = 32 \]
\[ L_{V1} \text{ after WD} = 1,599 - 32 = 1,567 \]
\[ L_{V2} \text{ before WD} = 1,567 \times (1 + 6.6\%) = 1,670 \]
\[ CF_2 = 1,670 \]
\[ PVCF_{sce2} = 32 \div 1.01 + 1,670 \div \left[ \frac{1}{(1.01)(1.02)} \right] = 1,653 \]
\[ FV = 0.3 \times 1,636 + 0.7 \times 1,653 = 1,648 \]
Solution 3

(a) LifeCo’s reported effective durations based on:
   • parallel yield curve shift
   • small (1bp) yield curve shift
They do reflect interest-sensitive cash flows, but:
   • cash flow models may not be perfect
   • may mean significantly higher convexity and also optionality, so impact under large shift may be very different than predicted by effective duration

(b) Impact of 100bp -drop- based on reported durations

Using effective duration * PV cash flows:
Assets: d=9.26  
   so impact = 416,600 * 9.3 * 1% = + 38,744 (or 38.7 million)
Liabs: d=4.00  
   so impact = 406,000 * 4.00 * 1% = + 16,240 (or 16.2 million)
Total: 38.7 - 16.2 = 22.5  i.e. economic gain of 22.5 million

(c) Reported “margin squeeze” impact based on 100bp drop was -10.3 million loss
   Estimate using effective duration in (b) was 22.5 million gain
Reasons for difference:
   • margin squeeze modeled under 100bp shift, not estimated from 1bp shift
   • other reasons from part (a) like convexity, optionality
   • convexity wouldn’t account for opposite direction of impact
   • but optionality could: may hit minimum guarantees under 100bp shift but not for 1bp
   • if min guarantees “in the money” under 100bp, would explain large negative “margin squeeze” impact

(d) What “bets” are there..
   • durations themselves indicate bet on parallel interest rate decrease, hopefully partials are consistent with this!
   • Partial sensitivities show $ change under a 1bp increase
   • Short (1-5 year) partials show a gain if short rates rise
   • Long (7-20 year) partials show a loss if long rates rise (gain if long rates fall)
   • So overall, bet is “flattening” of curve
Solution 3 (continued)

(e) Only 15-year partial affected, since going to cash (d=0) won’t recalculate 0.25-yr partial
Will show change to partial sensitivity since that’s what’s used in the case study
15-yr sensitivity is reported as -698,000 for 1bp increase
The 50 million of 15-yr zeros alone contribute approx 1bp * -15 * 50 million = -75,000
We should do exact calc using 15-yr spot rate of 5.42%....
   PV under 1bp increase is 50 million * (1.0542 ^ 15 / 1.0543 ^ 15) = 49,929,910
So zeros contribute -70,090
Selling them will *increase* 15-yr partial by about 70,000 to -628,000 (not much change)

(f) Revised margin squeeze... doing exact calc, the zeros contributed a *gain* under the 100bp drop:
   PV under 100 bp drop is 50 million * (1.0542 ^ 15 / 1.0442 ^ 15) = 57.7 million
   gain of 7.7 million for zeros
So selling them..... margin squeeze impact will be 7.7 million worse or about 18 million loss!
Proposed sale doesn’t help margin squeeze!

(g) Margin squeeze showed a “bet” that rates wouldn’t drop 100bp (or at least showed a loss if they did!)
Looking only at effective duration and partials said gain if longer rates drop 1bp... these didn’t tell the whole story
Solution 4

(a)

ELGIC: Call on 75% of increase of S&P500 over 5 year period

Variable annuity: put, contingent on death, on the invested funds with strike price \( S_0 = (1.05)^7 \)

Additional option provided by dollar for dollar partial surrender

(b)

Potential changes to reduce the impact of a large equity market decline just before maturity:
- Annual resets to lock-in gain at each policy anniversary
- Monthly averaging: use average increase over period
- High water mark to lock-in maximal gain

Reduce cost by using:
- Simple annual ratchet instead of compound one
- Averaging when calculating the actual return

(c)

Evaluate cost and efficiency of dynamic hedging by comparing the alternatives (no hedging or static hedging) over many economic scenarios. Use Monte Carlo simulation techniques to model hedging including:
- The impact of rebalancing and the hedging error introduced by the drift
- Transaction costs (proportional to change in stock position)

Stress testing is also important to highlight potential risks and exposure.

(d)

Dynamic hedging is a viable alternative for large blocks of business with embedded options that are difficult to replicate with standard options. This might be the case for Life Co ELGIC. With small notional amounts, transaction costs may deteriorate any benefit. The simplicity of the ELGIC option is such that it may be available in the market. One benefit of hedging is to combine offsetting exposure. Compare costs and availability of both alternatives.
Solution 4 (continued)

(e) Can hedge ELGIC by buying a bond and a 5 year European call option on 75% of the notional amount. No need to rebalance unless early withdrawals are very different from initial estimates.

Variable Universal Life investment options depend on timing and amount of premiums and withdrawals. A static hedge will not work. Dynamic hedging on the portfolio of investment options would be preferable.
Solution 5

(a)

1) Lognormal model
A standard model for evaluating equity returns, assumes percentage returns are normally distributed, which is a reasonable assumption in many situations. Allows analytical solution for European call and put options in a form that provides the way to construct a replicating portfolio consisting of underlying stocks and risk-free bonds. However, the model does not account for:
- big jumps in stock prices
- auto regression effects in returns
  volatility clustering effect
The limitations result from the assumption of constant volatility of return experience data shows that in practice volatility of returns is stochastic with some autoregressive features.

2) Regime Switching (RS)
One of the proposed models is a regime switching-between two lognormal distribution, with \( \mu_1, \sigma_1 \) and \( \mu_2, \sigma_2 \). The model is shown to fit well to actual results for returns on broad-based indices (like S&P500). The data shows that in fact there are periods in time, when volatility of index returns are relatively low which are then switch to periods with much higher volatility. The model give fatter tails for the distribution of returns and thus provides a better fit to the actual data than the simple lognormal distribution. The RS – distribution can be easily simulated, like the lognormal one, but – it does allow for analytical solutions. Also, the model does not incorporate auto regression. More parameters than ARCH or GARCH.
Solution 5 (continued)

3) **Time Series with GARCH volatility**
   In GARCH (1, 1) model the volatility is calculated from:
   \[ \sigma_t^2 = \gamma V + \alpha \sigma_{t-1}^2 + \beta (Y_{t-1} - \mu)^2, \]
   where \( V \) is the long-term average variance; \( \sigma_{t-1} \) and \( Y_{t-1} \) - values of the volatility and return at the previous time moment.
   Constants \( \alpha, \beta, \gamma \) are found by the regression to the actual data. This model is very general in nature and allows for:
   - mean reversion; through long-term average term \( V \)
   - auto regression, through dependence on the prior value of volatility; \( \sigma_{t-1} \)
   - effect on high volatility when returns are far from their long-term mean, \( \mu \), through term \( \beta (Y_{t-1} - \mu)^2 \),
   The model can be easily simulated but does not allow for analytical solutions.
   The volatility in the model is (unconditionally) stochastic due to \( Y_{t-1} \).

4) **Empirical**
   Actual data for returns are recorded for some period of time and then used as a sample space, from which the values for future returns are taken. The model is:
   - fit well to the past experience by definition; each observation is equally likely
   - is limited, since the data is limited
   - does not allow new developments in the future; only the returns that were recorded in the past are allowed for the future
   - If the return values are sampled with replacement randomly the model does not produce any auto correlations.
   This can be improved by sampling number “in bunches”.

5) **Wilkie**
   This is an econometric model, which combines processes for different economic factors, such as inflation level; short and long-term interest rates; divided yields and stock returns. The model has a cascade structure.
   A process for each new factor includes a term, connecting it to the parameters from the prior (upper) levels; and also a stochastic term.
   The model is very complex.
   **Main advantage** - combines different econometric parameters in one inter dependant model; for example, short interest rates and stock returns, which can be very useful for many actuarial applications (ALM).
   **Drawback** - very difficult to estimate model parameters. Contains a lot of them and requires a lot of experience data for estimation.
Solution 5 (continued)

6) Stable distributions
This is a particular class of functions, which satisfy specific conditions on their linear transformations. One example – normal distribution.

Advantages – wide class of functions with a convenient “convolution” feature; allows to model very “fat” tails

Drawbacks – difficult to simulate
- does not allow autocorrelation

b) GMDB in the simplest form is the return of premiums (less withdrawals) as the minimum death benefit. It is a put option on the fund value at the time of death. The payoff equals max (P-F, 0), where P=premiums net of withdrawals, F=fund value at the time of death. The value of the option should be adjusted for survivorship.

The total value of the GMDB at \( t = 0 \):

\[
\int_0^r p(t) \cdot p^*_t \cdot \mu^{(q)}_{t,t} \cdot dt;
\]

where the value of the European put option with term \( t \); \( p(t) \), is multiplied by probability of surviving to time \( t \) and dying in the interval \( dt \); and summarize for the term of the annuity, \( n \) years.

We assume that mortality and lapsing functions are known.

To hedge GMDB dynamically we need:
- to simulate the fund returns
- to calculate the value of the put option for each \( t \)
- to have a possibility to present the option value in terms of the replicating portfolio.

Of all the models, only the lognormal model gives the analytical value of the put option in terms of the hedge portfolio, split between the stock fund and risk-free bonds.

So - to calculate the parameters of the replicating portfolio, the lognormal model (Black-Scholes) should be used.
- to simulate the fund returns – lognormal, RS, GARCH or empirical can be used.

