Graph Edge Coloring

Graph edge coloring is a well established subject in the field of graph theory, it is one of the basic combinatorial optimization problems: color the edges of a graph $G$ with as few colors as possible such that each edge receives a color and adjacent edges, that is, different edges incident to a common vertex, receive different colors. The minimum number of colors needed for such a coloring of $G$ is called the chromatic index of $G$, written $\chi(G)$. By a result of Holyer, the determination of the chromatic index is an NP-hard optimization problem. The NP-hardness gives rise to the necessity of using heuristic algorithms. In particular, we are interested in upper bounds for the chromatic index that can be efficiently realized by a coloring algorithm. In this talk, we will start with the well-known Goldberg-Seymour conjecture and its proof, then talk about the recent development of recoloring techniques and its applications to a number of classic problems in critical class 2 simple graphs.