On regular set systems containing regular subsystems

Let $X, Y$ be finite sets, $r, s, h, \lambda \in \mathbb{N}$ with $s \geq r$, $X \subset Y$. By $\lambda(X^h)$ we mean the collection of all $h$-subsets of $X$ where each subset occurs $\lambda$ times. A coloring of $\lambda(X^h)$ is $r$-regular if in every color class each element of $X$ occurs $r$ times. A one-regular color class is a perfect matching. We are interested in the necessary and sufficient conditions under which an $r$-regular coloring of $\lambda(X^h)$ can be embedded into an $s$-regular coloring of $\lambda(Y^h)$ and nearly settle the case $h = 4$. 