To better represent actual fund behaviour - use RS or GARCH

Need - to rebalance frequently and calculate hedging errors and transaction costs.
Solution 6

(a)

\[ r_A = r_f + OAS - D_{OAS} \Delta OAS - \Sigma D_t (i) \Delta t (i) + r / c + pa - e_a \]

\[ r_e = r_f + ROAS - \Sigma D_t (i) \Delta t (i) + e_t \]

\[ r_f = 3\%; OAS = SObps; D_{OAS} \Delta OAS = 0; r / c, pa, e_a = 0; D(2) = 2 \]

Asset return: expected forward rate = 6.01%
actual rate = 4.00%

\[ r_A = 3.0\% + 0.5\% - 0 - (4\% - 6\%) \cdot 2 = 7.5\% \]

Liability Return

Liability Yield = \( \left( \frac{110}{100} \right)^{0.5} - 1 = 4.88\% \)

\[ ROAS = 88\% \]
expected forward rate = 5.01%
actual rate = 4.00%

\[ r_i = 3.0\% + 88\% - (4\% - 5\%) \cdot 1 = 4.88\% \]

(b)

<table>
<thead>
<tr>
<th>risk free</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3%</td>
<td>3%</td>
<td>0%</td>
</tr>
<tr>
<td>C1</td>
<td>( OAS = 5% )</td>
<td>0%</td>
<td>.5%</td>
</tr>
<tr>
<td>C2</td>
<td>0%</td>
<td>( ROAS = 88% )</td>
<td>- 88%</td>
</tr>
<tr>
<td>C3</td>
<td>( \Sigma D_t (i) \Delta t (i) )</td>
<td>( \Sigma D_t (i) \Delta t (i) )</td>
<td>4%</td>
</tr>
<tr>
<td>Total</td>
<td>7.5%</td>
<td>4.88%</td>
<td>2.62%</td>
</tr>
</tbody>
</table>
Solution 6 (continued)

(c) Total Return Approach:
- splits return into components (e.g., c risks)
- performance measurement
- setting consistent goals in managing both assets & liabilities
- prospective & retrospective analysis
- measure results relative to
  - bond selection
  - interest rate anticipation
  - sector rotation
  - r/c expenses
- market value measure
Solution 7

(a) Purpose is to single number \( V \), says that over the next \( N \) days, we will not lose more then \( V \) dollars of value with \( x\% \) confidence.

(b)

(i) using histogram (1-day)
   254 \( \times \) 5\% \( \approx \) 13 days
   the 15\textsuperscript{th} worst return is \(-1.5\%\)
   so \( VaR = -1.5\% - 0.04\% = -1.54\% \)
   or \( \$10m \times 1.54\% \approx 154K \)

(ii) using normal distribution (10-day)
    use \( N(x) = 0.95 \Rightarrow x = 1.645 \)
    So 10-day \( VaR = 10m \times 1.645\sigma \frac{1}{\sqrt{10}} \)
    \[ = 10m \times 1.645 \times 0.0107 \times \frac{1}{\sqrt{10}} \]
    \[ = 556K \]

(c) Histogram
    adv: avoid use cash-flow mapping
    use historical data
    disadv: Computing slow
    does not allow volatility updating
    sensitive to historical data

Normal:
    adv: quick to calculate
    can use volatility update scheme
    disadvan: normal distribution assumption
    give poor result for low-delta portfolios

Monte-Carlo:
    adv: any model can be used
    disadvan: Computing intensive
Solution 7 (continued)

(d) - assess use the estimation errors
- for quantile-based
\[ se(q) = \sqrt{\frac{c(1-c)}{Tf(q)^2}} \]
- for sigma-based
\[ se(\alpha s) = \alpha \times se(s) \]

For normal \( se(s(\Phi)) = \alpha \sqrt{\frac{1}{2T}} \)

Also, can use back testing and stress testing.

(e) Limitation:
- only 1 point, no tail distribution
- results depend on methodologies
- results depend on assumptions
- results depend on time horizon
- many factors not captured such as legal, operations
- may give management false sense of security

CTE:
- more robust
- consider the shape of tail distribution
- it is the expected loss give loss happens
- meet criteria

Stress testing:
- can test extreme cases not captured in \( VaR \)

(f) \[ CTE_\alpha (L) = E\left( I_0 / I_0 > V_\alpha \right) \]
13 worst losses
\[ CTE = 10m \times (1 \times 4.75\% + 1 \times 3.75\% + 2 \times 2.75\% + 4 \times 2.25\% + 5 \times 1.75\%) / 13 = 244K \]
Solution 8

(a)  
- A stable value fund if an options offered by 401(k) and other DC plans  
- It is typically the most conservative investment options  
- It is good with ERISA (fiduciary duty)  
- Invested mostly in GIC and other medium-term fixed-income contracts issued by a high-quality financial institution  
- Provide participants ability to withdraw and transfer funds (subject to plan rules) without penalty or market value risk. Principal is guaranteed by issuer (so there is credit risk)  
- Good for participant seeking a safety investment option (near retirement) / reduce volatility  
- Good for participant seeking to diversify their investment (low correlation with equity)

(b)  
- Credit risk of the GIC/BIC issuer  
- For Synthetic GIC / SA GIC we also have  
  Gains and losses amortized into the credited rate  
  Underlying asset default  
  Reinvestment, interest or market risk (performance)  
  Call/extension risk / withdrawal risk / competing funds

(c)  
- Asset risk:  
  (i) Default / credit risk  
  (ii) Call or extension risk  
  (iii) Performance, interest, market or Reinvestment risk  
- Liability risk:  
  (i) Contribution risk  
  (ii) Withdrawal / liquidity risk

(d)  
- Good Underwriting / reinsurance  
- Cash flow matching / Duration Matching / Convexity Matching  
- Risk management by ALM techniques / use derivatives  
- Computer monitoring / Stochastic projection (Monte Carlo)  
- Stress testing / scenarios testing  
- Contract design
Solution 9

(a) 1 year forward values are:

\[ V_{aa} = \frac{100}{(1.05)^4} = 82.27 \]

\[ V_a = \frac{100}{(1.053)^4} = 81.34 \]

\[ V_{bb} = \frac{100}{(1.057)^4} = 80.11 \]

\[ V_b = \frac{100}{(1.074)^4} = 75.16 \]

\[ V_c = \frac{100}{(1.09)^4} = 70.84 \]

(b) First percentile is at rating C

(since prob(C) = 0.5% and prob(B or C) = 1.25%)

\[ \Rightarrow 99\% \ VaR = 80.11 - 70.84 = -9.27 \]

(c) Capital charge = Expected forward value – First Percentile value

\[ EV = .01(82.27) + .02(81.34) + .9575(80.11) + .0075(75.16) + .005(70.84) = 80.07 \]

\[ \Rightarrow \text{charge} = 80.07 - 70.84 = 9.23 \]
Solution 10

(a)
1) Hedge investment has an underlying notional and is a derivative
2) Must be carried at Market Value
3) Hedge is matched to when underlying item affects income
4) Is only cash flow or Fair Value hedge
5) Risk is Market Price, Interest Rate, or foreign currency
6) Must have well documented use and purpose
7) Must be judged to be effective

(b)
1) DM Life pays fixed payments as premium and will receive at time of defined credit event a payoff of par or other agreed delivery of protection

Risks being hedged move with change in credit quality of assets and is a fair value hedge

CDS is a derivative
Solution 11

(a) Black-Scholes model’s assumptions, which differ from real world:
- Geometric Brownian motion for stock price changes
- Smooth price changes
- Constant interest rate
- Constant volatility
- no penalty for short selling
- no penalty for borrowing at risk-free rate
- fractional securities are allowed
- European option (Exchange-traded options are mostly American)
- no dividends
- no takeover
- no taxes
- no transaction costs

(b) No, Black-Scholes model does not use Expected Return $E(r)$. $E(r)$ is used in hedging, but not option valuation. Investor decisionmaking is based on $E(r)$ & risk (proxied by variance) as per Markowitz.

use a Generalized Actuarial model

Assumption: Normal Distribution for Stock Return
Can handle combination of securities
Calculates $E(r)$ for the investment strategy, we then compare alternative strategies
Solution 12

(a) His statement is false in the sense that nobody could possibly "know" exactly which properties are great performers. To measure his performance, you need to compare his returns to the index return. Compare in two ways: his individual property selection and property-type (market timing).

1. Property type weighting
   \[ \text{SUM(portf wght} \times \text{ indx wght} - \text{SUM(indx wght} \times \text{ indx rets} \]
   He got 6.05% vs. 5.70% = 35bps better than index
   He outperformed the index by increasing his weight in the high-returning retail segment and decreasing his weight in the lower-returning office segment

2. Individual property selection-compare his property type returns to the index returns using the index weights
   \[ \text{SUM(indx wght} \times \text{ portf rets} - \text{SUM(indx wght} \times \text{ indx rets} \]
   He got 5.70% vs. 5.70% for the index, so he exactly matched the index return. His better performance in picking retail and office properties was offset by his poorer performance in picking warehouse and apartment properties.

So his skill in weight in the portfolio to higher – returning segments and not in individual property selections.

(b) On the recommendation about selling the apartment holdings:
Reducing the apartment holdings might be a good idea since apartments have been low yielding. But if demand is high (since occupancy rates are high) you may be able to increase rent (depending on lease agreements) and improve yields. Plus, the property management must be fairly good if occupancy rates are high. I would stay in this holding and increase rent to see what would happen.

On the recommendation about investing in the new retail complex:
Retail is more risky and already over-weighted
Would change risk/return profile.
New complex means higher risk and return versus the apartment, especially since it is only a proposal.
Should perform scenario analysis
Solution 13

(a) Statement is incorrect. Current rates are not the sole determinate of prepayments. The pace of prepayments is also driven by general housing turnover and refinancing.

General housing turnover is driven by relocation, seasonal variations, the aging process and curtailments.

Refinancing. Rate of refinancing is influenced by the shape of the yield curve, credit quality of borrower, mortgage characteristics (i.e. LTV, equity build up) and is path dependent (burnout).

(b)

i) This bond has a 6 year period before first principal payment is made. Principal payments will follow a schedule as long as prepayment stay within 100 PSA to 250 PSA. This bond has more certain cashflows then pass-thru or standard CMOs, it provides call protection and has better convexity than most CMO structures.

ii) this bond will not start paying principal until the earlier tranches have been paid down to zero. This bond will have less prepayment variability than support tranches but more volatility than PAC’s. It typically offers higher yield than a comparable PAC.

iii) The bond is not paid until all senior bonds are paid. It has a period of principal and interest lockout or an accrual phase and payment phase. It has long duration and is good for long liabilities.

iv) Similar to Z-bond but based on some event it will stop accruing and begin paying P&I. This jump can be sticky or non-sticky.

