Discrete Mathematics Seminar

Illinois State University

2:00–3:30 pm, April 22

Antimagic Orientation of Subdivided Caterpillars

Jessica Ferraro and Geneieve Newkirk, Illinois State University

Let $m \geq 1$ be an integer and D be a digraph with m arcs. We say that D has an antimagic labeling if there is a bijection $\tau : A(D) \to \{1, \ldots, m\}$ such that no two vertices in D have the same vertex-sum under τ, where the vertex-sum of a vertex v in D under τ is the sum of labels of all arcs entering v minus the sum of labels of all arcs leaving v. We say a graph, G, has an antimagic orientation if G has an orientation that has an anitmagic labelling. Hefetz, Mtze and Schwartz [J. Graph Theory, 64: 219-232, 2010] conjectured that every connected graph admits an antimagic orientation. The conjecture was confirmed for certain classes of graphs such as dense graphs, regular graphs, and trees including caterpillars and complete k-ary trees. We prove that every subdivided caterpillar admits an antimagic orientation, where a subdivided caterpillar is obtained from a caterpillar by subdividing each of its legs the same number of times.

Hamiltonicity of 4-tough $(K_2 \cup 3K_1)$-free graphs

Elizabeth Grimm and Andrew Hatfield, Illinois State University

Chvátal conjectured in 1973 the existence of some constant t such that all t-tough graphs on at least 3 vertices are hamiltonian. While the conjecture has been proven for some special classes of graphs, it remains open in general. We say that a graph is $(K_2 \cup 3K_1)$-free if it contains no induced subgraph isomorphic to $K_2 \cup 3K_1$, where $K_2 \cup 3K_1$ is the disjoint union of an edge and three isolated vertices. We show that every 4-tough $(K_2 \cup 3K_1)$-free graph on at least three vertices is hamiltonian.

Hamiltonian Cycles in Tough $(2K_2 \cup K_1)$-Free Graphs

Jeremy Corry, Jesse Hayes-Carver, Nick Klecki, Illinois State University

Let $G = (V, E)$ be a graph. We say G is t-tough if $|S| \geq t \cdot c(G - S)$ for each cutset $S \subseteq V(G)$. The concept of toughness, a measure of graph connectivity and "resilience" under the removal of vertices, was introduced by V. Chvátal in 1973. Chvátal later conjectured that there exists a constant t_0 such that every t_0-tough graph is Hamiltonian. The conjecture was confirmed for some special classes of graphs but remains wide open in general. In this talk, we will present our findings regarding Hamiltonian cycles in tough $(2K_2 \cup K_1)$-free graphs.