Discrete Mathematics Seminar

Illinois State University

2:00–2:50 pm, February 20@ STV 120

Speaker: John Goldwasser, West Virginia University

Polychromatic Colorings of the Integers

We show that if S is any set of 4 integers then there is a 3-coloring of $\mathbb{Z}$ such that every additive translate of S gets all 3 colors. This proves a conjecture of Newman that the codensity of any set of 4 integers in $\mathbb{Z}$ is at most $1/3$ (the codensity of a finite set S of integers is the minimum density of a set T in $\mathbb{Z}$ such that $S + T = \mathbb{Z}$).

Loosely speaking, if L is any large structure consisting of some elements, and $\mathscr{F}$ is a family of substructures of L, we say a k-coloring of the elements of L is $\mathscr{F}$-polychromatic if every substructure in $\mathscr{F}$ gets all k colors. The polychromatic number of $\mathscr{F}$ in L is the largest k such that there exists an $\mathscr{F}$-polychromatic k-coloring. We will talk briefly about polychromatic colorings in other settings besides the integers and suggest some questions for further thought.