Buy the newly issued PAC bond with lock out period for this interest forecast. This PAC has the least negative convexity. The rising interest rates will cause all the other bonds to extend more than the PAC. The PAC’s support or companion bonds will help re-direct prepayments and keep the PAC on schedule.
Solution 14

(a) The student’s response is incorrect. This product has the effective duration less
than 5 years as embedded options are included
  • right to surrender policy at book value
  • rate reset feature after the initial 5 year guarantee period
  • minimum crediting rate
  • interest-sensitive cash flows
    • higher new money rate leads to higher lapse
    • lower new money rate leads to lower lapse

(b) The MVA mitigates the disintermediation risk to the policy holders in the event of
rising interest rates by discouraging anti-selective surrender which requires capital
loss on asset sales.
The MVA removes the embedded put option from SPDA which reduces the
convexity.
The company can match the liability better with option-free bonds.
The MVA allows the company to invest longer which enables the company to
credit higher rates.

(c) Without the MVA, the put option embedded in the SPDA reduces the effective
duration. Without the MVA, when the interest rates rise, higher lapse/surrender
shortens the duration.
In other word, the value of the liability doesn’t decrease much comparing to the
SPDA with MVA.

(d) The return of premium feature would be in the money if the value of contract after
applying the MVA factor is less than the initial deposit.
Let initial deposit to be P
Account value at the end of the year 1=P×(1.04)
The MVA factor = \( \left( \frac{1+j}{1+i} \right)^{T-t} \)
where \( j \) = the current fixed crediting rate = 4%
\( i \) = the current market rate
\( T-t \) = the fixed rate period remaining = 4 yr

Solve \( i \) for Premium (Deposit) ≥ Account Value × (MVA factor)
\[ P \geq P \left( 1.04 \right) \left( \frac{1.04}{1+i} \right)^4 \]
Solution 14 (continued)

\[(1+i)^4 \geq 1.04^5\]
\[1+i \geq 1.04^{\frac{5}{4}}\]
\[i \geq 1.04^{\frac{5}{4}} - 1 \approx 0.0502\]

If the current market rate is higher than 5.02\% at the end of year 1, the option is in-the-money.

(e) Since the policyholders could have a higher first year cash surrender value, the policyholders are more likely to surrender in the year 1 and Return of Premium decreases the effective duration. ROP works like a put option.

(f) Since there is no MVA when the current market rate \( j \) is lower than fixed rate \( i \), the minimum guarantee has no effect during the initial guarantee period. After the 5 year initial guarantee period, the minimum guarantee should extend the effective duration when the current rate is below the minimum crediting rate. The policyholders would keep their contract. It acts similar to the interest floor.
Solution 15

(a) risk free rate = 0.05
V0=market value of company's assets today = 21
D=Company's debt interest and principal due to be
σ v=volatility of assets = 0.20
I=1 year

d2=[ln(V0/D)+(r - σ^2/2)T]/σ√(T/2)
d2=[ln(21/17) + (.05 - (0.2^2)/2)*1 ] / 0.2*1
   =1.2065

Calculate N(-d2) to obtain risk-neutral probability of default
N(-d2) = 1-N(d2)
N(d2) = N(1.20) + .65 * [N(1.21) - N(1.20)]
   =0.8849 + 0.65(0.8869-0.8849)
   =0.1138

(b) A=De^-ri
A=17e^(-0.05*1)
A=16.1709

B=V0-E0
B=21-5=16

Expected loss on the debt = (16.1709-16)/16 1709=1.06%

0.0106=1 1138*(1-R)
(1-r)=0.0106/0.1138
R=1-0.0106/0.1138=(0.1138-0.0106)/0.1138
R=90.7%
Solution 15 (continued)

\( Q(T) = (1 - \exp[-(y(T) - y^*(t))T]) / (1 - R) \)
\( Q(5) = (1 - \exp[-0.035*5]) / (1 - 0.4) \)
\( Q(5) = 0.26757 \)
\( Q(1) = 0.26757/5 = 5.35\% \)

Lower than Merton's default probability

Reasons for discrepancy:
- Provision for liquidity premium
- Provision for possibility of recession or depression scenario
- Merton's model impacted by volatility
Solution 16

(a)

\[ p = K \cdot e^{rt} \cdot N(-d_2) - S_0 \cdot N(d_1) \]

\[ d_1 = \left[ \ln(S_0/K) + \left( r + \frac{\sigma^2}{2} \right) T \right] / \sigma \sqrt{T} \]

\[ = 1100e^{-0.05 \cdot 0.37336} - 1200(0.28331) \]

\[ = 400.56 - 339.972 \]

\[ = 60.58 \]

\[ 60.58 \times 5000 = 302,919 \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

\[ d_2 = 0.5730 \]

\[ \frac{d_1}{d_2} = 0.3230 \]

\[ N(d_1) = 0.71669 \quad N(d_2) = 0.62664 \]

\[ \text{profit} = 302919 - 242900 \]

\[ = 60,019 \]

(b) Delta is \( N(d_1) - 1 \) for a put option

\[ d_1 = \left[ \ln(1200/1100) + 0.025 + \frac{0.22^2}{2} \right] / 0.22 = 0.6191 \]

\[ N(d_1) = 0.7321 \]

\[ \text{delta} = 0.7321 - 1 = -0.2679 \]

\[ \times 5000 = -1339 \]

need to buy 1339 notional amt of index = 1339 \times 1200 = 1,607,382
Solution 16 (continued)

(c) At each quarter ⇒ they need to recalculate Δ of option, then sell or purchase shares of index to make Δ of portfolio = 0.

<table>
<thead>
<tr>
<th>Time</th>
<th>Index</th>
<th>N(d1)</th>
<th>Δ (option) = ( \frac{d1}{1-N(d1)} )</th>
<th>Shares purchased</th>
<th>Cost (# shares × index level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1200</td>
<td>0.7321</td>
<td>0.2679</td>
<td>-1339.5</td>
<td>-1,607,400 negative</td>
</tr>
<tr>
<td>1</td>
<td>1250</td>
<td>0.8051</td>
<td>0.1949</td>
<td>365</td>
<td>456,250 implies profit</td>
</tr>
<tr>
<td>2</td>
<td>1150</td>
<td>0.6700</td>
<td>0.3300</td>
<td>-675.5</td>
<td>-776,800 profit</td>
</tr>
<tr>
<td>3</td>
<td>1050</td>
<td>0.3783</td>
<td>0.6217</td>
<td>-1458.5</td>
<td>-1,531,400</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>1.000</td>
<td>1.000</td>
<td>-1891.5</td>
<td>-1,891,500</td>
</tr>
</tbody>
</table>

in the money total=−5000

at time = 1, \( Δ = 0.1949 \Rightarrow \) so you want \((5000)(^-0.1949) = -974.5\) shares,
so you need to buy \((^-974.5) - (-1339.5) = 365\) shares

time = 2 ⇒ sell\((5000)(0.33) - 974.5 = 675.5\) shares

time = 3 ⇒ sell \((5000)(0.6217) - (5000)(0.33) = 1458.5\) shares

time = 4 ⇒ sell \((5000)(1.000) - (5000)(06217) = 1891.5\) shares

(negative = profit)

<table>
<thead>
<tr>
<th>Time</th>
<th>Stock Purchase Cost</th>
<th>Trading Cost</th>
<th>Cum Cost</th>
<th>Int Cost = ( \frac{0.025}{4} ) (Cum Cost)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1,607,400</td>
<td>1607</td>
<td>-1,605,800</td>
<td>-10036</td>
</tr>
<tr>
<td>1</td>
<td>456,250</td>
<td>456</td>
<td>-1,159,130</td>
<td>-7245</td>
</tr>
<tr>
<td>2</td>
<td>-776,800</td>
<td>777</td>
<td>-1,942,398</td>
<td>-12140</td>
</tr>
<tr>
<td>3</td>
<td>-1,531,400</td>
<td>1531</td>
<td>-3,484,407</td>
<td>-21777</td>
</tr>
<tr>
<td>4</td>
<td>-1,891,500</td>
<td>1892</td>
<td>-5,395,942</td>
<td></td>
</tr>
</tbody>
</table>

so firm has cumulative cost of −5,395,792 ⇒ profit =$5,395,792

Now, to settle put, you’re obligated to purchase 5000 shares at price of 1100 ⇒ so, you need 5000 (1100) = 5,500,000 (You’ve sold and then purchased 5000 shares so net gain in shares)

Therefore, net loss=5395,792-5,500,000=104,200

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Solution 16 (continued)

(d) Securitization package CF's expected and sell at market to offset risky CF's

Market maker - sell products that counterbalance risks of this product

Reinsurance - hard to find good price and willing counterparty

Naked position - do nothing. Okay if option is out-of-the-money, but in trouble if it is in-the-money

Covered position - A hedge initially, but do nothing afterwards-risky if \( \Delta \) changes dramatically

Stoploss - only change holdings if option is in or out of the money

Gamma hedging - make portfolio gamma neutral \( \Rightarrow \) requires position in another instrument

Rho, vega hedging - similar to gamma hedging
Solution 17

(a) Risks in Global investing:
legal protection for investor
corporate objectives of management
communication with shareholders
political risk
currency risk
credit risk

Solutions:
understand local market conditions
understand local legal framework
understand reliability of communications with corporations
understand corporate objectives
use experienced staff
do research
currency hedging strategies
credit hedging strategies

(b) Political Risk model: 10 variables, correlations show:
1. Democracy: if lack democracy & legitimacy, then less stable
2. quality of life: higher means more stable
3. GDP: higher means more stable
4. Rental Income: higher means less stable
5. Distribution of income: if inequality, then less stable
6. Predictability of wholesale prices: higher means more stable
7. Agriculture (as a % of GDP): higher means less stable
8. Trauma: countries had trauma can be successful
9. Competition (measure=$\frac{(\text{Import} + \text{Export})}{\text{GDP}}$): higher means more stable
10. Human Capital: higher means more stable
### Solution 17 (continued)

