

Discrete Mathematics Seminar

Illinois State University

2:00–2:50 pm, January 23@ STV 120

Speaker: Songling Shan, Illinois State University

Proof of the Core Conjecture of Hilton and Zhao

Let G be a simple graph with maximum degree Δ . We call G *overfull* if $|E(G)| > \Delta \lfloor |V(G)|/2 \rfloor$. The *core* of G , denoted G_Δ , is the subgraph of G induced by its vertices of degree Δ . A classic result of Vizing shows that $\chi'(G)$, the chromatic index of G , is either Δ or $\Delta + 1$. It is NP-complete to determine the chromatic index for a general graph. However, if G is overfull then $\chi'(G) = \Delta + 1$. Hilton and Zhao in 1996 conjectured that if G is a simple connected graph with $\Delta \geq 3$ and $\Delta(G_\Delta) \leq 2$, then $\chi'(G) = \Delta + 1$ if and only if G is overfull or $G = P^*$, where P^* is obtained from the Petersen graph by deleting a vertex. This conjecture, if true, implies an easy approach for calculating $\chi'(G)$ for graphs G satisfying the conditions. The progress on the conjecture has been slow: it was only confirmed for $\Delta = 3, 4$, respectively, in 2003 and 2017. We confirm this conjecture for all $\Delta \geq 4$.

This is joint work with Yan Cao, Guantao Chen, and Guangming Jing.

