

Discrete Mathematics Seminar

Illinois State University

2:00–2:50 pm, March 11

Speaker: Hao Huang, Emory University

On local Turán problems

Since its formulation, Turán's hypergraph problems have been among the most challenging open problems in extremal combinatorics. One of them is the following: given a 3-uniform hypergraph \mathcal{F} on n vertices in which any five vertices span at least one edge, prove that $|\mathcal{F}| \geq (1/4 - o(1))\binom{n}{3}$. The construction showing that this bound would be best possible is simply $\binom{X}{3} \cup \binom{Y}{3}$ where X and Y evenly partition the vertex set. This construction satisfies the following more general $(2p+1, p+1)$ -property: any set of $2p+1$ vertices spans a complete sub-hypergraph on $p+1$ vertices.

In this talk, we will show that, quite surprisingly, for all $p > 2$ the $(2p+1, p+1)$ -property implies the conjectured lower bound. Furthermore, we will prove that for integers $r, a \geq 2$, the minimum edge density of an r -uniform hypergraph satisfying the $(ap+1, p+1)$ -property tends to $1/a^{r-1}$ when p tends to infinity.

Joint work with Peter Frankl and Vojtěch Rödl.