(b) | Country: | A | B | C |
--- | --- | --- | --- | --- |
Agriculture (as % GDP) | Rank: middle | $\frac{1+3}{26} = 15.4\%$ | $\frac{150 + 450}{1500} = 40\%$ | $\frac{15 + 40}{600} = 9\%$ |

\begin{align*}
\text{Competition} & \quad \text{Rank: middle} \\
\left(\frac{\text{Import} + \text{Export}}{GDP}\right) & \quad \frac{1+1+8}{26} = 38.4\% \\
& \quad \frac{150 + 150 + 150}{1500} = 30\% \\
& \quad \frac{5 + 15 + 300}{600} = 53\% \\
\\
\text{Democracy} & \quad \text{Rank: Stable} \\
& \quad \text{Stable} \\
& \quad \text{Worst} \\
& \quad (\text{: Emerging}) \\
\\
\text{Infant deaths} & \quad \text{Rank: Best} \\
& \quad \text{middle} \\
& \quad \text{worst} \\
\\
\text{Rental:Oil/GDP} & \quad \text{Rank: middle} \\
& \quad 3.8\% \\
& \quad 10\% \\
& \quad 0.8\% \quad \text{Best} \\
\\
\text{Life Expectancy} & \quad \text{Rank: Best} \\
& \quad \text{middle} \\
& \quad \text{Worst} \\
\\
\text{Inflation range} & \quad \text{Rank: middle} \\
& \quad \text{worst} \\
& \quad \text{Best} \\
\\
\text{D=GDP/Capita} & \quad \text{Rank: middle} \\
& \quad \frac{26}{3} = 8.7 \\
& \quad \frac{14}{4} = 3.8 \\
& \quad \frac{600}{50} = 12 \quad \text{Best}
\end{align*}

\text{Country C is the most stable} \quad \text{A is the next most stable} \quad \text{B is the least stable}
Solution 18

(a) Reduces volatility of CFs
Reduces cost of financial distress
   Legal & Accounting costs
   Higher costs with customers, employers and suppliers
Reduces Taxes
   Taxes reduced if tax schedule convex
Improve Investment Decision
   Improve incentives to undertake only profitable projects
Improve Debt Capacity
   Reduces conflicts with stockholders & bond holders
Dividend Policy
   Important means to express confidence in company’s growth
Managerial Self Interest
   Management has incentive to manage strategic exposure otherwise will not
   have job
Imperfect Market Conditions
   External capital more expensive than internal capital

(b) Purchase 1000 call options with strike=400 at $t = \frac{1}{2}$
Purchase 1000 call options with strike=400 at $t=1$
Enter into swap to receive LIBOR and pay 3.25%
   Assume company is payor of LIBOR
Enter a forward contract to hedge forward currency rate
   $= 75 \left( \frac{1.05}{1.03} \right) = .764$ This guarantees US payoff of $\frac{7M}{.764} = 9.156M$

(c) Value of Risk = Cost to hedge
Call options = 1000(4)+1000(6) = 10,000
LIBOR swap = no cost to hedge however.
   In one year earn $10M \times 3\%$ risk free
   pay $10M \times 3.25\%$ through swap
   cost = (3.25\%-3.0\%) \times 10M/1.03 = 24,272
Foreign currency exchange forward
   Receive 9.16M in 1 yr (hedged at no cost)
   Expect $\frac{7M}{.75} = 9.33M$ in 1 yr
   Risk = $177,777 \times 1.03 = 172,600$
   Total Risk = 10,000+24,272+172,600 = 206,872
Solution 19

(a) From Ito’s lemma, if \( G \) is a function of \( S \) and \( t \), the process is
\[
dG = \left( \frac{\partial G}{\partial S} \right) \mu S + \frac{\partial G}{\partial t} + \frac{1}{2} \left( \frac{\partial^2 G}{\partial S^2} \right) \sigma^2 S dt + \frac{\sigma S}{\partial S} \sigma S dz
\]
Since \( G = S \exp\left( r(T-t)\right) \)
\[
\frac{\partial G}{\partial S} = \exp\left( r(T-t)\right)
\]
\[
\frac{\partial G}{\partial t} = -rS \exp\left( r(T-t)\right)
\]
\[
\frac{\partial^2 G}{\partial S^2} = 0
\]
This gives
\[
dG = \left( \exp\left( r(T-t)\right) \right) \mu S - rS \exp\left( r(T-t)\right) + 0 \right) \sigma S dz + \exp\left( r(T-t)\right) \sigma S dz
\]
or
\[
dG = \left( \mu G - rG \right) dt + \sigma G dz = \left( \mu - r \right) G dt + \sigma G dz
\]

(b) If \( G \) is a stock with an instantaneous dividend payout, \( \mu \) would be the risk free rate, \( r \) is the instantaneous dividend yield, and \( \sigma \) the volatility of the stock. The stock is growing at the rate of \( \mu - r \) at the risk-neutral world.

(c) \[dG = \left( r - r_f \right) G dt + \sigma G dz\]

where \( r \) is the domestic risk-free rate and \( r_f \) is the foreign risk-free rate.

(d) Each unit of call option on 1 Japanese Yen is worth
\[C = G_0 \exp\left( -r_f (T-t)\right) \left( d_1 \right) - K \left( d_2 \right) \]
\[d_1 = \left[ \ln\left( \frac{G_0}{K}\right) + \left( r - r_f + \sigma^2 / 2\right) (T-t) \right] / \left( \sigma \ast (T-t)^{0.5} \right) \]
\[d_2 = = d_1 - \sigma \ast (T-t)^{0.5} \]

Where \( r \) is the domestic risk-free interest rate,
\( r_f \) is the foreign (Japanese) risk-free rate,
\( \sigma \) is the exchange rate’s annualized volatility,
\( T-t \) is the time to maturity of the option
\( S_0 \) is the current price of Japanese Yen,
and \( K \) is the strike price on the Japanese Yen of the option
\( G_0 = K = 1/110 \text{ US}\$
\]
r=0.01
\[r_f = 0.0005\]
\( T-t = 0.25 \)

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Solution 19 (continued)

Annualized volatility \( \sigma = (\# \text{ of trading days in a year})^{0.5} \times \text{daily volatility} \)
\[ = (4 \times 65)^{0.5} \times 0.62\% = 10\% \]

\[ d_1 = \frac{\ln(1) + (0.01 - 0.0005 + 0.1^2/2) \times 0.25}{0.1 \times 0.25^{0.5}} = 0.0725 \]
\[ d_2 = 0.0725 - 0.1 \times 0.25^{0.5} = 0.0225 \]

\[ C = \frac{1}{110} \times \exp(-0.0005 \times 0.25) \times N(0.0725) - \frac{1}{110} \times \exp(-0.01 \times 0.25) \times N(0.0225) \]
\[ = (1/110) \times 0.999875 \times 0.528898 - (1/110) \times 0.997503 \times 0.5090 \]
\[ = 0.004808 - 0.004616 = 0.000192 \text{ US}\$ \]

Value of the contract on 220 billion Yen
\[ = 0.000192 \times 220 \text{ billion US}\$ \]
\[ = 42,240,000 \text{ US}\$ \text{ or about 42 million US}\$ \]
Solution 20

The guarantee at maturity: \( L_t^* = L_0 \cdot e^{\gamma t} \)
\[ = 66,673,640 \times e^{0.03 \times 10} \]
\[ = 90,000,000 \]

\[ \alpha = \frac{L_0}{A_0} = \frac{66,673,640}{74,081,822} = 0.9 \]

\[ = (1 - \alpha) A_0 = C_e(A_o, L_t^*) - \delta \alpha C_e \left( A_o, \frac{L_t^*}{\alpha} \right) \]

\( \delta \) is the equilibrium participation level

\[ \frac{L_t^*}{\alpha} = \frac{90,000,000}{0.9} = 100,000,000 \]

From the given table we have,

\[ : C_e(A_o, L_t^*) = C_e(A_o, 90,000,000) = 7,625,000 \]

\[ : C_e \left( A_o, \frac{L_t^*}{\alpha} \right) = C_e \left( A_o, 100,000,000 \right) = 1,204,330 \]

\[ : \delta = \frac{C_e(A_o, L_t^*) - (1 - \alpha) A_0}{\alpha C_e \left( A_o, \frac{L_t^*}{\alpha} \right)} = \frac{7,625,000 - (1 - 0.9) \times 74,081,822}{0.9 \times 1,204,330} \]
\[ = 0.2 \]

hence the equilibrium participation level is 0.2
Solution 1

(a)

Accounting
Effects on different accounting basis

Tax
Tax gain/loss carry forward position
Netted within calendar year
Netted between affiliates

EV/EVA
Need to reproduce cash flows

ALM

Credited Rate
Effect on credited rate

Policyholder Equity
Effect on segment’s credit quality, maturity structure, concentration

(b)

Description of UL product characteristics:
Product has embedded options
Liability cash flows are interest sensitive

Asset Mix

Duration of 12 is CRAZY long. The company is exposed to
HUGE disinter mediation risk if interest rates rise.
A/L dollar-duration mismatch is way outside Investment Policy
Constraints
Sell much of the government and public corporate portfolios,
reinvest much shorter, to bring duration down toward 4 0

Need to consider product liquidity needs
Given privates’ low liquidity a higher quality would be appropriate
As the pvts would be very difficult to sell, reinvest any excess cash
flow on maturity in short-dur, non-callable publics

Derivatives

Policy is in place to permit use
Solution 1 (continued)

Use to quickly correct duration mismatch

Protect with a floor or pay-float swap
Solution 2

(a) **Option pricing method:**
- Also known as direct method or multi-scenario method
- Generate stochastic economic scenarios
- Project liability CF along each scenario path
- Calculate pathwise liability PV by discounting liability CF along each path
- May reflect risk by adjusting either the discount rate or the projected liability CF
- If adjusting discount rate, may add a spread that reflects
  ⇒ Issuer’s credit
  ⇒ Quality of issuer’s bond portfolio
  ⇒ Risk contingency margin of the liability
  ⇒ Liquidity of the liability
- Assign probability to each scenario path
- Calculate fair value of liability as weighted sum of pathwise liability present values
- Not often used for transfer pricing for a block of insurance liabilities

