Rainbow solutions to the Sidon equation in cyclic groups and in the interval

Given a coloring of group elements, a rainbow solution to an equation is a solution whose every element is assigned a different color. The rainbow number of \( \mathbb{Z}_n \) for an equation \( eq \), denoted \( rb(\mathbb{Z}_n, eq) \), is the smallest number of colors \( r \) such that every exact \( r \)-coloring of \( \mathbb{Z}_n \) admits a rainbow solution to the equation \( eq \). In this talk we show that for every exact 4-coloring of \( \mathbb{Z}_p \), where \( p \geq 3 \) is prime, there exists a rainbow solution to the Sidon equation \( x_1 + x_2 = x_3 + x_4 \). Furthermore, we determine the rainbow number of \( \mathbb{Z}_n \) for the Sidon equation. We also discuss the rainbow number of the set of integers \( [n] = \{1, \ldots, n\} \) for an equation \( eq \), denoted \( rb([n], eq) \), and determine the rainbow number of \( [n] \) for the Sidon equation. This is joint work with Jürgen Kritschgau.