**Actuarial Appraisal Methods:**
- Also known as indirect method
- Generate stochastic economic scenarios
- Project free CF under each scenario
- Free CF may reflect:
  ⇒ Investment earnings
  ⇒ income tax
  ⇒ RBC
- Calculate pathwise PV by discounting free CF at risk adjusted firm’s cost of capital
- Calculate DDE as average of pathwise PVs
- FV of liability = MV of assets – DDE – DIL
- Method of choice for transfer pricing for a block of insurance liabilities
- Reconcilable with Option Pricing Method under certain assumptions
Solution 2 (continued)

(b)  
- Volume of computation could be prohibitive  
- System may not be available to do this routinely  
- Expertise may be lacking  
- Market data for calibration may not exist for certain insurance options

(c)  
Scenario 1:  
$LV_0$ (liability value at year 0) = 1,500  
$LV_1$ before withdrawal (WD) = 1,500 \times (1 + 6.6\%) = 1,599  
$CF_1 = 1,599 \times 4\% = 64$  
$LV_1$ after WD = 1,599 - 64 = 1,535  
$LV_2$ before WD = 1,535 \times (1 + 6.6\%) = 1,636  
$CF_2 = 1,636$  
$PVCF_{scen1} = 64/1.01 + 1,636/[(1.01)(1.03)] = 1,636$

Scenario 2:  
$LV_0 = 1,500$  
$LV_1$ before WD = 1,599 - 32 = 1,599  
$CF_1 = 1,599 \times 2\% = 32$  
$LV_1$ after WD = 1,599 - 32 = 1,567  
$LV_2$ before WD = 1,567 \times (1 + 6.6\%) = 1,670  
$CF_2 = 1,670$  
$PVCF_{scen2} = 32/1.01 + 1,670/[(1.01)(1.02)] = 1,653$

$FV = 0.3 \times 1,636 + 0.7 \times 1,653 = 1,648$
Solution 3

(a) LifeCo’s reported effective durations based on:
   • parallel yield curve shift
   • small (1bp) yield curve shift
They do reflect interest-sensitive cash flows, but:
   • cash flow models may not be perfect
   • may mean significantly higher convexity and also optionality, so impact under large shift may be very different than predicted by effective duration

(b) Impact of 100bp -drop- based on reported durations

Using effective duration * PV cash flows:
Assets: \( d = 9.26 \) so impact = 416,600 * 9.3 * 1% = + 38,744 (or 38.7 million)
Liabs: \( d = 4.00 \) so impact = 406,000 * 4.00 * 1% = + 16,240 (or 16.2 million)
Total: 38.7 - 16.2 = 22.5 i.e. economic gain of 22.5 million

(c) Reported “margin squeeze” impact based on 100bp drop was -10.3 million loss
Estimate using effective duration in (b) was 22.5 million gain
Reasons for difference:
   • margin squeeze modeled under 100bp shift, not estimated from 1bp shift
   • other reasons from part (a) like convexity, optionality
   • convexity wouldn’t account for opposite direction of impact
   • but optionality could: may hit minimum guarantees under 100bp shift but not for 1bp
   • if min guarantees “in the money” under 100bp, would explain large negative “margin squeeze” impact

(d) What “bets” are there...
   • durations themselves indicate bet on parallel interest rate decrease, hopefully partials are consistent with this!
   • Partial sensitivities show $ change under a 1bp increase
   • Short (1-5 year) partials show a gain if short rates rise
   • Long (7-20 year) partials show a loss if long rates rise (gain if long rates fall)
   • So overall, bet is “flattening” of curve

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Solution 3 (continued)

(e) Only 15-year partial affected, since going to cash (d=0) won’t recalculate 0.25-yr partial
Will show change to partial sensitivity since that’s what’s used in the case study
15-yr sensitivity is reported as −698,000 for 1bp increase
The 50 million of 15-yr zeros alone contribute approx 1bp * -15 * 50 million = -75,000
We should do exact calc using 15-yr spot rate of 5.42%....
PV under 1bp increase is 50 million * (1.0542 ^ 15 / 1.0543 ^ 15) = 49,929,910
So zeros contribute −70,090
Selling them will *increase* 15-yr partial by about 70,000 to −628,000 (not much change)

(f) Revised margin squeeze... doing exact calc, the zeros contributed a *gain* under the 100bp drop:
PV under 100 bp drop is 50 million * (1.0542 ^ 15 / 1.0442 ^ 15) = 57.7 million
gain of 7.7 million for zeros
So selling them..... margin squeeze impact will be 7.7 million worse or about 18 million loss!
Proposed sale doesn’t help margin squeeze!

(g) Margin squeeze showed a “bet” that rates wouldn’t drop 100bp (or at least showed a loss if they did!)
Looking only at effective duration and partials said gain if longer rates drop 1bp... these didn’t tell the whole story
Solution 4

(a) ELGIC: Call on 75% of increase of S&P500 over 5 year period

Variable annuity: put, contingent on death, on the invested funds with strike price $S_0 = (1.05)^7$
Additional option provided by dollar for dollar partial surrender

(b) Potential changes to reduce the impact of a large equity market decline just before maturity:
- Annual resets to lock-in gain at each policy anniversary
- Monthly averaging: use average increase over period
- High water mark to lock-in maximal gain

Reduce cost by using:
- Simple annual ratchet instead of compound one
- Averaging when calculating the actual return

(c) Evaluate cost and efficiency of dynamic hedging by comparing the alternatives (no hedging or static hedging) over many economic scenarios. Use Monte Carlo simulation techniques to model hedging including:
- The impact of rebalancing and the hedging error introduced by the drift
- Transaction costs (proportional to change in stock position)

Stress testing is also important to highlight potential risks and exposure.

(d) Dynamic hedging is a viable alternative for large blocks of business with embedded options that are difficult to replicate with standard options. This might be the case for Life Co ELGIC. With small notional amounts, transaction costs may deteriorate any benefit. The simplicity of the ELGIC option is such that it may be available in the market. One benefit of hedging is to combine offsetting exposure. Compare costs and availability of both alternatives.
Solution 4 (continued)

(e)

Can hedge ELGIC by buying a bond and a 5 year European call option on 75% of the notional amount. No need to rebalance unless early withdrawals are very different from initial estimates.

Variable Universal Life investment options depend on timing and amount of premiums and withdrawals. A static hedge will not work. Dynamic hedging on the portfolio of investment options would be preferable.
Solution 5

(a)

1) **Lognormal model**
A standard model for evaluating equity returns, assumes percentage returns are normally distributed, which is a reasonable assumption in many situations.
Allows analytical solution for European call and put options in a form that provides the way to construct a replicating portfolio consisting of underlying stocks and risk-free bonds. However, the model does not account for:
- big jumps in stock prices
- auto regression effects in returns
  volatility clustering effect
The limitations result from the assumption of constant volatility of return experience data shows that in practice volatility of returns is stochastic with some autoregressive features.

2) **Regime Switching (RS)**
One of the proposed models is a regime switching-between two lognormal distribution, with $\mu_1, \sigma_1$ and $\mu_2, \sigma_2$. The model is shown to fit well to actual results for returns on broad-based indices (like S&P500). The data shows that in fact there are periods in time, when volatility of index returns are relatively low which are then switch to periods with much higher volatility. The model give fatter tails for the distribution of returns and thus provides a better fit to the actual data than the simple lognormal distribution. The RS – distribution can be easily simulated, like the lognormal one, but – it does allow for analytical solutions. Also, the model does not incorporate auto regression. More parameters than ARCH or GARCH.
Solution 5 (continued)

3) Time Series with GARCH volatility
In GARCH (1, 1) model the volatility is calculated from:
\[ \sigma_i^2 = \gamma V + \alpha \sigma_{i-1}^2 + \beta \left( Y_{i-1} - \mu \right)^2, \]
where \( V \) is the long-term average variance; \( \sigma_{i-1} \) and \( Y_{i-1} \) - values of the volatility and return at the previous time moment
Constants \( \alpha, \beta, \gamma \) are found by the regression to the actual data. This model is very general in nature and allows for:
- mean reversion, through long-term average term \( V \)
- auto regression, through dependence on the prior value of volatility; \( \sigma_{i-1} \)
- effect on high volatility when returns are far from their long-term mean, \( \mu \), through term \( \beta \left( Y_{i-1} - \mu \right)^2 \),
The model can be easily simulated but does not allow for analytical solutions
The volatility in the model is (unconditionally) stochastic due to \( Y_{i-1} \)

4) Empirical
Actual data for returns are recorded for some period of time and then used as a sample space, from which the values for future returns are taken. The model is:
- fit well to the past experience by definition; each observation is equally likely
- is limited, since the data is limited
- does not allow new developments in the future; only the returns that were recorded in the past are allowed for the future
- If the return values are sampled with replacement randomly the model does not produce any auto correlations.
This can be improved by sampling number “in bunches”.

5) Wilkie
This is an econometric model, which combines processes for different economic factors, such as inflation level; short and long-term interest rates; divided yields and stock returns. The model has a cascade structure
A process for each new factor includes a term, connecting it to the parameters from the prior (upper) levels; and also a stochastic term.
The model is very complex.
Main advantage – combines different econometric parameters in one inter dependent model; for example, short interest rates and stock returns, which can be very useful for many actuarial applications (ALM).
Drawback – very difficult to estimate model parameters. Contains a lot of them and requires a lot of experience data for estimation.
6) Stable distributions
This is a particular class of functions, which satisfy specific conditions on
their linear transformations. One example – normal distribution.
Advantages – wide class of functions with a convenient “convolution”
feature; allows to model very “fat” tails
Drawbacks – difficult to simulate
- does not allow autocorrelation

b) GMDB in the simplest form is the return of premiums (less withdrawals)
as the minimum death benefit. It is a put option on the fund value at the
time of death. The payoff equals max (P-F, 0), where P=premiums net of
withdrawals, F=fund value at the time of death. The value of the option
should be adjusted for survivorship.
The total value of the GMDB at $t = 0$:
$$\int_0^t p(t) \cdot p_t^* \cdot \mu_{it}^{(d)} \cdot dt;$$
where the value of the European put option with
term t, $p(t)$, is multiplied by probability of surviving to time $t$ and dying
in the interval $dt$; and summarize for the term of the annuity, n years.
We assume that mortality and lapsing functions are known.
To hedge GMDB dynamically we need:
to simulate the fund returns
- to calculate the value of the put option for each $t$
to have a possibility to present the option value in terms of the
replicating portfolio.
Of all the models, only the lognormal model gives the analytical value of
the put option in terms of the hedge portfolio, split between the stock fund
and risk-free bonds.
So - to calculate the parameters of the replicating portfolio, the
lognormal model (Black-Scholes) should be used.
- to simulate the fund returns – lognormal, RS, GARCH or empirical
can be used.
To better represent actual fund behaviour - use RS or GARCH
Need - to rebalance frequently and calculate hedging errors and
transaction costs.
Solution 6

(a)

\[ r_A = r_f + OAS - D_{OAS} \Delta OAS - \Sigma D(i) \Delta t(i) + \frac{r}{c} + pa - e_a \]
\[ r_e = r_f - ROAS - \Sigma D(i) \Delta t(i) + e_t \]
\[ r_f = 3\%; OAS = SObps; D_{OAS} \Delta OAS = 0; r/c, pa, e_a = 0; D(2) = 2 \]

Asset return: expected forward rate = 6.01%
actual rate = 4.00%
\[ r_A = 3.0\% + 0.5\% - 0 - (4\% - 6\%) \cdot 2 = 7.5\% \]

Liability Return

Liability Yield = \( \left( \frac{110}{100} \right)^{-5} - 1 = 4.88\% \)
\[ ROAS = 88\% \]
expected forward rate = 5.01%
actual rate = 4.00%
\[ r_i = 3.0\% + 88\% - (4\% - 5\%) \cdot 1 = 4.88\% \]

(b)

<table>
<thead>
<tr>
<th>risk free</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>OAS = .5%</td>
<td>0</td>
<td>.5%</td>
</tr>
<tr>
<td>C2</td>
<td>0</td>
<td>ROAS = 88%</td>
<td>-88%</td>
</tr>
<tr>
<td>C3</td>
<td>( \Sigma D(i) \Delta t(i) )</td>
<td>( \Sigma D(i) \Delta t(i) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4%</td>
<td>1%</td>
<td>3%</td>
</tr>
<tr>
<td>Total</td>
<td>7.5%</td>
<td>4.88%</td>
<td>2.62%</td>
</tr>
</tbody>
</table>

COURSE 8: Fall 2004
Investment
Morning Session
Solution 6 (continued)

(c) Total Return Approach:
- splits return into components (e.g., c risks)
- performance measurement
- setting consistent goals in managing both assets & liabilities
- prospective & retrospective analysis
- measure results relative to
  - bond selection
  - interest rate anticipation
  - sector rotation
  - r/c expenses
- market value measure
Solution 7

(a) Purpose is to single number $V$, says that over the next $N$ days, we will not lose more then $V$ dollars of value with $x\%$ confidence.

(b) 
(i) using histogram (1-day)
$254 \times 5\% \approx 13$ days
the 15th worst return is $-1.5\%$
so $VaR = -1.5\% - 0.04\% = -1.54\%$
or $10m \times 1.54\% \approx 154k$

(ii) using normal distribution (10-day)
use $N(x) = 0.95 \Rightarrow x = 1.645$
So 10-day $VaR = 10m \times 1.645\sigma \sqrt{10}$
$= 10m \times 1.645 \times 0.0107 \sqrt{10}$
$= 556k$

(c) Histogram
adv: avoid use cash-flow mapping
use historical data

disadv: Computing slow
does not allow volatility updating
sensitive to historical data

Normal:
adv: quick to calculate
can use volatility update scheme

disadv: normal distribution assumption
give poor result for low-delta portfolios

Monte-Carlo:
adv: any model can be used

disadv: Computing intensive
Solution 7 (continued)

(d)  
- assess use the estimation errors
- for quantile-based

\[ se(q) = \frac{c(1-c)}{Tf(q)^3} \]

- for sigma-based

\[ se(\sigma_s) = \alpha \times se(s) \]

For normal \[ se(s(\Phi)) = \alpha \sqrt{\frac{1}{2T}} \]

Also, can use back testing and stress testing.

(e)  
Limitation:
- only 1 point, no tail distribution
- results depend on methodologies
- results depend on assumptions
- results depend on time horizon
- many factors not captured such as legal, operations
- may give management false sense of security

CIE:
- more robust
- consider the shape of tail distribution
- it is the expected loss give loss happens
- meet criteria

Stress testing:
- can test extreme cases not captured in VaR

(f)  
\[ CTE_\alpha(L) = E(L_c / L_o > V_\alpha) \]
13 worst losses
\[ CTE = 10m*(1*4.75% + 1*3.75% + 2*2.75% + 4*2.25% + 5*1.75%)/13 \approx 244K \]
Solution 8

(a)  
- A stable value fund if an options offered by 401(k) and other DC plans
- It is typically the most conservative investment options
- It is good with ERISA (fiduciary duty)
- Invested mostly in GIC and other medium-term fixed-income contracts issued by a high-quality financial institution
- Provide participants ability to withdraw and transfer funds (subject to plan rules) without penalty or market value risk. Principal is guaranteed by issuer (so there is credit risk)
- Good for participant seeking a safety investment option (near retirement) / reduce volatility
- Good for participant seeking to diversify their investment (low correlation with equity)

(b)  
- Credit risk of the GIC/BIC issuer
- For Synthetic GIC / SA GIC we also have  
  - Gains and losses amortized into the credited rate
  - Underlying asset default
  - Reinvestment, interest or market risk (performance)
  - Call/extension risk / withdrawal risk / competing funds

(c)  
- Asset risk:
  (i) Default / credit risk
  (ii) Call or extension risk
  (iii) Performance, interest, market or Reinvestment risk
- Liability risk:
  (i) Contribution risk
  (ii) Withdrawal / liquidity risk

(d)  
- Good Underwriting / reinsurance
- Cash flow matching / Duration Matching / Convexity Matching
- Risk management by ALM techniques / use derivatives
- Computer monitoring / Stochastic projection (Monte Carlo)
- Stress testing / scenarios testing
- Contract design
Solution 9

(a) 1 year forward values are:

\[ V_{aa} = \frac{100}{(1.05)^4} = 82.27 \]

\[ V_a = \frac{100}{(1.053)^4} = 81.34 \]

\[ V_{bb} = \frac{100}{(1.057)^4} = 80.11 \]

\[ V_b = \frac{100}{(1.074)^4} = 75.16 \]

\[ V_c = \frac{100}{(1.09)^4} = 70.84 \]

(b) First percentile is at rating C

(since prob(c) = 0.5% and prob(B or C) = 1.25%)

\[ \Rightarrow 99\% \ VaR = 80.11 - 70.84 = -9.27 \]

(c) Capital charge = Expected forward value – First Percentile value

\[ EV = .01(82.27) + .02(81.34) + .9575(80.11) + .0075(75.16) + .005(70.84) = 80.07 \]

\[ \Rightarrow \text{charge} = 80.07 - 70.84 = 9.23 \]
Solution 10

(a)
1) Hedge investment has an underlying notional and is a derivative
2) Must be carried at Market Value
3) Hedge is matched to when underlying item affects income
4) Is only cash flow or Fair Value hedge
5) Risk is Market Price, Interest Rate, or foreign currency
6) Must have well documented use and purpose
7) Must be judged to be effective

(b)
1) DM Life pays fixed payments as premium and will receive at time of defined credit event a payoff of par or other agreed delivery of protection

Risks being hedged move with change in credit quality of assets and is a fair value hedge

CDS is a derivative
Solution 11

(a) Black-Scholes model's assumptions, which differ from real world:
- Geometric Brownian motion for stock price changes
- Smooth price changes
- Constant interest rate
- Constant volatility
- no penalty for short selling
- no penalty for borrowing at risk-free rate
- fractional securities are allowed
- European option (Exchange-traded options are mostly American)
- no dividends
- no takeover
- no taxes
- no transaction costs

(b) No, Black-Scholes model does not use Expected Return $E(r)$. $E(r)$ is used in hedging, but not option valuation.
Investor decisionmaking is based on $E(r)$ & risk (proxied by variance) as per Markowitz.
- use a Generalized Actuarial model

Assumption: Normal Distribution for Stock Return
Can handle combination of securities
Calculates $E(r)$ for the investment strategy,
we then compare alternative strategies
Solution 12

(a) His statement is false in the sense that nobody could possible "know" exactly which properties are great performers. To measure his performance, you need to compare his returns to the index return. Compare in two ways: his individual property selection and property-type (market timing).

1. Property type weighting
   \[ \text{SUM}[\text{portf wght} \times \text{idx rets}] - \text{SUM}[\text{idx wght} \times \text{idx rets}] \]
   He got 6.05% vs. 5.70% = 35bps better than index
   He outperformed the index by increasing his weight in the high-returning retail segment and decreasing his weight in the lower-returning office segment

2. Individual property selection-compare his property type returns to the index returns using the index weights
   \[ \text{SUM}[\text{idx wght} \times \text{portf rets}] - \text{SUM}[\text{idx wght} \times \text{idx rets}] \]
   He got 5.70% vs. 5.70% for the index, so he exactly matched the index return. His better performance in picking retail and office properties was offset by his poorer performance in picking warehouse and apartment properties.

So his skill in weight in the portfolio to higher – returning segments and not in individual property selections.

(b) On the recommendation about selling the apartment holdings:
Reducing the apartment holdings might be a good idea since apartments have been low yielding. But if demand is high (since occupancy rates are high) you may be able to increase rent (depending on lease agreements) and improve yields. Plus, the property management must be fairly good if occupancy rates are high. I would stay in this holding and increase rent to see what would happen.

On the recommendation about investing in the new retail complex:
Retail is more risky and already over-weighted
Would change risk/return profile.
New complex means higher risk and return versus the apartment, especially since it is only a proposal.
Should perform scenario analysis
Solution 13

(a) Statement is incorrect. Current rates are not the sole determinate of prepayments. The pace of prepayments is also driven by general housing turnover and refinancing.

General housing turnover is driven by relocation, seasonal variations, the aging process and curtailments.

Refinancing. Rate of refinancing is influenced by the shape of the yield curve, credit quality of borrower, mortgage characteristics (i.e. LTV, equity build-up) and is path dependent (burnout).

(b)

i ) This bond has a 6 year period before first principal payment is made. Principal payments will follow a schedule as long as prepayment stay within 100 PSA to 250 PSA. This bond has more certain cashflows than pass-thru or standard CMOs, it provides call protection and has better convexity than most CMO structures.

ii ) This bond will not start paying principal until the earlier tranches have been paid down to zero. This bond will have less prepayment variability than support tranches but more volatility than PAC’s. It typically offers higher yield than a comparable PAC.

iii ) The bond is not paid until all senior bonds are paid. It has a period of principal and interest lockout or an accrual phase and payment phase. It has long duration and is good for long liabilities.

iv ) Similar to Z-bond but based on some event it will stop accruing and begin paying P&I. This jump can be sticky or non-sticky.

Buy the newly issued PAC bond with lock out period for this interest forecast. This PAC has the least negative convexity. The rising interest rates will cause all the other bonds to extend more than the PAC. The PAC’s support or companion bonds will help re-direct prepayments and keep the PAC on schedule.
Solution 14

(a) The student’s response is incorrect. This product has the effective duration less than 5 years as embedded options are included
   - right to surrender policy at book value
   - rate reset feature after the initial 5 year guarantee period
   - minimum crediting rate
   - interest-sensitive cash flows
     - higher new money rate leads to higher lapse
     - lower new money rate leads to lower lapse

(b) The MVA mitigates the disintermediation risk to the policy holders in the event of rising interest rates by discouraging anti-selective surrender which requires capital loss on asset sales
   The MVA removes the embedded put option from SPDA which reduces the convexity.
   The company can match the liability better with option-free bonds.
   The MVA allows the company to invest longer which enables the company to credit higher rates.

(c) Without the MVA, the put option embedded in the SPDA reduces the effective duration. Without the MVA, when the interest rates rise, higher lapse/surrender shortens the duration.
   In other word, the value of the liability doesn’t decrease much comparing to the SPDA with MVA.

(d) The return of premium feature would be in the money if the value of contract after applying the MVA factor is less than the initial deposit.
   Let initial deposit to be $P$
   Account value at the end of the year \(1=P \times (1.04)\)
   
   The MVA factor = \(\left(\frac{1+j}{1+i}\right)^{7-i}\)
   
   where \(j = \text{the current fixed crediting rate} = 4\%\)
   \(i = \text{the current market rate}\)
   \(T - t = \text{the fixed rate period remaining} = 4 \text{ yr}\)

   Solve \(i\) for Premium (Deposit) \(\geq\) Account Value \(*\) (MVA factor)
   \[P \geq P(1.04)\left(\frac{1.04}{t+i}\right)^4\]
Solution 14 (continued)

\[(1+i)^4 \geq 1.04^4\]
\[1 + i \geq 1.04^{\left(\frac{3}{4}\right)}\]
\[i \geq 1.04^{\left(\frac{3}{4}\right)} - 1 \approx 0.0502\]
If the current market rate is higher than 5.02% at the end of year 1, the option is in-the-money.

(e) Since the policyholders could have a higher first year cash surrender value, the policyholders are more likely to surrender in the year 1 and Return of Premium decreases the effective duration. ROP works like a put option.

(f) Since there is no MVA when the current market rate \((j)\) is lower than fixed rate \((i)\), the minimum guarantee has no effect during the initial guarantee period. After the 5 year initial guarantee period, the minimum guarantee should extend the effective duration when the current rate is below the minimum crediting rate. The policyholders would keep their contract. It acts similar to the interest floor.
Solution 15

(a)  
Risk free rate = 0.05  
V0=market value of company’s assets today = 21  
D=Company’s debt interest and principal due to be  
σ=volatility of assets = 0.20  
T=1 year  

\[ d_2 = \frac{\ln \left( \frac{V_0}{D} \right) + \left( r - \sigma^2/2 \right) T}{\sigma \sqrt{T}} \]  
\[ d_2 = \frac{\ln (21/17) + (.05 - (0.2^2)/2) \times 1}{0.2 \times 1} \]  
\[ d_2 = 1.2065 \]  

Calculate N(-d2) to obtain risk-neutral probability of default  
N(-d2) = 1 - N(d2)  

\[ N(d_2) = N(1.20) + .65 \times \left[ N(1.21) - N(1.20) \right] \]  
\[ = 0.8849 + 0.65 \times (0.8869 - 0.8849) \]  
\[ = 0.1138 \]  

(b)  
A=De^(-rt)  
A=17e^(-0.05 \times 1)  
A=16.1709  

B=V0-E0  
B=21-5=16  

Expected loss on the debt = (16.1709-16)/16 1709=1.06%  

0.0106=1 1138*(1-R)  
(1-r)=0.0106/0.1138  
R=1-0.0106/0.1138=(0.1138-0.0106)/0.1138  

R=90.7%
Solution 15 (continued)

(c) \[ Q(T) = \frac{1 - \exp[-(y(T) - y^*(t))T]}{(1 - R)} \]
\[ Q(5) = \frac{1 - \exp[-0.035*5]}{(1 - 0.4)} \]
\[ Q(5) = 0.26757 \]
\[ Q(1) = 0.26757/5 = 5.35\% \]

Lower than Merton’s default probability

Reasons for discrepancy:
- Provision for liquidity premium
- Provision for possibility of recession or depression scenario
- Merton’s model impacted by volatility
Solution 16

(a)

\[ p = K \cdot \exp[rt] \cdot N(-d_2) - S_0 \cdot N(d_1) \]

\[ d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma \sqrt{T}} \]

\[ = 1100e^{-0.025} \cdot 0.37336 - 1200 \cdot 0.28331 \]

\[ = 400.56 - 339.972 \]

\[ = 60.58 \]

\[ 60.58 \times 5000 = 302.919 \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

\[ = 0.5730 \]

\[ d_2 = 0.3230 \]

\[ N(d_1) = 0.71669 \]

\[ N(d_2) = 0.62664 \]

\[ N(-d_1) = 0.28331 \]

\[ N(-d_2) = 0.37336 \]

\[ \text{profit} = 302919 - 242900 \]

\[ = 60019 \]

(b) Delta is \( N(d_1) - 1 \) for a put option

\[ d_1 = \frac{\ln(1200/1100) + 0.02S + .22^2/2}{0.22} = 0.6191 \]

\[ N(d_1) = 0.7321 \]

\[ \text{delta} = 0.7321 - 1 = -0.2679 \]

\[ \times 5000 = -1339 \]

\[ \text{need to buy} \ 1339 \text{ notional amt of index} = 1339 \times 1200 = 1,607,382 \]
Solution 16 (continued)

(c) At each quarter \( \Rightarrow \) they need to recompute \( \Delta \) of option, then sell or purchase shares of index to make \( \Delta \) of portfolio = 0.

<table>
<thead>
<tr>
<th>Time</th>
<th>Index</th>
<th>( N(d_1) )</th>
<th>( \Delta ) (option) = ( \Delta ) ((1-N(d_1)) )</th>
<th>Shares purchased</th>
<th>Cost (# shares \times index level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1200</td>
<td>0.7321</td>
<td>0.2679</td>
<td>-1339.5</td>
<td>-1.607,400 negative</td>
</tr>
<tr>
<td>1</td>
<td>1250</td>
<td>0.8051</td>
<td>0.1949</td>
<td>365</td>
<td>456,250 implies profit</td>
</tr>
<tr>
<td>2</td>
<td>1150</td>
<td>0.6700</td>
<td>0.3300</td>
<td>-675.5</td>
<td>-776,800 profit</td>
</tr>
<tr>
<td>3</td>
<td>1050</td>
<td>0.3783</td>
<td>0.6217</td>
<td>-1458.5</td>
<td>-1,531,400</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>1.000</td>
<td>1.000</td>
<td>-1891.5</td>
<td>-1,891,500</td>
</tr>
</tbody>
</table>

in the money total = 5000

at time = 1, \( \Delta = 0.1949 \Rightarrow \) so you want \((5000)(-0.1949) = -974.5 \) shares,

so you need to buy \((-974.5) - (-1339.5) = 365 \) shares

time = 2 \( \Rightarrow \) sell \((5000)(0.33)-974.5 = 675.5 \) shares

time = 3 \( \Rightarrow \) sell \((5000)(0.6217) - (5000)(0.33) = 1458.5 \) shares

time = 4 \( \Rightarrow \) sell \((5000)(1.000) - (5000)(0.6217) = 1891.5 \) shares

(negative = profit)

<table>
<thead>
<tr>
<th>Time</th>
<th>Stock Purchase Cost</th>
<th>Trading Cost</th>
<th>Cum Cost</th>
<th>Int Cost = ( \frac{0.025}{4} )(Cum Cost)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1,607,400</td>
<td>1607</td>
<td>-1,605,800</td>
<td>-10036</td>
</tr>
<tr>
<td>1</td>
<td>456,250</td>
<td>456</td>
<td>-1,159,130</td>
<td>-7245</td>
</tr>
<tr>
<td>2</td>
<td>-776,800</td>
<td>777</td>
<td>-1,942,398</td>
<td>-12140</td>
</tr>
<tr>
<td>3</td>
<td>-1,531,400</td>
<td>1531</td>
<td>-3,484,407</td>
<td>-21777</td>
</tr>
<tr>
<td>4</td>
<td>-1,891,500</td>
<td>1892</td>
<td>-5,395,942</td>
<td></td>
</tr>
</tbody>
</table>

so firm has cumulative cost of \(-5,395,792 \Rightarrow \) profit = $5,395,792

Now, to settle put, you’re obligated to purchase 5000 shares at price of 1100 \( \Rightarrow \) so, you need 5000 (1100) = 5,500,000 (You’ve sold and then purchased 5000 shares so net gain in shares)

Therefore, net loss = 5395,792 - 5,500,000 = 104,200
Solution 16 (continued)

(d) Securitization package CF’s expected and sell at market to offset risky CF’s

Market maker - sell products that counterbalance risks of this product

Reinsurance - hard to find good price and willing counterparty

Naked position - do nothing. Okay if option is out-of-the-money, but in trouble if it is in-the-money

Covered position - A hedge initially, but do nothing afterwards - risky if \( \Delta \) changes dramatically

Stoploss - only change holdings if option is in or out of the money

Gamma hedging - make portfolio gamma neutral \( \Rightarrow \) requires position in another instrument

Rho, Vega hedging - similar to gamma hedging
Solution 17

(a) Risks in Global investing:
legal protection for investor
Corporate objectives of management
Communication with shareholders
Political risk
Currency risk
Credit risk

Solutions:
understand local market conditions
understand local legal framework
Understand reliability of communications with corporations
Use experienced staff
Do research
Currency hedging strategies
Credit hedging strategies

(b) Political Risk model: 10 variables, correlations show:
1. Democracy: if lack democracy & legitimacy, then less stable
2. Quality of life: higher means more stable
3. GDP: higher means more stable
4. Rental Income: higher means less stable
5. Distribution of income: if inequality, then less stable
6. Predictability of wholesale prices: higher means more stable
7. Agriculture (as a % of GDP): higher means less stable
8. Trauma: countries had trauma can be successful
9. Competition (measure=\(\frac{\text{Import} + \text{Export}}{\text{GDP}}\)): higher means more stable
10. Human Capital: higher means more stable
Solution 17 (continued)

(b)

<table>
<thead>
<tr>
<th>Country</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture (as % GDP)</td>
<td>(\frac{1 + 3}{26} = 15.4%)</td>
<td>(\frac{150 + 450}{1500} = 40%)</td>
<td>(\frac{15 + 40}{600} = 9%)</td>
</tr>
<tr>
<td>Rank:</td>
<td>middle</td>
<td>worst</td>
<td>Best</td>
</tr>
<tr>
<td>Competition (= \frac{\text{Import + Export}}{\text{GDP}})</td>
<td>(\frac{1 + 1 + 8}{26} = 38.4%)</td>
<td>(\frac{150 + 150 + 150}{1500} = 30%)</td>
<td>(\frac{5 + 15 + 300}{600} = 53%)</td>
</tr>
<tr>
<td>Rank:</td>
<td>middle</td>
<td>worst</td>
<td>Best</td>
</tr>
<tr>
<td>Democracy</td>
<td>Rank:</td>
<td>Stable</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>Stable</td>
<td>Stable</td>
<td>Worst</td>
</tr>
<tr>
<td>Infant deaths</td>
<td>Rank:</td>
<td>Best</td>
<td>middle</td>
</tr>
<tr>
<td>Rental:Oil/GDP</td>
<td>Rank</td>
<td>3.8%</td>
<td>middle</td>
</tr>
<tr>
<td>Life Expectancy</td>
<td>Rank:</td>
<td>Best</td>
<td>middle</td>
</tr>
<tr>
<td>Inflation range</td>
<td>Rank:</td>
<td>middle</td>
<td>worst</td>
</tr>
<tr>
<td>D=GDP/Capita</td>
<td>Rank:</td>
<td>(\frac{26}{3} = 8.7)</td>
<td>(\frac{14}{4} = 3.8)</td>
</tr>
<tr>
<td></td>
<td>middle</td>
<td>worst</td>
<td>Best</td>
</tr>
</tbody>
</table>

Country C is the most stable
A is the next most stable
B is the least stable
Solution 18

(a) Reduces volatility of CFs
Reduces cost of financial distress
   Legal & Accounting costs
   Higher costs with customers, employers and suppliers
Reduces Taxes
   Taxes reduced if tax schedule convex
Improve Investment Decision
   Improve incentives to undertake only profitable projects
Improve Debt Capacity
   Reduces conflicts with stockholders & bond holders
Dividend Policy
   Important means to express confidence in company’s growth
Managerial Self Interest
   Management has incentive to mange strategic exposure otherwise will not
   have job
Imperfect Market Conditions
   External capital more expensive than internal capital

(b) Purchase 1000 call options with strike=400 at $t = \frac{1}{2}$
Purchase 1000 call options with strike=400 at $t=1$
Enter into swap to receive LIBOR and pay 3.25%
   Assume company is payor of LIBOR
Enter a forward contract to hedge forward currency rate
\[ = .75 \left( \frac{1.05}{1.03} \right) = .764 \] This guarantees US payoff of $\frac{7M}{.764} = 9.156M$

(c) Value of Risk = Cost to hedge
Call options = 1000(4)+1000(6) = 10,000
LIBOR swap = no cost to hedge however.
   In one year earn $10M \times 3\%$ risk free
   pay $10M \times 3.25\%$ through swap
   cost = (3.25\%-3.0\%) \times 10M / 1.03 = 24,272
Foreign currency exchange forward
   Receive 9.16M in 1 yr (hedged at no cost)
   Expect $\frac{7M}{.75} = 9.33M$ in 1 yr
   Risk = $177,777 / 1.03 = 172,600$

Total Risk = 10,000+24,272+172,600 = 206,872
Solution 19

(a) From Ito’s lemma, if \( G \) is a function of \( S \) and \( t \), the process is
\[
dG = \left( (\partial G / \partial S) \mu S + \partial G / \partial t + 0.5 (\partial^2 G / \partial S^2) dt + (\partial G / \partial S) \sigma S dz \right)
\]

Since \( G = S \exp (r (T-t)) \),
\[
\partial G / \partial S = \exp (r (T-t)) \\
\partial G / \partial t = r S \exp (r (T-t)) \\
\partial^2 G / \partial S^2 = 0
\]

This gives
\[
dG = \left( \exp (r (T-t)) \mu S - r S \exp (r (T-t)) + 0 \right) dt + \exp (r (T-t)) \sigma S dz
\]
or
\[
dG = (\mu G - r G) dt + \sigma G dz = (\mu - r) G dt + \sigma G dz
\]

(b) If \( G \) is a stock with an instantaneous dividend payout, \( \mu \) would be the risk free rate, \( r \) is the instantaneous dividend yield, and \( \sigma \) the volatility of the stock. The stock is growing at the rate of \( \mu - r \) at the risk-neutral world.

(c) \( dG = (r - r_f) G dt + \sigma G dz \)

where \( r \) is the domestic risk-free rate and \( r_f \) is the foreign risk-free rate.

(d) Each unit of call option on 1 Japanese Yen is worth
\[
C = G_0 \text{ Exp} \left( -r_f (T-t) \right) N(d_1) - K \text{ Exp} \left( -r_f (T-t) \right) N(d_2)
\]
\[
d_1 = \left[ \ln (G_0/K) + (r - r_f + \sigma^2/2) (T-t) \right] / \left( \sigma (T-t)^{0.5} \right)
\]
\[
d_2 = d_1 - \sigma (T-t)^{0.5}
\]

Where \( r \) is the domestic risk-free interest rate,
\( r_f \) is the foreign (Japanese) risk-free rate,
\( \sigma \) is the exchange rate’s annualized volatility,
\( T-t \) is the time to maturity of the option
\( S_0 \) is the current price of Japanese Yen,
and \( K \) is the strike price on the Japanese Yen of the option
\( G_0 = K = 1/110 \) US$

\begin{align*}
\text{r} &= 0.01 \\
r_f &= 0.005 \\
T-t &= 0.25
\end{align*}
Solution 19 (continued)

Annualized volatility \( \sigma = (\# \text{ of trading days in a year})^{0.5} \times \text{daily volatility} \)
\[ = (4 \times 65)^{0.5} \times 0.62\% = 10\% \]
\[ d_1 = \left[ \ln(1) + (0.01 - 0.0005 + 0.1^2/2) \times 0.25 \right] / (0.1 \times 0.25^{0.5}) = 0.0725 \]
\[ d_2 = 0.0725 - 0.1 \times 0.25^{0.5} = 0.0225 \]
\[ C = \frac{1}{110} \times \text{Exp}(-0.0005 \times 0.25) \times N(0.0725) - \frac{1}{110} \times \text{Exp}(-0.01 \times 0.25) \times N(0.0225) \]
\[ = \left( \frac{1}{110} \right) \times 0.999875 \times 0.528898 - \left( \frac{1}{110} \right) \times 0.997503 \times 0.5090 \]
\[ = 0.004808 - 0.004616 = 0.000192 \text{ US$} \]

Value of the contract on 220 billion Yen
\[ = 0.000192 \times 220 \text{ billion US$} \]
\[ = 42,240,000 \text{ US$ or about } 42 \text{ million US$} \]
Solution 20

The guarantee at maturity: \( L_T^* = L_0 \cdot e^{\gamma} \)
\( = 66,673,640 \times e^{0.03 \times 10} \)
\( = 90,000,000 \)

\[ \alpha = \frac{L_0}{A_0} = \frac{66,673,640}{74,081,822} = 0.9 \]

\[ \therefore (1 - \alpha) A_0 = C_E(A_0, L_T^*) - \delta \alpha C_E \left( A_0, \frac{L_T^*}{\alpha} \right) \]

\( \delta \) is the equilibrium participation level

\[ \frac{L_T^*}{\alpha} = \frac{90,000,000}{0.9} = 100,000,000 \]

From the given table we have,
\[ C_E(A_0, L_T^*) = C_E(A_0, 90,000,000) = 7,625,000 \]

\[ C_E \left( A_0, \frac{L_T^*}{\alpha} \right) = C_E \left( A_0, 100,000,000 \right) = 1,204,330 \]

\[ \therefore \delta = \frac{C_E \left( A_0, L_T^* \right) - (1 - \alpha) A_0}{\alpha C_E \left( A_0, \frac{L_T^*}{\alpha} \right)} = \frac{7,625,000 - (1 - 0.9) \times 74,081,822}{0.9 \times 1,204,330} \]
\( = 0.2 \)

hence the equilibrium participation level is 0.2